

MATHEMATICS CONFIDENCE: REFLECTIONS ON PROBLEM-SOLVING EXPERIENCES

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The purpose of this paper is to present the findings of a qualitative inquiry into Grade 8 and 9 learners' mathematics confidence. Learners reflect on experiences with mathematics that have influenced their levels of confidence in the social, psychological and intellectual domains. A convenient purposive sample of four participants, individually interviewed with reflective questioning, provides insight into their reflective accounts on experiences with mathematics problem solving. Through content analysis, the researchers' findings confirm the dimensionality of mathematics confidence and present sources of participants' mathematics confidence.

Keywords: *Mathematics, mathematics confidence, reflection, metacognition, anxiety*

INTRODUCTION

Maree et al. (1997, p.95) classifies problems in study orientation as cognitive, external, internal and intrapsychological causes influencing achievement in mathematics. Sherman and Wither (2003:138) documented a case where a psychological factor, mathematics anxiety, caused an impairment of Grade 8 learners' mathematics achievement. There is a scarcity on literature focusing on metacognitive aspects and its link with affect in mathematics learning and teaching (Efklides, 2006, p.6). However, aspects of affect, such as values, beliefs and motivation do not primarily form part of this paper, although these aspects are considered in the larger study. Instead the focus is on mathematics confidence and the metacognitive reflections learners have on problem-solving experiences.

The findings of Bormotova (2010) confirm this link between metacognitive reflection and mathematics confidence. The current paper explores the nature of four Grade 8 and 9 learners' mathematics confidence by focusing on their reflections on experiences with mathematics problem solving.

In mathematics, an affect such as anxiety causes personal distrust of intuition and a consequent lack of effort, which are seen as learners' greatest barrier to achievement (Aschraf & Kirk, 2002, p.2; Thijsse, 2002, p.18). John (2009) proposes a model for metacognitive reflective practice and explains this as looking into one's own thoughts and feelings as a guide and aid to problem solving. According to John (2009), reflective practice consists of two phases. The first, where the learner recalls problem solving experiences, identifying the goals and achievements as reflected upon. Through such reflection, learners begin to understand their use of approaches towards mathematics problem solving. In the second phase they describe accompanying feelings and emotions that surrounds their memories and selected strategies. A deeper

form of reflective account is established once the learner recognises how he or she feels, what they did and why. These emotions are metacognitively collective and could lead the learner along either of two different paths (Bormotova, 2010, p.29). The first gives rise to faulty beliefs that cause apprehensive thoughts about mathematics and threaten performance, thus interfering with the learner's thinking, memory processing and reasoning. Along the other road, confident learners may experience symptoms that are opposite to mathematics anxiety (Strawderman, 2010; Sheffield & Hunt, 2007, p.2). These include feelings of relief, fun, enjoyment and support, which aid with the solving of mathematical problems in a wide variety of contexts. Metacognitive reflection could ease mathematics anxiety, foster confidence and promote achievement in mathematics.

Some conscious and unconscious social, cognitive and personal evaluation practices are explored. The sources for low and high forms of mathematics confidence are scrutinised and contextualised.

CONCEPTUAL FRAMEWORK

Experiences with mathematics problem solving are either successes or failures and are accompanied by feelings of high or low confidence. Three integrated components feature in Strawderman's model for mathematics anxiety. In her study she adapted the model to include the opposite affect – mathematics confidence – in the social, intellectual and psychological domains (Strawderman, 2010, p.1). A natural overlap occurs between the boundaries where these domains coincide and our current study reflects upon these overlaps as portrayed in the conceptual framework. The discussion that follows provides insight into the theoretical influences in each domain. Metacognitive reflection, in this study, was seen as summative cognitive processes and involves thinking, examining, differentiating, detaching and serves as a self-exploratory action (Kaune, 2006, p.350) and regards social, psychological and intellectual aspects of experiences with problem solving.

Social influences

Bergh and Theron (2009, p.86) explain the social domain in terms of Bronfenbrenner's model for ecological systems. The environment includes four levels where human influences develop. The first is a micro system, consisting of persons and organisations with the most frequent contact with the individual. A second is the meso system, consisting of interactions with groups such as schools. The exo system involves aspects outside immediate contacts, such as hospitals, social groups, chat rooms and clubs. The fourth system involves the micro system. This includes habits, socio-economic and political influences. The social domain represents external factors outside of the individual's control, which are contributed by persons such as family, peers and teachers (Strawderman, 2010, p.2).

Psychological influences

Reinforcement, from the psychological behaviourist view, has a positive or negative motivational effect on individuals and the outcomes of their actions (Bergh & Theron, 2009, p.156). Rewarding the correct action, approach or behaviour and punishing the incorrect has completely diverse consequences. Positive motivation involves setting goals and helping problem solvers to achieve those goals, which leads to the successful handling of more tasks that are complex. However, negative motivation can cause undesirable effects such as hostile behaviour and avoidance of certain tasks. It may also suggest alternative ways of doing the same wrong thing differently and instigate fear, which reduces the willingness to continue with a task. Hattie (2009), however, has found low correlations between motivation and achievement. The psychological region linked to internal processes involves the individual's behaviour and varies between becoming involved in and avoiding mathematics problem solving situations. The psychological domain extends further, relating affective factors – emotional history, familiar experiences and stimulus reactions associated with the individual's feelings of confidence, anxiety or discomfort, and pleasurable experiences. This includes skills and knowledge of the problem-solving procedures and strategies selected. Personal performance measured in this domain associates with the region of personal achievement and the perception thereof.

Intellectual influences

The intellectual domain entails cognitive influences. Reflecting on success or failure, the individual evaluates the appropriateness, acquirement and use of mathematics skills and concepts learnt, and consider aspects of their higher-order reasoning and reflective questioning. Guilford's structural model of intellect (Bergh & Theron, 2009, p.149) reviews the intellectual domain as a combination of various aspects involving visual, auditory, symbolic, semantic and behavioural constructs. The operations involved with these aspects appear mainly to be cognitive (memory, divergent production, convergent production) and metacognitive (evaluation and monitoring) in nature. The product of these operations is found to influence group settings, relationships, social systems, transformation of knowledge and it has implications for problem solving.

The three domains have individual characteristics as well as a natural overflow between their components; due to the cause and effect that one component has on another. In the social domain, family members, friends (peers) and society influence the values, beliefs and views the learner might develop. The result is a motivated or demotivated individual with positive or negative emotions. The engaging or avoiding of mathematics tasks and the intellectual components involved, either allows the individual to succeed or fail in his/her use of strategies or approaches.

Combining the components of social, psychological and intellectual domains, lead to an affective product in mathematics problem solving, namely mathematics

confidence. This overflow and connection of the three domains can be observed in the theoretical framework illustrated below.

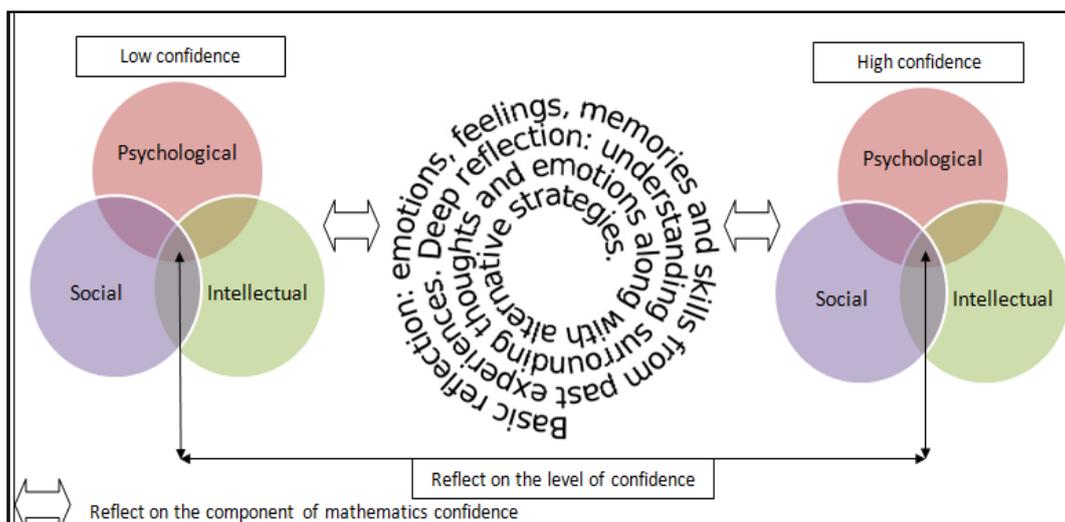


Figure 1 Conceptual framework for reflection on mathematics confidence domains
Source: Adapted from Strawderman (2010) and John (2009)

The primary research question this study seeks to answer is: what does learners' mathematics confidence entail when reflecting on mathematics problem solving experiences? To answer this question, the following methodology was employed.

METHOD

Three invited schools from the North West province in South Africa took part in the current study. This paper focuses on one aspect of this study, namely learners' confidence emanating from experiences with mathematics problem solving. As part of this qualitative, interpretive research design, data was gathered by interviewing four participants from the invited schools. According to their teachers, the purposively selected participants could express themselves verbally and would not be shy to share information about themselves. As arranged with appropriate authorities, the interviews were conducted in one of the classrooms at the participants' respective schools. The participants included three girls and one boy. Some biographical information of the participants is summarised in Table 1.

Table 1 Biographical information of participants

Participant	Age	Grade	Average achievement in 2010
Learner A	15	9	70%
Learner B	15	8	80%
Learner C	14	8	76%
Learner D	13	9	96%

Insight into their experiences with problem solving allowed the researchers to explore the sources and constructs of mathematics confidence. Interview questions, taken from the work of Wilson and Clarke (2004), Bormotova (2010) and Swanson (2006) included questions such as: Are you good at solving mathematical problems? How do you feel when solving problems in the classroom? Do your parents help or advice

you in problem solving? The questions promoted participants' thinking about experiences with problem solving and determined their perceptions about whether the sources contributed to their psychological state before, during and after such as experiences. The verbatim transcribed responses, analysed and coded, supported the themes identified by Strawderman (2010).

A-priori categories were determined in the theoretical framework and included the themes of the three domains: social, psychological and intellectual. The study involved an interpretive natural approach (Bormotova, 2010, p.73) towards an understanding of the research topic. Through content analysis, responses were categorised according to low and high traits in the three dimensions of mathematics confidence.

The results that are discussed in the next section point out that mathematics confidence has at least some roots in mathematics tuition during the primary school years – the effect of such tuition therefore has implications for senior phase mathematics problem solving.

RESULTS

Through reflection on past and more recent occurrences, participants 'made connections' with their feelings regarding mathematics problem solving. They identified the commencement of their confidence in the problem solving and developed an understanding and awareness of its foundations and variations. When asked when they first noticed a change in their mathematics confidence, all participants reflected on experiences in Grades 5 and 6. Their reflections on mathematics confidence could be traced back to incidents that had occurred between the ages of ten and fourteen years. Reflective responses included low and high confidence regarding social, psychological and intellectual aspects of mathematics problem solving.

Reflections on social aspects

Social interaction related to the context within and around mathematics problem solving. Participants compared their self-reflective knowledge with that of their peers. The social structure of the classroom, the arrangement of desks and teaching practices influenced the confidence of learners as social aspects.

Participants mentioned various teaching factors that they had experienced and to which they attributed their achievement in and understanding of mathematics during problem solving. One participant claimed:

Learner A: I couldn't understand, she [teacher] jumped around using different methods

The contributions of peers and family members were of a contrasting nature. According to the participants, parents who motivate and support their children and assist with homework do not always help to improve the latter's maths understanding.

Parental influences seemed to be minimal as one participant appeared to believe:

Learner C: I didn't enjoy the work, even if my parents explained it to me.

Comparing themselves with peers, participants fused beliefs and views about their own capabilities. One participant stated a personal disliking of this quality:

Learner B: I don't like to compete against someone.

This self-reflective knowledge promotes low and high confidence in problem solving. It is noteworthy that one participant in this study completely disagreed with the statements made by his peers and confidently said:

Learner D: I was like one with the group. Most kids would say that I don't like it and it's boring. Other people are still doing it, but not me. They don't have the passion, so they think, why should I do it? This year everybody tells us, our marks will drop because its high school...

Teachers' role in the social domain is evident:

Learner A: Most of them taught me something that wasn't maths...my math teacher was a real down-to-earth person that I could relate to.

Reflections on psychological aspects

Psychological traits of the learners' behaviour included a variation of feelings and emotions. Key aspects of the psychological state of participants' mathematics confidence included emotions and preferences with regard to likes and dislikes. Reflections on experiences integrated positive and negative emotions, feelings and associated thoughts, and an understanding of those sentiments. Participants disliked word problems the most. When asked what they liked and disliked about mathematics, three of the four participants indicated that their past and current teachers' teaching approaches and methodology contributed to their like or dislike of mathematics. Some of their reflections on the instruction methods included the following:

Learner A: Some say what you must do, they just give you the facts. Others make it practical.

Learner B: I liked the teacher but not the way he did it.

Participants seemed to understand, from experience, the characteristics of mathematics anxiety. It appears that their view on mathematics confidence revolves around their self-belief, pride and being scared of mathematics or failing the subject. One participant mentioned a high-confidence trait when comparing problem solving tasks in mathematics with tasks such as tests in other subjects:

Learner D: I just don't feel stressed like in other subjects. Maths is just there, it comes natural. I don't feel stressed. I feel excited to see what questions to expect. Anxious to see what's happening in the test [task].

Reflections on intellectual aspects

When asked what learners thought would make problem solving difficult or easy, all four participants responded with some regard to metacognitive components. Participants reasoned why they did not avoid mathematics:

Learner A: Even if you get it wrong, you still try to get it right. I just keep trying over and over again.

Learner B: I think it will boost my marks and I will become a better person.

A metacognitive trademark source of confidence was noted:

Learner A: When I know I can do it in another way, it's easy. When I know there's another way to solve the problem, I know I understand it.

Reviewing the steps used in problem solving contributed to the learners' selection and employing of strategies:

Learner C: I go through my work, wondering what I did wrong.

Avoiding mathematics was also an issue, as Learner C remarked:

Learner C: My body just gets into sleep mode. When the bell rings, I'm all energetic again.

It appears from the statement of at least one participant that the dislike (psychological) and understanding (intellectual) of mathematics problem solving is related:

Learner D: Word problems are just something I don't like. I don't like it, because I don't understand it.

It is noteworthy that Learner D, in particular, seems to experience both low and high levels of confidence depending on the psychological and intellectual sources.

DISCUSSION

The findings suggest that it is possible that metacognitive reflection may regulate mathematics confidence in the social, psychological and intellectual domains. According to their reflections, participants recalled not only low confidence connected to unsuccessful experiences with problem solving, but also moments when their confidence was high and performance successful. The following discussion focuses on Strawderman's (2010) three domains of mathematics confidence.

Social aspects of mathematics confidence

Learners' attitude towards the subject can either be positive or negative (Maree et al., 1997) with strong correlations to achievement throughout the senior and further education phase (Grades 7-12) (Hannula, Maijala & Pekhonen, 2004:18). A number of other researchers (Ernest, 2002; Bormotova, 2010; Strawderman, 2010) confirm social aspects as a psychological factor empowering mathematical problem solving. It therefore seems that the academic environment must be of such a nature that it allows learners to learn how to learn.

Learners face and have to cope with the social context at school. According to McFarland (2011, p.3), learners in the senior phase experience changes associated with puberty, and have to make social and psychological adjustments in their everyday lives. Stankov and Lee (2009) argue that social attributes include values and

attitudes originating from society, and in particular from peers, parents and teachers. McFarland (2011, p.4) explains that adolescents focus on their peers and are greatly concerned about social acceptance. The findings of the current study seem to agree. The effect that peers might have on an individual's mathematics confidence involves a comparing of attitudes towards mathematics in general and problem solving in particular.

Psychological aspects of mathematics confidence

Experiences involving Grade 5 and 6 learners indicate that, for these participants, mathematics confidence seems to start to change at around age ten or eleven, having its foundations in primary school years. Learners' dislike of teachers' approaches (Thijsse, 2002) towards problem solving and their preference for participating in group activities attest to the constructive role that reflection on mathematics confidence plays in respect of achievement in problem solving. Bormotova (2010, p.32) found that positive feelings and emotions enhance the learning process. These feelings keep the learner focused on the task and inspire new learning. Malmovuori (2006) found a connection, in terms of positive experiences, between regulating skills and strategies, and participants' enjoyment of mathematics problem solving.

Intellectual aspects of mathematics confidence

The knowledge and skills of problem solving strategies, as well as content knowledge, is vital in order to perform in mathematics. Participants reflected on their successes and failures with problem solving, and identified the level of difficulty or ease of using approaches learned and taught. Varying between complex and diverse components (Legg & Locker, 2009, p.471), learners reflected on the approaches used during problem solving and the experiences that contributed to their selection and use of strategies and knowledge. Typical traits (Learner C) displayed by learners with low mathematics confidence included avoidance of problem solving or metacognitive components such as evaluating and monitoring. Reflecting on metacognitive and mathematics skills occurred on a subconscious level. This regulative skill (Garrett, Mazzocco & Baker, 2006) differs between individuals. Strategies selected are acquired from knowledge of experiences in similar contexts, as suggested by Pantziara and Philippou (2011).

RECOMMENDATIONS

Several aspects of mathematics confidence and reflection require further investigation. These aspects include possible affective and metacognitive connections to mathematics problem solving. The following might serve as potential considerations for both researchers and teachers involved in mathematics education.

For researchers

The assessment of reflection on affect in mathematics – by using explorative and convergent mixed methods – may confirm and increase literature on the traits of the origins of mathematics confidence. It is suggested that the development of assessment methods to measure mathematics confidence might include social,

psychological and intellectual factors. According to Bormotova (2010), journal-writing experiences as self-reflective reports may serve as a basis for reflection and could provide a narrative of the participants' mathematics confidence. Cultural and socio-economic differences could also be explored to determine the possible relationships between environmental backgrounds and their relation to mathematics confidence and reflection.

For teachers

Learners who experience low mathematics confidence or anxiety may build confidence in the presence of a supporting teacher (Thijsse, 2002, p.25). Symptomatic descriptions and identification of participants' mathematics confidence, especially during primary school years (Hannula, Maijala & Pekhonen, 2004), could alter the foundational challenges that learners experience with mathematics and possibly improve their attitude towards the subject. The use of standardised tests can assist with the measuring of learners' confidence in mathematics, identify and diagnose causes and resources to reduce the level of anxiety, and increase confidence. This, in turn, could guide the planning and implementation of relevant interventions. Curriculum development could perhaps cater for the social, psychological and intellectual needs of learners and aim to nurture ambition for mathematics, mathematics problem solving and math teaching.

CONCLUSION

Some of the most noticeable social sources include teachers' approaches to the teaching of problem solving strategies and learners' self-comparison with peers. Psychological aspects include feelings of stress when the learner does not understand the problem or what approach to use. A feeling of excitement is expected when maths comes *naturally*. As learner B mentioned, the social and psychological domains are connected with liking the teacher, but disliking the approach. Intellectual sources for learners' confidence appear to include persistence and an understanding and use of multiple strategies.

The results of this study indicate that confidence in mathematics has certain implications for the individual, and that the social, psychological and intellectual success of the community as a whole has its roots in diverse experiences with mathematics.

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