The mathematical knowledge for teaching (MKT) measures have become widely used among researchers both within and outside the U.S. The measures as well as the underlying framework have, however, also been subject to criticism. One aspect of the criticism relates to the MKT framework failing to include teachers’ beliefs. This paper has a methodological focus and discusses in which ways focused discussions based on MKT items can be used to tap into teachers’ beliefs about aspects of MKT. Two example studies will be used to indicate how analyses of teachers’ focused discussions can provide researchers with important information about teachers’ epistemic beliefs related to aspects of MKT.

Keywords: MKT, epistemic beliefs, focus group interviews

INTRODUCTION

Familiarity with the participating teachers’ mathematical knowledge for teaching (MKT) is beneficial for teacher educators when planning and implementing professional development (e.g., Ball, Thames, & Phelps, 2008). Since beliefs influence teachers’ interpretation, application and implementation of pedagogical approaches (Philipp, 2007)—as well as their approach to professional development and eventual gains from attendance (Fives & Buehl, 2010)—teacher educators will also benefit from being familiar with the participating teachers’ beliefs. The teachers’ beliefs about teaching knowledge—their epistemological beliefs (e.g., Fives & Buehl, 2010)—are particularly important. For researchers, it is relevant to investigate methodologies that can be used to gain insight into teachers’ knowledge and epistemological beliefs.

In our research, we study the adaptation and use of the MKT measures (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012). These measures have been used among researchers both within and outside the U.S. The measures as well as the underlying framework have been used and referred to by many, but they have also been subject to criticism. One aspect of the criticism is related to how the MKT framework fails to acknowledge the importance of teachers’ beliefs (Petrou & Goulding, 2011; Schoenfeld, 2011). The importance of including beliefs in studies of teachers’ knowledge has been emphasized, and some even argue for the equivalence of beliefs and knowledge (Beswick, 2011). Beswick (ibid.) suggests that beliefs about mathematical content and pedagogy should be included in the MKT framework. Schoenfeld (2011) supports the idea of
including teachers’ beliefs, and he argues that this would increase the validity of studies on teachers’ knowledge.

Despite the amount research on teachers’ beliefs (e.g., Philipp, 2007), relatively few studies have focused on teachers’ beliefs about teaching knowledge in general (e.g., Fives & Buehl, 2010); even fewer studies focus on teachers’ beliefs about the knowledge they need to teach mathematics in particular. The MKT framework does not acknowledge the importance of teachers’ beliefs (Petrou & Goulding, 2011), and we intend to investigate if and how teachers’ reasoning—expressed in writing as well as in focus group discussions—may elicit their epistemic beliefs about MKT.

We invited teachers to participate in our studies, where we aimed at exploring how discussions based on MKT items can be used to study teachers’ beliefs about aspects of MKT (See figure 2 for an example item). For the purpose of this paper, we will focus on methodological issues related to these FGIs in order to answer the following research question:

In which ways can focused discussions based on MKT items be used to tap into teachers’ epistemic beliefs?

We use two of our own studies as starting point for making this methodological discussion. Seven focus group interviews (FGIs) were conducted as part of the first example study, and the 15 participating teachers in these FGIs were selected from a larger sample of teachers. They had all given their responses to a set of 30 MKT items in a testing situation prior their participation in the FGIs. In the second study, six FGIs were conducted with 26 teachers based on their responses and written reflections related to ten MKT items.

**CONCEPTUAL FRAMEWORK**

Ball and colleagues (2008, p. 395) define MKT as “the mathematical knowledge needed to carry out the work of mathematics”, and this represents a further development of Shulman’s (1986) theories of teacher knowledge. In his seminal paper, Shulman (ibid.) distinguished between ‘Subject Matter Knowledge’ (SMK) and ‘Pedagogical Content Knowledge’ (PCK). The MKT model builds directly upon this initial distinction (Figure 1).

In the MKT “egg”, SMK is divided into three sub domains. ‘Common Content Knowledge’ (CCK) describes the mathematical knowledge that is common outside as well as inside the teaching profession. ‘Specialized content knowledge’ (SCK), on the other hand, represents a mathematical knowledge that is unique to the work of teaching. The third subdomain of SMK, ‘horizon content knowledge’, is mathematical knowledge not directly deployed in instruction.
Figure 1. Domains of MKT (Ball et al., 2008, p. 403).

The right side of the oval contains three different categories of knowledge related to Shulman’s PCK. The first category—‘knowledge of content and students’ (KCS)—is focused on students’ mathematical thinking, knowledge and learning of mathematics. ‘Knowledge of content and teaching’ (KCT), which is the second category, refers to the knowledge used by teachers when designing mathematics lessons. The final category, ‘knowledge of content and curriculum’, includes (but is not exclusively related to) knowledge of grade levels where particular topics are typically taught, assessments and educational goals.

The MKT framework does not include beliefs. This has been criticized, and Beswick (2011) argued that beliefs about mathematical content and pedagogy should be included in the MKT framework. Philipp (2007), on the other hand, suggested that beliefs are closely related to knowledge, but he argued that a distinction should still be made between the terms. In this paper, we follow Philipp’s suggestion and distinguish between knowledge and beliefs. We focus on the beliefs that teachers have about MKT, and this is an aspect of teachers’ epistemological beliefs. Teachers’ epistemological beliefs are considered important by several researchers and several competing models that describe the nature of epistemological beliefs have been proposed, but general epistemological beliefs still seem to refer to “individuals’ belief about the nature of knowledge and the processes of knowing” (Hofer & Pintrich, 1997, p. 112). In their attempt to clarify the research in this area, these researchers proposed that epistemological theories are composed of “certainty of knowledge, simplicity of knowledge, source of knowledge, and justification for knowing” (ibid., p. 133).

In order to avoid confusion, we use ‘epistemological beliefs’ with reference to teachers’ general beliefs about knowledge and knowing in this paper. ‘Epistemic beliefs’ refer to teachers’ domain-specific beliefs about knowledge and knowing—in particular teachers’ beliefs about aspects of MKT.
Most of the research on MKT has been related to the measurement of MKT by the use of multiple-choice items, and the MKT items themselves are strongly connected with—and can even be seen as manifestations of—the MKT construct. When attempting to investigate teachers’ epistemic beliefs about MKT, it might therefore be a good idea to use MKT items as a starting point for focused discussions. In the following, two studies will be presented as examples of ways in which focused discussions based on MKT items can be used to tap into teachers’ epistemic beliefs.

Bryman (2004, p. 348) suggests that FGIs may give “more realistic accounts of what people think, because they [interviewees] are forced to think about and possibly revise their views”. The use of activity-oriented questions in FGIs is highly recommended as productive supplements to oral questions and the importance of asking the participants to do something before the FGI is emphasized (Colucci, 2007). Inspired by other researchers who have used similar items in this way we included MKT items that were well known for the teachers to focus the discussions. Below is an example from the public released MKT items that focuses on definitions. In the first example study the teachers discussed definitions, and related to one of the items the main issue was whether or not 1 is defined as a prime number. The MKT item in Figure 2 is not the exact same, but it also has a focus on the definition of prime numbers.

2. Ms. Chambreaux’s students are working on the following problem:

   Is 371 a prime number?

   As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

   a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
   b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
   c) Check to see whether 371 is divisible by any prime number less than 20.
   d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Figure 2. Item 2 from the set of released items (Ball & Hill, 2008, p. 4).

In the second example study an item focusing on place value and decomposing numbers was discussed. Since we are discussing two example studies in this paper, we have chosen to make separate presentations of the methodological issues and results for each study below.
STUDY 1—METHODS AND RESULTS

In the first study, 15 teachers participated in seven semi-structured FGIs. These teachers were selected from a convenience sample of 142 teachers. All the participants had a special interest in mathematics and mathematics teacher education and worked individually with a set of MKT items before they participated in the interviews. A complete form (Elementary form A, MSP_A04) with items from the LMT project\textsuperscript{2} was used. This form had been translated and adapted for use among Norwegian teachers (Fauskanger et al., 2012), and it contained 30 item stems and 61 items in total. The form consisted of the following three sets of MKT items: number concepts and operations (27 items), geometry (19 items), and patterns, functions and algebra (15 items).

After the teachers had taken the test, they were given a short break before groups of two or three teachers’ were invited to discuss the items. The initial aim with these discussions was to investigate whether or not our adaptation of the MKT measures was successful by bringing in the voices of the test-takers. In our previous analyses of these interviews, we learned that the practicing teachers also discussed different aspects of the knowledge they found relevant and irrelevant for their work as teachers—including aspects related to mathematical definitions (Fauskanger, 2012). This inspired us to analyze the FGIs with a focus on epistemic beliefs about MKT definitions. The results from this study was presented at the 2012 AERA conference in Canada (Mosvold & Fauskanger, 2012), and the main focus then was on what teachers’ reflections on MKT items reveal about their epistemic beliefs concerning mathematical knowledge for teaching definitions.

The transcripts from these interviews were analyzed in two steps: First, directed content analysis (Hsieh & Shannon, 2005) was applied to the data. We began by identifying all that was discussed related to MKT-items focusing on definitions, and all that was said related to definitions when discussing other items as well. Both authors first searched the transcripts for occurrences of the words ‘define’, ‘definition’ and derived terms. When reading the transcripts, we discovered that words like ‘concept’ and ‘formula’ were used more or less as synonyms of ‘definition’. We therefore searched the transcripts for these terms as well. In our separate analyses, we ended up with an almost perfect overlap of excerpts from the transcripts. Second, these excerpts were subject to further qualitative analysis to uncover subcategories related to what was said about definitions. For a subcategory to be established, the aspect in focus had to be discussed by the teachers in at least 2 separate interviews. Two researchers carried out independent content analysis of the data to ensure reliability.

Mathematics teachers need to know something about mathematical definitions, and the focus on definitions was present as one of the mathematical tasks of teaching in Ball and colleagues’ (2008) presentation of the MKT framework.
When “choosing and developing useable definitions”, teachers need to know the actual definitions.

A directed content analysis approach followed by qualitative analyses of data from these FGIs revealed different epistemic beliefs from the teachers concerning the relevance of MKT definitions as well as the task of teaching mathematical definitions. In our analysis, two categories emerged. The first category was: Knowledge of definitions is an important part of teachers’ MKT? This category included the following two subcategories: Definitions are important and Remembering definitions is not important. Below is an excerpt from the transcripts that illustrates the first subcategory:

Interviewer: You suggest, in a way, more of the kind of tasks that focus on definitions, and less of the kind of tasks that focus on calculations, then?

Betty: Yes, I think that is correct.

Benjamin: Definitions are incredibly important as a pre-requisite, because if you don’t have clear definitions and know a little about it, then you will easily be out of track.

Betty: And, what was said after the TIMSS study, what I have heard anyway, is that we score low on concepts. So, I believe it is more important to be clear about this than to be able to calculate correctly.

The teachers in our study seemed to agree that knowledge of definitions is important. Quite a few teachers suggested, however, that teachers do not have to remember the actual definitions—only know about them. Some teachers even claimed that knowledge of definitions—although it is arguably part of teachers’ knowledge—is not crucial.

The second main category that emerged from our analysis was: Choosing and developing useable definitions. It contained the following subcategories: Adjusting to different groups of students and Inclusive definitions are confusing. In the FGIs, some teachers suggested that certain definitions are less suitable for the lower grades—which is an example of the first subcategory—as represented by the following excerpt from the transcripts:

Karen: I think they [the MKT measures] should have been differentiated... As an example if one can have a rectangle that is not a parallelogram and that stuff [definitions of quadrangles]. (...) But we do not have [teach] it [definitions of different quadrangles] for the younger ones [students] we teach.

Ken: No, exactly.

This can be seen as supporting the claim that “choosing and developing useable definitions” is a relevant mathematical task of teaching, however the definitions
chosen should be adjusted to different groups of students. This example study indicates that such a methodological approach—i.e. inviting teachers to respond to MKT items in a testing situation followed by focused discussions—might be a fruitful way to tap into teachers’ epistemic beliefs.

**STUDY 2—METHODS AND RESULTS**

The second study was presented at the Norwegian conference “FoU i praksis” (Fauskanger & Mosvold, in review). In that study, our focus was on teachers’ beliefs about MKT place value. This second study was conducted on the basis of what we had learned from the first study—indicating that FGIs based on teachers’ responses to MKT items are suitable for eliciting teachers’ views on which aspect of the MKT they consider relevant and/or irrelevant. In the second study, 26 teachers were asked to give written responses to ten MKT items from the “number concepts and operations” scale at home. Follow-up questions were added to the items, and they were asked to reflect upon which items best captured MKT that was important for them.

The interviews were recorded and transcribed, and the transcripts were analyzed in three steps. First, a double content analysis was made in order to reduce data. The first author read through all the transcripts and identified all that was said related to the item in focus. The second author conducted an independent content analysis in which the transcripts were analyzed by counting words in the text with the purpose of understanding the contextual use of words related to place value. Second, utterances were chosen as coding units, and the context unit was defined to be two utterances before and after each coding unit in which a key word was found. This was followed by a comparison of what was included in the first sample and not in the second, and the union of the two samples was used in the further analyses. To analyze teachers’ epistemic beliefs about MKT, we decided to use the aspects of MKT as presented in Figure 1 as codes in this second part of our content analysis, and the sample was then coded by both researchers. In a third and final step, all the coded parts were analyzed in relation to aspects of MKT place value as highlighted in the research literature.

Our analyses indicate that the teachers as a group emphasize all aspects of MKT (Figure 1) when describing what is important for them as teachers. They agree that place value is an important base for students’ future learning of mathematics, and when trying to make the teachers identify why it is important, one of the teachers responded:

Interviewer: What makes the place value system so important?

Doris: It is the foundation for counting and arithmetic. It is the basis for everything. The numbers wouldn’t even have the names they have if it wasn’t for the place value system.
On the other hand, our findings also indicate that the teachers’ utterances—in the FGIs—in some cases contrast important research findings. As an example, the teachers indicate column value to be more important than quantity value, whereas research emphasize that both are important related to multi-digit calculation (e.g., Thompson, 2003). A second example is that the teachers in this example study emphasize a decomposition of multi-digit numbers following the positions, whereas research emphasizes non-standard decomposition as an important base for multi-digit calculation (e.g., Jones et al., 1996). This is interesting since e.g. the standard Norwegian algorithm(s) for multi-digit subtraction includes non-standard decomposition, and it might indicate that this aspect is important to discuss in future professional development. Teachers’ beliefs about MKT place value might thus provide a relevant starting point for professional development initiatives.

As in the first example study, this second study also indicates that using MKT items in FGIs might be a fruitful way to tap into teachers’ epistemic beliefs about important aspects of MKT—such as their MKT place value. A three step content analysis of data from these FGIs as described above revealed different epistemic beliefs concerning the MKT place value. In conclusion, this example study indicates that inviting teachers to respond to MKT items and reflect upon the items in writing, followed by FGIs including the items might be a fruitful way to tap into teachers’ epistemic beliefs.

CONCLUSIONS

Preliminary analyses of teachers’ focused discussions related to MKT items in these two example studies indicate that such analyses can provide researchers with important information about teachers’ epistemic beliefs related to aspects of MKT—in particular about the knowledge needed to teach place value (example study 2) and their epistemic beliefs about mathematical definitions (example study 1).

Different methodological approaches can be used when studying beliefs and the two example studies presented in this paper exemplify how focused discussions based on MKT items can be used to tap into teachers’ epistemic beliefs about aspects of MKT. We have learned that analyzing these FGIs using content analysis can illuminate aspects of teachers’ epistemic beliefs about MKT important for future professional development. Knowing, for instance, that teachers believe decomposition following the positions to be most important, professional development can be developed that allows us to discuss teachers’ epistemic beliefs about decompositions.

So far, there is a scarcity of research related to epistemic beliefs about MKT (Fives & Buehl, 2010), and more research is needed. Our preliminary results indicate that analyses of teachers’ reflections in FGIs based on MKT items can be particularly useful in this connection. Despite the obvious limitations of these
two example studies, they support the argument that epistemic beliefs about MKT are important. Further studies of teachers’ epistemic beliefs about aspects of MKT are called for, and we believe that such studies might have a potential to inform the further development of the MKT construct as such.

NOTES

1. Our research project has been supported by OLF, The Norwegian Oil Industry Association.

2. See http://sitemaker.umich.edu/lmt/home.


REFERENCES


Colucci, E. (2007). “Focus groups can be fun”: The use of activity-oriented questions in focus group discussions. Qualitative Health Research, 17(10), 1422-1433.


