AN INVESTIGATION INTO THE TENSION ARISING BETWEEN NATURAL LANGUAGE AND MATHEMATICAL LANGUAGE EXPERIENCED BY MECHANICAL ENGINEERING STUDENTS

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This investigation is grounded within the concept of embedded cognition where the mind is considered to be part of a biological system. A relationship between conceptual metaphors and modules is proposed to account for how students learn and use mathematics. A first year undergraduate Mechanical Engineering cohort of students was tasked with explaining the behaviour of three balls of different masses being rolled down a ramp. The explanations given by the students highlighted the cognitive conflict between the everyday use of the word energy and its scientific one. The results showed that even after many years of schooling, students were still unable trust the mathematics they had learned and relied upon pseudo-scientific notions to account for the behaviour of the balls.

Keywords: metaphor, modules, concept, communicate, reductionist**.**

INTRODUCTION

The purpose of this paper is to report upon a study carried out with first year undergraduate Mechanical Engineering students. They were presented with a physical system which was comprised of a ramp on which three different mass balls were rolled down. One half of the class was asked to predict where the balls would land relative to one another, the other half were given a composite photograph of the balls landing and asked to explain this behaviour. The students' analysis of the system was used to investigate the tension that arises from the everyday use of scientific terms and the precise mathematics definition. In this case the concept and interpretation of the word energy was used to facilitate this study.

BACKGROUND

Mathematics, as an intellectual discipline, can be divided into two sections: the language element and the computational aspect. This dichotomy is at the root of the difficulties experienced by many learners and indeed by mathematicians at all levels. The difficulties faced by learners can be eloquently summed up by (Davis & Jones, 2006):

> 'To put it in a simple form that highlights the students' dilemma: they need to know the language of mathematics in order to know what mathematics is about; conversely, they need to know what mathematics is about in order to know how to use the language'.
>
> (P 117)

Natural language is used as the medium to convey and learn mathematics. The development of natural language relies upon forming a link between the three dimensional 'concrete' world we live in and the conceptual world of our minds. In the early stages of development a child learns to form the link between an object and the name given to it by the society the child lives in. This is evident when a child first learns to count in that a number name (a numeral) is attached to an object. The child also learns that once all of the objects have been counted, the last number indicates the quantity of the objects (the cardinal number).

Lakoff & Núñez (2000) suggested that this process of learning can be explained by the use of metaphors and conceptual metaphors. A metaphor is defined to be the linguistic construction used to describe a subject in terms of another unrelated object and conceptual metaphors to be where one idea is understood in terms of another unrelated idea. Lakoff & Núñez would explain the scenario of the child counting objects as an 'arithmetic as object collection' conceptual metaphor. As the child progressed more conceptual metaphors would be developed in order to sensibly interpret the world. This process of developing conceptual metaphors is reinforced by the use of metaphorical language extensively used in the teaching of mathematics. It is quite common when teaching children about numbers to employ the use of a 'measuring stick' metaphor to explain the relationship between numbers. In a similar way, the use of an 'arithmetic as motion along a path' metaphor is employed to explain negative numbers. The use of conceptual metaphors has been shown to be problematic within the classroom (Doritou & Gray 2007).

This use of natural language and its reliance upon metaphors to communicate mathematical ideas can be problematic. Within the Philosophy of Language community there is an ongoing debate concerning the interpretation and meaning assignment to metaphors. Searle (1993) suggested that the metaphorical meaning of a predicate on a particular occasion depends upon the literal meaning of the predicate. Stearn (2000) argued that metaphors are context dependent in that they can convey different meanings in different situations and that the interpretation of the metaphor depends upon the recipient's environment. With regard to the learning of mathematics this external use of metaphors, that is, the metaphors used to teach, reinforce the development of conceptual metaphors. The learner at some stage has to realise that concrete objects are not necessarily the subject of mathematical operations. For example, the arithmetic as object collection metaphor can prove to be problematic when the learner is faced with set theory where the natural numbers are described in terms of the empty set, that is, $\emptyset$ maps to 0, $\{\emptyset\}$ maps to 1, $\left\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\right\}$ maps to 0, 1 and 2, since the empty set is the fundamental 'unit' and not a physical object. This highlights the problem that the learner can, in some instances, attaching a name to a concept at one stage in their mathematical development, but at a later stage the concept is modified but the familiar name is still used or a familiar concept is given a different name (eg. the equation $y = 3x + 2$ becomes the function $f(x) = 3x + 2$).

This learned ability to perform, for example, arithmetic builds upon an innate ability to 'subitise'. Butterworth (1999) and Dehaene (1997) have provided evidence for the existence of a region of the brain, the inferior parietal lobe, which is capable of processing numbers up to approximately four without the need for counting (numerosity, subitising). Techniques such as counting, ordering and grouping have, of necessity, been developed from this innate ability to allow humans to make sense of and control their world.

LINGUISTIC AND MATHEMATICAL COMMUNICATION

Two models of linguistic communication that stand in sharp contrast are the classical code model where sentences are comprised of sound-meaning pairs and the more recent inferential model where the sentence provides a semantic structure from which the meaning can be inferred (Sperber & Origgi, 2010). In order to communicate, according to the code model, the speaker and listener must interpret the sentences in precisely the same way. In other words, the speaker and listener have identical lexicons and both interpret the words and how they are arranged in the same way. In contrast to this the inferential model acknowledges that ambiguity can arise from the listener misinterpreting the speaker's message. In this model, understanding the speaker's meaning is an inferential process using a common grammar which assigns semantic properties to the sentence and uses context to aid interpretation. Anderson (1997) studied two working environments: a garage and a postal service. He discovered that in many cases the workers would use words and phrases that did not conform to the standard natural language, but instead used abbreviated sentences and sometimes just words to make a request or convey an instruction. Since the workers were in the same environment, ambiguities did not arise.

In the context of mathematics, the language used is often inferential. In one respect the novice mathematician develops a form of proto-language where a lexicon of coded concepts is built where these concepts, to a certain extent, are independent of one another. This developmental or learning process eventually leads to the acquisition of a language governed by a grammar or grammars associated with a comprehensive mathematical lexicon. The developed mathematical language is a much more concise language than a natural one and gains its analytical power from the ability to use a symbol to represent a complex concept. It is not until a later stage that links are formed between these concepts and then super concepts are created (Sfard, 1991). It could be said that the symbol reifies the concept in the sense that it can be manipulated and used to investigate both physical artefacts and abstract concepts. From the learners' perspective, the interpretation of these symbols can be a difficult and cognitively demanding process especially when the fundamental concepts rely upon a conceptual metaphor grounded in the 'real' world.

Nowak, Kamarova and Niyogi (2001), in their discussion of cognitive development in humans, suggested that the brain developed as a modular system. For example, the skill of reading is relatively new on an evolutionary time scale whereas coarse grained object recognition would have been required from early on in order to survive. This example provides an instance of modular adaptation where the object recognition module developed the capability to recognise abstract symbols. They also suggested that skills like reading and other more sophisticated object recognition processes that are constantly being used, develop their own modules based on ones in which similar processes already exist.

Fodor (1983) suggested that modules must fulfil certain properties to a lesser or greater degree: they must be domain specific, they use encapsulated information i.e. that do not have to use other psychological systems in order to operate, they are activated without conscious control (Fodor called them mandatory), their outputs are shallow, they are quick to process pertinent data, they have characteristic breakdown patterns and the neural architecture is fixed. Pinker (1998) suggested that a module should be defined by the specific operations they perform on the information they receive. Fodor himself recognised that his original definitions were too rigid since some processes, to a greater or lesser extent, may not adhere to the criteria he set for a process to be considered modular. Two of the key criteria suggested by him were domain specificity and encapsulation. Domain specificity is defined to be the class of information that the module is designed to accept or operate upon. Encapsulation refers to the notion that modules are not influenced by information or processes external to it. An analogy given by Barrett & Kurzan (2006) of encapsulation is that of a pipe. The inputs are fed into one end of the pipe and, since the information is protected by the structure of the pipe, the outcome cannot be influenced. Fodor's idea of encapsulation therefore prohibits the influence of other modules, such as memory or other higher cognitive functions, having an effect. This strict definition would prohibit the process of adaptation to fulfil newer processes. It must be possible, therefore, for the modules to interlink and form interdependencies. This would be more cognitively efficient than having to develop specific modules to fulfil a particular task.

If the ideas of conceptual metaphors and modules are combined, a relationship between them can be established. The 'activation' of a module relies upon domain specificity. The data activating the module at a fundamental level comes from the sensory system. This data in turn becomes information upon which an action can be taken. For example, in the classic 'fight or flight' scenario a decisive point is reached where a decision has to be made regarding the most appropriate action to take. A similar emotional response is elicited when someone watches, for example, a horror film. Although the viewer knows intellectually it is just a fiction, the human limbic system translates this to a pseudo-reality where some life preserving response is necessitated. The extension of this idea of abstract scenarios to pseudo-physical ones within mathematics seems perfectly feasible. For example, vectors can be used to represent objects moving in a particular direction. In order for a learner to make sense of vectors, some idea of what 'moving' means and what 'direction' means is crucial. This 'real world' experience is internalised as a conceptual metaphor thus enabling a cognitive link to be made in order to interpret and make sense of a mathematical expression or equation. Unfortunately, the over reliance upon conceptual metaphors, as illustrated earlier, presents problems when strict mathematic definitions are introduced. At a 'folk' definition level the concepts of speed and velocity are interchangeable and indeed in many mathematics problems the learner is able to rely upon this 'loose' definition. The challenge comes when circular motion has to be studied. In this instance the object moving along a circular path is subject to continuous directional changes and hence must be described by velocity not speed. This creates a conflict for the learner who now either has to adapt a deeply ingrained metaphor to include the notion of changing direction or else has to separate the two concepts and rely upon semantic processing to correctly resolve any problems. This conflict was evident in an investigation carried by Peters & Graham (2009) where a group of trainee teachers were asked to explain the forces acting on an object suspended without any support in mid-air. This investigation demonstrated the cognitive conflict experienced by some teachers when attempting to use Newton's third law of motion to explain the problem. They could not identify the relationship between the object and the Earth in terms of forces and attempted to modify their conceptual metaphors to allow for an object to be suspended without physical support.

The question arises as to how a conceptual metaphor can be developed from a real life experience. Corballis (2010) stated that the discovery of mirror neurons provided strong support for the theory that language evolved from manual gestures rather than primate calls. The extension of this enables the explanation of how a conceptual metaphor is developed. The combination of modules and mirror neurons could provide an explanation as to how a life experience is mapped to an abstract one. For example, an observer watches someone walking down a street. The observer knows from experience that the walker must have had a starting point, will travel a certain distance and then stop. In the suggested model this equates to the observer parsing the data, activating a mirror neuronal structure which in turn activates pertinent modules resulting in the development of a metaphor for moving. If the abstract case is considered of a mathematician parsing the equation $s = d/t$ where s is speed, d is distance and t time, the mathematician is first able to equate the physical entities of speed, distance and time to the respective symbols. These symbols invoke the necessary conceptual metaphor and a resolution is formulated. There seems an obvious link between concrete experiences and conceptual metaphors, but what about when the concept itself is abstract and the possibility exists for there to be a cognitive conflict between the 'scientific definition' and the 'folk definition'? A good example of this is the use of the term 'energy'.

The issues around the learning and teaching of energy are well reported at the secondary level of education (Sefton 1998, Millar 2005, Trumper & Gorsky 1993, Solomon, 1983). As Sefton points out there is no unique definition of energy and the one normally taught in schools is 'the capacity of a system to do work'. In this paper energy is treated as a value which cannot change (first law of thermodynamics). The approach to teaching science and indeed mathematics is to start with the components of a system (ie. definitions, single concepts) and build towards an overall analytical model ie. a bottom up approach. This approach often obscures the 'end point' of learning about, for example energy, leaving the learners with an assortment of disconnected concepts. This form of teaching along with the pseudo-scientific use of the word 'energy', results in the learner not knowing when or what to apply the abstract, scientific conception of energy.

Experimental Design

The group for this research comprised of 44 first year undergraduate Mechanical Engineering students. They had all studied mathematics to an advanced level during their secondary education. The investigation was carried out in the second semester of their first year.

The group was divided into two subgroups and placed in different rooms. Once in their designated rooms they were seated so that they were unable to see their immediate neighbour's response. The first group were shown a small ball (tennis ball) rolling down a ramp (Figure 1). They were then asked to predict where a 'heavier' ball would land relative to the first one. Finally they were asked to predict where a third ball ('heaviest') would land relative to the other two. They were given the opportunity to request additional information. To facilitate this, the researcher had brought along a tape measure and a set of electronic scales which were initially hidden from view so that the learners would not be given a false prompt that they required additional information. In order to record their responses they were given prepared sheets with the pertinent questions. A typical question was, "You have seen a ball being rolled down a ramp and made a note of where it landed. If another heavier ball was to be rolled down the ramp, would it land: (a) In front of the lighter ball or (b) behind the lighter ball?" The option 'land in the same place' was not given since the idea behind the investigation was to test the students' deep learning of interpreting equations. They could have stated in the space provided for them to explain their answers that the balls would land in the same place. The individual sheets were handed out at the appropriate time and collected in after each response. This was done so that as the investigation progressed the learners would be unable to modify their previous answers.

The second group were shown a composite photograph of all three balls leaving the ramp and landing in the same place (Figure 2). Their tasks were centred on them knowing the end result and having to provide reasons why this outcome occurred. In a similar way to the first group, their answer sheets were collected in after each task.

Discussion of results

It was evident from their responses that the students were used to using a particular form of language inherited from their previous studies. It seemed that when studying mathematics and in particular mechanics that they were used to a reductionist approach and analysing the individual components rather than analysing the system as a whole. One student wrote "The mass of the ball, the initial and final velocity and the displacement are the key values…." This suggests that the student was going to apply a series of steps, rather than taking the more holistic energy approach. The approach that would have been best for them to have adopted was to focus on the transfer of energy and its conversion from potential energy to kinetic energy.

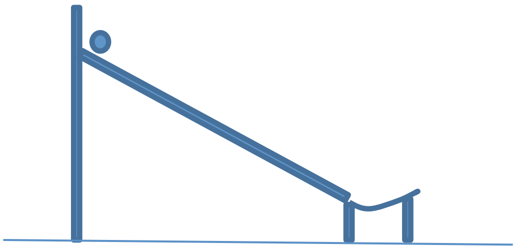

Figure 1, Ramp Configuration.

Figure 2, Composite Photograph of Landing Balls

In their answers some of them talked about the energy of the ball as if energy was a physical thing, for example like a mass, rather than an abstract mathematical concept used to facilitate the discussion of how physical systems operate. In effect these students had been taught in a way that the concept of energy had become reified and was a commodity that was physically transferrable between the different components of a system. This is not surprising since in every day conversations the word energy is used in a variety of contexts. For example, it is quite common to talk about how much energy a person gains from eating certain foods, or the cost of energy to heat a home. In terms of using conceptual metaphors, the students employed a container metaphor in that energy was contained within a vessel and the only way it could be transferred was by emptying the contents from one container into another. For example, a typical response was "When the balls reached the bottom of the ramp, the potential energy was transferred to the kinetic energy of the ball…". Most of the students were aware of potential and kinetic energies and how energy can be converted from one form to the other. Although the equation for potential energy includes height, the students seemed unable to realise that this implied that for potential energy to exist a system is required and also a reference level. In other words, potential energy cannot exist within a physical object in isolation. If one looks at the two equations, $PE = mgh$ and $= \frac{1}{2}mv^2$, it is evident that the common factor of m (mass) has no effect on the transfer of energy in this context and consequently the velocity of the ball is only reliant upon the initial height from which the ball was released. Since the mass can be eliminated from their equations, the students should have been able to infer that energy could not be a physical entity because there would not be a physical object to possess energy.

Typical approaches when asked to justify their answers mathematically, were to calculate the PE when the ball was stationary at the top of the ramp and use this to calculate a value for the KE and subsequently find the velocity of the balls. One student followed this procedure but still insisted that the mass was the major factor in determining how far the balls would travel. This insistence was linked to earlier statements made when they were asked to explain the behaviour of the balls without the use of mathematics. The context of the investigation, that is a mechanics class, seemed to have primed the students, in that they thought they were expected to calculate values, rather than to think conceptually. This was evident from the way many of the students wrote down a list of every equation they could remember involving forces (Newton's 2nd law), displacement, time and acceleration equations (eg. $s = ut + \frac{1}{2}at^2$), energy (PE and KE) and attempted to fit the data they had to one or more of the equations rather than use the relevant equations to explain the scenario. One student wrote "This equation covers all aspects of this experiment such as height of the object above ground level, acceleration due to gravity, etc to make the balls behave the way they do."

The second group were shown a composite photograph of all three balls leaving the ramp and landing in the same place. They were asked, using their knowledge of mathematics, how this could be explained. This form of the experiment involved the students in being able to 'reverse engineer' a system given the final outcome. The semantic process should have been breaking down a 'super-concept' into the relevant individual ones thus enabling the key facts to be identified. The majority of responses from this group were similar to the ones given by the first group. For example, one student wrote "The heavier ball has to overcome more friction and has more gravity acting on it, meaning it falls to the ground faster." Another student wrote, even after seeing the outcome, "If the ball has more mass it will gain more momentum on the way down the slope and so will travel further." When asked which were the most important factors influencing the balls, the height the balls were released from, the distance the balls travelled and their mass were considered the key ones. When asked to write down any conclusions regarding the behaviour of the balls, such things as the final velocity of the balls, the time of travel, mass, initial velocity and acceleration were considered the factors which determined where the balls would land.

CONCLUSIONS

The use of metaphors and metaphorical language, especially in the early stages of learning mathematics, is, to some degree, unavoidable. Imagine trying to teach a five year old child the concept of number without relating it to counting physical objects. Even after many years of compulsory education and further education, this investigation revealed that the pseudo-scientific explanations still dominate. The everyday use of the word energy has obscured, for many students the scientific notion of energy in the sense that they reify the concept of energy and employ a container metaphor to analyse a system involving energy transfer. The wording and optional answers given on the answer sheets, no doubt influenced the answers given by the students. In particular, since there was not an option stating the "balls could land in the same place" had a strong influence on the answers the students gave. This in itself demonstrated that the depth of learning was superficial in the sense that their scientific knowledge and their ability to interpret the pertinent equations was not strong enough to overcome their intuitive interpretation and pseudo-scientific interpretation of the given scenario. In addition to this the students were primed by the fact they were in a mechanics class at a university and felt obliged to use a number of equations to endeavour to find a numerical solution. Their previous experience of mechanics classes, where they would expect to perform calculations to produce a solution to a well formed, contrived problem conditioned them to employ a reductionist approach, rather than investigating the system has a whole.

The influence of natural language and way it is used to describe and explain everyday phenomena has a very profound effect on how learners interpret mathematics. This paper used the concept of energy transfer to explore the tension between natural language and mathematical language. Invariably, since natural language and the interpretation of scientific words used in everyday conversations are more common, the contest is often won by the ingrained pseudo-scientific concepts developed through these everyday interactions.

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