PLACE OF THE CONVERSION OF SEMIOTIC REPRESENTATIONS IN THE DIDACTIC FRAMEWORK R^2C^2

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This article is about the place that we have allocated to the conversion of semiotic representations at the heart of the didactic framework R^2C^2. Ten 3rd year primary school classes were implicated in this study: eight in France and two in Czech Republic. This framework was tested in four of the eight French classes. After having explained the theoretical framework, we describe the methodology, based essentially on the analysis of professional practice of teachers, using video recordings of sessions as well as interviews. Then we try to highlight to what extent the principle of conversion of representations can encourage the conceptualization process among pupils.

Keywords: Semiotic register, conceptualization, conversion of representation, elementary school

INTRODUCTION

Whereas in more than 3 classes out of 4, the teachers declare that at least once a week they present their pupils to numerical problems solving (Priolet, 2000), the pupils' competences in this field remain very variable, as this was reminded at the national conference on mathematics teaching at secondary and primary school.

Those pupils' recurrent difficulties in learning numerical problems solving led us to focus our research works on the observation and analysis of the teaching practices in primary school classes and on the implementation of a didactic framework.

THEORETICAL FRAMEWORK

As soon as we started our research works on numerical problems solving at primary school (Priolet, 2000), we focused on the interest attached by the teachers to the conversion of semiotic representations.

We wonder to what extent the pupils are effectively exposed to this conversion, considered as fundamental by Duval (1995, 2001) in the comprehension process which enables to access mathematical objects.

Indeed, mathematical objects, numbers for example, are not directly accessible by perception or observable with instruments (Duval, 2001); their accessibility is possible by semiotic representations belonging to a system with its own meaning and functioning constraints.

These semiotic representations can be discursive (in the native language, in formal language) or non-discursive (figures, graphs, charts…). Duval classifies these representations in registers and, according to him, the originality of the mathematical activity lies in the mobilisation of at least two registers of representation and in the possibility of changing register at any time.

He defines conversion as the transformation that consists in changing register while keeping the same objects and he defines treatment as the transformation internal to a register. He considers as fundamental for the construction of a concept the conversion tasks between several registers of semiotic representation of a same mathematical object.

Each time a change of register proves to be necessary or that two registers must be mobilised simultaneously, we face a rise in the number of pupils' failures or blocks, at all teaching levels.

Novotná (2003) observes that if the graphic language is not presented previously by the teacher, it is rarely used by the pupils. She shows, in particular, the help that a chart can bring to problem-solving (Novotná, 2001). However, she warns of the risk when imposing such support, given the fact that individual differences exist in the manner of treating semiotic representations.

In mathematical didactics, apart from Novotná's work, our theoretical framework is also based on Glaeser's work (1973) which insists on the necessity of effectively putting pupils in situations to search solutions to a given problem as well as the research developed by Brousseau since the seventies and his theory of didactic situations (Brousseau, 1990), in particular for managing the teaching and learning experiences by the teacher and for the concept of devolving some decision making to students. Our approach also encapsulates the cognitive psychology contributions as well as Vergnaud's (1990) on the importance of the categorisation of problem-situations according to the mathematical relations at stake.

R^2C^2 DIDACTIC FRAMEWORK AND RESEARCH HYPOTHESIS

Our research, which considers the interface of learning and teaching the resolution of mathematical problem-solving in primary school leads us to venture the following hypothesis.

Numerical data problem-solving learning can be enhanced if the pupil is exposed to four principles: P1: Looking for solutions to problems; P2: networking of knowledge; P3: conversion of semiotic representations; P4: categorisation of problem-situations.

Linked to an analysis of practice, putting together these four principles and their devolution to the pupil contribute to develop a reflective attitude from teachers on their problem-solving teaching and contribute to their professionalization.

Linked to the various theoretical works mentioned, and in order to test our research hypothesis we have conceived and tested a didactic framework named under the acronym R^2C^2 (Research, networking, Conversion, Categorization). This framework is based on the implementation of the four principles (P1, P2, P3, P4) and this, under the conditions of their coexistence, of their regularity and of their devolution to the pupil.

This article aims to analyse and describe more specifically the implementation of principle 3.

EXPERIMENTATION

Ten third year primary school classes (eight in France and two in the Czech Republic) were involved in this study. The experiment followed 3 main phases:

In a first stage, at the start of the school year, we observed and analysed problem solving sessions laid out by ten teachers. The observations of these sessions, known as initial sessions, were filmed (type n°1 recording). The pupils have been submitted to a pre-test made up of 12 numerical problems solving.

In a second stage, from January to March, four classes in France selected randomly amongst the 4 were exposed to our didactic framework R^2C^2. Problem-solving sessions carried out in these four classes forming the experimental group were recording again (recording n°2).

The four other classes which formed the control group in France continued the work planned by the teacher in the framework of their usual problem solving teaching. At the end of the academic year, the pupils have then been submitted to a post-test which content was identical to the pre-test.

Simultaneously to this experimentation in French classes and further to the reading of an article written by Novotnà (2001), we extended our observations to two Czech classes. The pupils in both classes were submitted to the same test (translated into Czech language) as the one presented to the French eight classes' pupils.

Artefacts introduced in the framework of the operationalization of the R^2C^2 framework

The implementation of R^2C^2 framework includes the introduction and the use of artefacts named as « reference-boxes » and of « reference-dictionaries » which lead teachers to submit their pupils to the implementation of the 4 principles P1, P2, P3 and P4 in the conditions mentioned earlier on.

Each pupil in each experimental class has a « reference-box ». They contain forms, envelops, cassettes, exercise-books, these «reference-boxes» are aimed to receive and enable, without imposing it, the linking of verbal statements and various representations : operation, drawing, graph, text… in order to foster the conversion of representation of a register in a new register (Duval, 1995). The basis of solved problems which will grow little by little will link to networking and categorisation activities thus giving each « reference-box » the status of reference for each class of problems (Vergnaud, 1990).

Another artefact, named « reference-dictionary » was used in organising teaching and learning, as conceived by Brousseau (1990). A dictionary was developed collectively in each of the four classes of the experimental group as verbal difficulties arose.

DISCUSSION

Description and analysis of usual problem-solving teaching in the 8 classes in France. The place of the conversion of representations.

The observation of Type 1 sessions, as well as the self-reflective interviews, show that pupils were mainly exposed to the use of textual and numeric register, registers between which conversions occur when problem-solving. The iconic register was essentially mobilised during correction stages, under the teacher's control, it was introduced by the teachers themselves when collecting pupils' work, or by a pupil showing at the board the iconic representations traced in their book during the resolution of the problem. In conclusion, and in the limits of our experiment, it turns out that teachers endeavour most of the time but not exclusively to put their pupils in a position to look for solutions to problems.

On the other hand, the linking of previous knowledge as well as the conversion of representations were mostly placed under the guidance of teachers. However, during the sessions, we have noticed that this networking and this conversion did not coexist in the same teacher. No categorisation activities were noticed during our observations.

After the analysis of the usual problem-solving teaching practices of these 8 teachers we focus on the implementation of the P3 principle in the didactic framework R^2C^2 in the four classes of the experimental group.

Interpretation of the implementation of the principle of the Conversion of Semiotic Representations (principle P3 of the didactic framework R^2C^2)

In the type 2, due to the implementation of the didactic framework R^2C^2, we notice that it is the teacher that defines the rule and that it is the pupil who realises himself the representations during his searching stage.

Through the use of « reference-boxes » pupils are invited to use the iconic register, on one hand to elaborate a drawn representation of the situation described in the problem title and on another hand, to schematise and model the situation. Before coming back to more general considerations on the implementation of this conversion principle (principle P3), we will deal first with the recourse to drawings and then the recourse to the graph drawing.

Resorting to a drawing:

Pupils are in presence of a textual utterance which presents a situation to solve. In referring to the work from the learning psychology, we consider that this situation, even in the case where it is about a daily life situation, must be reconstructed by the reader in order to elaborate a mental representation (Johnson-Laird, 1983). Resorting to the drawing seems to force the pupil to do a step to step reading of the textual utterance, or several readings which could suggest a better identification of the data. We concur with Novotná (2003) who develops the view point according to which the construction of the mental representation of the situation could thus be facilitated by resorting to a drawing which would allow relieve the work memory burden.

Resorting to drawn graphs:

If one is interested in the representation of diagram types in the Vergnaud sense, one can consider (Novotná, 2003), that resorting to a drawn graph is going to facilitate the heuristic process thanks to the manipulation, in written form, of mathematical relationships.

In our didactic framework, the material form of the « reference box » with a blank box planned for a diagram invites the pupil to produce it. We know indeed that without a previous presentation by the teacher, the graphic language is rarely used by pupil (Novotná, 2003). Here, in the R^2C^2 framework, the use of the artefact "referent box" makes the appeal to the recourse to this kind of conversion explicit, nevertheless, we drew the teachers' attention on the possibility to run the risk, as Novotná points it out, to compel the pupils to fill in all the squares of the "referent box".

After having considered more particularly the traces belonging to the iconic register we consider the implementation conditions of the P3 principle.

Problem 3 (translation) :

Last Friday, the president of the Amicale Laïque association came to give us sweets to congratulate us of the good results we had at the Foulées Vertes. On the packet was written 100 sweets. That day, there were 3 absentees in our class.

In order to share equitably the sweets between all the children present on that day, how many sweets did we have to give to each pupil?

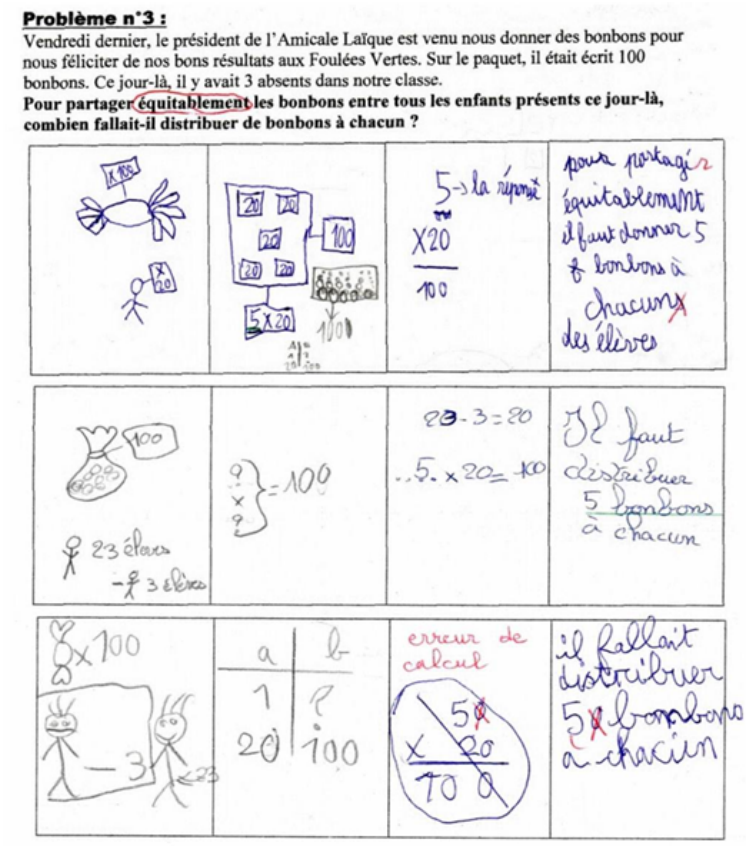

Figure 1: Resorting to the conversion of representations : productions of 3 pupils of a same class of the experimental group.

We have chosen to illustrate the variety of the recourse to conversions of representations (calculation, drawing, diagram, text), through the work of three pupils, selected randomly, among those which all squares of the referent box have been completed (figure 1). This selection must not hide the fact that the instructions in relation to the use of the artifact "referent box" quite specify that it is not imperative to fill in all the squares and then the recourse to the set of the mentioned registers is not needed.

The regularity imposed in the use of « reference boxes » aims also to induce regularity in linking several registers. This regular implementation condition, mixing a variety of treatment tasks and register conversions is mentioned by Pluvinage (1998) from the moment we are interested in the place of these operations in problem-solving learning.

Duval (1995) points out the fundamental role played by conversion tasks in between several registers of semiotic representations of a same mathematical object for the construction of a concept.

The film recordings n°2 show a more personal involvement from pupils, some involving themselves more in the conversion of representations, others not resorting to it because they proceed directly to problem-solving using an expert process;

whereas others still feel authorised to use drawings or graphs. This set of observations show the taking into account of the devolution to the pupil of the conversion of representations P3 principle.

It also seems to us that the implementation of this P3 principle can influence on the didactic contract (Brousseau, 1980) insofar as the pupil is now exposed to different representation possibilities recognised by the teacher. The analysis of Type 1 sessions revealed homogeneity in the initial practices of teachers, which demanded going through the normal presentation form « solution / operation ». This form centred on a unique approach of resolution based on resorting to calculating mode seems to us to encourage the use of the arithmetical operational technique during the learning stage (Priolet, 2008).

One can then consider that the fact of establishing, during Type 2 sessions, the use to different registers of representation modifies the didactic contract.

However, the analysis of the filmed documents, show that some pupils produce an iconic type representation after having given the solution to the problem. You can see here, again, the effect of the didactic contract. The pupil wants to respond to the request to use different registers and fill in all the available boxes. It seems essential to consider, in referring to Vergnaud (1997), that these drawings or graphs have a transitory status and that they are meant to be forgotten as gradually problems get mastered.

MAIN RESULTS

The quantitative analysis of the effects of the R^2C^2 didactic framework implemented in the four classes of the experimental group in France has been carried out by comparing the results of the pupils belonging to the control-group to those of the pupils belonging to the experimental-group during a pre-test and a post-test made up of a same set of twelve numerical problems solving. The series of problems was composed of six problems concerning simple proportionality, of one problem concerning multiplying comparison and of five problems requiring intermediate calculations. The score of each pupil was calculated by taking the number of attainments into account.

		Number of pupils	Total score		Average		Difference
			Pre-test	Post-test	Pre-test	Post-test	
Groups	Control	65	304	364	4,68	5,60	0,92
	Experimental	72	315	457	4,38	6,35	1,97

Figure 2 : Number of pupils, total score and average (pre-test and post-test), difference according to the groups.

The difference (1,05) between the averages obtained at the pre-test and the ones obtained at the post-test (1,97 for the experimental group and 0,92 for the control-group) is significant:

$t_{Student} = 2,94 > 2,61$; ddl = 135 ; $p < 0,01$).

This study showed convincing results: the performances noticed in the control-group increase by about one more problem solved, whereas the ones observed in the experimental-group increase by about two more problems solved.

The pupils in the two Czech classes, not submitted to the experimentation of the R^2C^2 didactical framework, have also been submitted to take this same test of twelve numerical problems.

For these two Czech classes, the results to this test are better compared to those of the French experimental-group (7,67 vs 6,35).

CONCLUSION

The main qualitative and quantitative results which emerge from this study come to reinforce the hypotheses we have stated.

The possibility of the recourse to referent-boxes encourages the pupil to the conversion (principle P3) between the literal, numerical and iconic registers, and thus favors the production of representations. We can consider that these productions, associated with a reading of the wording and the drawing of diagrams allowing to visualize the mathematical relationships at play facilitate the construction of a mental pattern (Johnson-Laird, 1993).

The introduction of the artefact (referent box) allows to consider a double networking: the networking of problems, according to underlying mathematical relationships (Vergnaud, 1997) which leads the pupils to an activity of categorization; the networking of diagrams which aims at the drafting of a referent-pattern with a view to an establishment of a model of these mathematical relationships. In that, we can consider that the principle P3, thanks to the categorization and the establishment of a model, constitutes an element allowing to generate the process of conceptualization among pupils.

It has to be reminded that the results of the experimental group in France are inferior to those of the two Czech Republic classes observed in the framework of ordinary practices and not submitted to the R^2C^2 didactic framework experiment. The pupils of these two Czech Republic classes considered as natural to research similar problems, to try to use the register of representation the best adapted to their problem-solving strategy, to break down a new problem in several simple sub problems for which they already knew the resolution algorithms (Priolet & Novotná, 2007). We notice similarities with the principles of the R^2C^2 didactic framework of which the implementation in the four classes of our experimental group in France can explain the significant improvement compared to the four classes of the control group. However, the pupils of the French experimental group have only been exposed for three months to the implementation of the four principles inherent to the R^2C^2 didactic framework whereas the practices observed in Czech Republic were implemented each year and regularly throughout the school year. We attribute the superiority of the performances of the two Czech Republic classes to the long-term systematic work undertaken since the start of compulsory schooling.

The example below is extracted from a reflective interview and shows that the implementation of the P3 principle (the conversion of representations) had effectively brought to the teachers' awareness the consequences of the use of the presentation « solution-operation » that they imposed to their pupils. Indeed, this normal presentation, limited to textual and numeric registers leaves little place for conversions between registers.

> Teacher 5: I gave up the rigorous presentation « solution – operations » because I became aware that they thought they had to do operations with the numbers and that it translated by an operation and not necessarily by the right answer.

In conclusion, by a systemic approach, we can consider that the implementation of the R2C2 didactical framework favored the process of conceptualization among pupils, however on approval that the principles be intended to the pupil.

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