RECONSTRUCTING TEACHERS´ BELIEFS ON CALCULUS

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The teachers' instructional planning, their classroom practices that impact on their students' knowledge and beliefs could be understood as individual beliefs systems dependent from their actual teaching and learning experience. This individual experience might be contradictory when we regard different teachers or one teacher concerning different mathematical disciplines. For this reason, this report focuses on teachers´ beliefs with different levels of experience about their teaching of a specific mathematical domain, i.e. calculus that is a central part of the (German) curriculum at upper secondary level. After a brief outline of the theoretical framework and methodology of this research project, results of the qualitative reconstruction of different types of teachers´ beliefs on calculus will be explained.

INTRODUCTION

The importance of gaining knowledge towards mathematics teachers' thinking or beliefs has been emphasised by many researchers in mathematics education in various settings and projects for some reasons: on the one hand teachers' beliefs about mathematics and the teaching and learning of mathematics have a high impact on their instructional practice (Philipp, 2007; Eichler, 2011), on the other hand teachers' instructional practice, which is considerably determined by teachers' beliefs about their professional world (Calderhead, 1996), has a high impact on students' learning and beliefs concerning mathematics (Hiebert & Grouws, 2007). Moreover, the possibilities of changing the teachers' thinking about mathematics education depend on the teachers' beliefs towards teaching and learning mathematics (Franke, Kazemi & Battey, 2007). On these grounds there has been much effort in investigating mathematical beliefs of teachers all over the world in the recent two decades (Philipp, 2007). In this huge body of research, most of the study approaches concern teachers' beliefs on *mathematics* and the learning and teaching of *mathematics*. It is rarely considered though that – similar to the classification of mathematical subjects into fields such as algebra or probability theory – teachers´ beliefs on different mathematical domains such as geometry, stochastics or calculus may vary and may be associated with specific beliefs (Franke et al., 2007).

Research on (intended) curricula of experienced calculus teachers is rare (e.g. Tietze, 2000). Recent works aim primarily at investigating calculus lessons with the use of technology (e.g. Kendal et al., 2005) or look at calculus curricula of undergraduate courses at university. Although there is plenty of research on algebra teaching (Kieran, 2007), deeper investigations which focus on the development of teachers´ intended curricula independent of technology aspects are scarce.

As there are few investigations about calculus, which is a central part of the German secondary curriculum, domain-specific beliefs of secondary teachers referring to the teaching and learning of calculus are the main focus in this paper. Our specific interest primarily concerns the structure of beliefs that characterise calculus teachers' instructional planning (teachers' intended curricula) and, thus, impact on the teachers' classroom practice (teachers' enacted curricula), and the students' learning (Eichler, 2011). Reconstructing this structure possibly facilitates to partially identify which constituents are more central than others (Green, 1971). Further, regarding the system on calculus teachers' beliefs, we search for relations between different clusters of calculus teachers' beliefs that we call views (Grigutsch et al., 1998). We refer our findings to existing results for teachers of other mathematical disciplines. As part of a larger research programme it can be asked whether the degree of professional experience might have an impact on characteristics of the teachers' belief systems. Before we address the mentioned questions for this paper, an outline is given about the theoretical framework of the research programme, and a brief description of those parts of the method being relevant for this paper.

THEORETICAL FRAMEWORK

Research on teaching and curriculum has brought forward that a significant difference exists between the curriculum as represented in specifications by national or regional governments, sometimes accompanied by instructional materials, and the curriculum as it is actually enacted in the classroom by teachers and students. The various meanings of curriculum have been conceptualized by the work of Stein, Remillard and Smith (2007), who provide a curriculum model including four phases of a transformation process to describe a mathematical teacher's planning, the teacher's classroom practice and his students learning (see fig. 1).

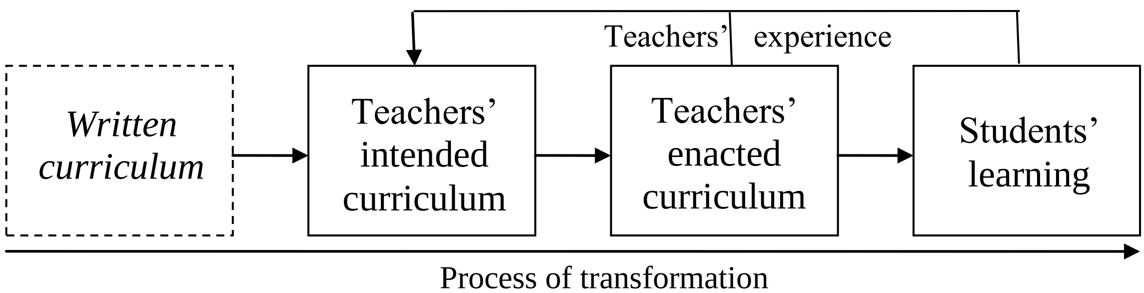

Figure 1: Four phases of the curriculum according to Stein et al. (2007)

The *written curriculum* involves both instructional content and teaching goals, often prescribed by national governments. The way the teachers interpret a written curriculum concerning content and goals is called the *intended curriculum*. The classroom practice involving interactions of a teacher with his or her students, and the instructional content "create something different than what could exist […] in the teacher's mind" (Stein et al., 2007, p. 321). This transformation of the intended curriculum is called the *enacted curriculum*. The individual students learning outcomes that are not necessarily intended by their teachers is called *students' learning*. In this article, the discussion is restricted to teachers' intended curricula.

A teacher's intended curriculum represented by content and goals can be understood as specific form of beliefs, if beliefs are defined as an individual's personal conviction concerning a specific subject, which shapes an individual's way of both receiving information about a subject and acting in a specific situation (Pajares, 2007). Regarding this definition, content and goals portray a teacher's conviction about an appropriate way of teaching mathematics. Thus a teacher's intended curriculum can be understood as a belief system that is characterised by a quasi-logical system of beliefs with different grades of importance or centrality (Green, 1971, Philipp, 2007). Another theoretical feature of a belief system involves that belief systems consist of several clusters of beliefs that may be contradictory. We hypothesise that a teacher's intended curriculum concerning the teaching of mathematics consists of clusters representing different mathematical domains that need not coincide with each other if the teaching goals are regarded, but are mostly consistent, if a specific mathematical domain, e.g. calculus, is regarded (Girnat & Eichler, 2011). In this paper, we restrict discussion of results on teachers' intended curricula concerning the teaching and learning of calculus.

In the following, belief systems of secondary teachers are characterised towards the teaching and learning of calculus as views representing the main instructional goals of the teachers (Philipp, 2007). There are four main categories which can be characterized by different features regarding the perception of mathematics in general (Grigutsch, Raatz & Törner, 1998) and calculus in particular:

- A formalist view stresses that mathematics/calculus is characterized by a strongly logical and formal approach. Accuracy and precision are most important and a major focus is put on the deductive nature of mathematics or calculus.
- A process-oriented view is represented by statements about mathematics being experienced as a heuristic and creative activity that allows solving problems using different and individual ways.
- An instrumentalist view places emphasis on the "tool box"-aspect which means that mathematics is seen as a collection of calculation rules and procedures to be memorized and applied according to the given situation.
- An application oriented view accentuates the utility of mathematics for the real world and the attempts to include real-world problems into mathematics classrooms.

Usually teachers' belief systems might consist of a mixture of all the four views outlined above. However the weight of each aspect varies from teacher to teacher. Although we assume that a teacher's belief system consists of parts of different views, our research shows patterns of views when we investigate teachers' beliefs referring to a specific mathematical domain: e.g., the research of Eichler (2011) yields that stochastics teachers primarily refer to application oriented teaching goals when they analyse both their instructional planning and their classroom practice. By contrast, teachers mostly disregard a direct connection of school mathematics and real world problems, if geometry is considered (Girnat & Eichler, 2011). Accepting the hypothesis, like Franke et al. (2007), that teachers' beliefs referring to the teaching and learning of mathematics differ, if they have geometry, stochastics or calculus in mind, it is worthwhile to investigate teachers' beliefs concerning the calculus domain to understand their teaching practice.

METHOD

Concerning the teaching and learning of calculus, we analyse teachers in different stages of their professional development including 10 pre-service teachers, 10 teachers in a practical phase after university studies (teacher-training college) that lasts 18 months and 10 teachers with a professional experience of at least five years. All the teachers are employed or would be employed at upper secondary schools (Gymnasium) in Germany.

In the first part of our research that we address in this paper, data were collected by semi-structured interviews comprising clusters of questions concerning several subjects: contents and goals of instruction, teaching and learning calculus, institutional boundaries and the nature of mathematics and calculus at school level in particular. Within these obligatory clusters, the teachers gave distinction to the interviews by describing their own typical examples but also had to react to specific prompts. These prompts include, for example, evaluating statements about adequate teaching, students' statements about calculus or different tasks from calculus text books. In addition, the teachers were asked to respond to questionnaires concerning mathematical beliefs (Grigutsch et al. 1998) and their teaching orientation (Staub & Stern, 2002).

The interviews were transcribed verbatim. Each transcript has a length of 30 to 40 pages. The first step of the analysis was to split the transcripts into episodes and label them in terms of the question clusters outlined above. Further, the episodes were analyzed by coding. Coding guidelines were adapted in compliance with qualitative data analysis (Kuckartz, 2012). First the relevant coding units were determined and then supplemented by inductive codings. This approach enables both the consideration of empirically-based individual content items (inductive aspects) and the significance and reconstruction of contents and goals (deductive aspects) that have been shown to be relevant in previous work on individual curricula e.g. of stochastics teachers (Eichler, 2011). Finally, the characteristics of each teacher were systematically arranged and summarized with respect to the projects´ research questions. Furthermore case summaries were compiled to highlight essential characteristics of each individual teacher.

RESULTS

In order to categorize and illustrate teachers´ beliefs concerning the planning and teaching of calculus by means of qualitative analysis, the deductive aspects of four different views (see above) were chosen. This method has been efficient in previous studies of intended curricula for other mathematical domains with respect to patterns of beliefs as well as to describe these views in depth. This involves the subjective teachers' definition of a specific view that represents the teachers' overarching teaching objectives. We illustrate a coherent view, in this case a formalist view concerning the subjective definition of Mr. C.

Mr. C.: „In general, exactness is crucial for me. That means to fit a necessary formalism as I know from my university studies. This also means that it must be possible to recognise a logical rigor. Sometimes I do more in that sense than the textbook actually demands."

Taking this teacher as a paradigmatic example, he did not mention aspects like to apply mathematics in real world problems or to learn problem solving, which means to emphasise the process of developing mathematical concepts. By contrast, for Mr. C., the main goal of calculus teaching seems to be emphasising the stringent and logical construction of a mathematical domain.

The identification of specific teachers´ views is always established in various parts of a single interview and we report only teachers´ views that are in some sense coherent throughout the whole interview. We illustrate this concerning this exemplified teacher. When Mr. C was asked to regard the expectations and needs of his students, he agrees consistently with a formalist view. For instance, Mr. C was asked to rate the statements of students shown in figure 2.

Which of the following comments on calculus would your students most often make?
Why? Which of these are the most important in your view?

I like calculus, because there is a connection to real life problems.

I like calculus, because hard nuts must be cracked and difficult problems can be solved.

I like calculus, because many exercises can be solved by similar procedures/patterns.

I like calculus, because the logic is clear and it follows strict mathematical rules.

Figure 2: Statements of students concerning calculus

As expected, Mr. C attaches particular importance to the last statement that represents a formalist view. Mr. C further explains his goals concerning his students' beliefs towards calculus:

Interviewer: How should your students characterise calculus?

Mr. C: Precise mathematics. Thus, on the one side that it is possible to understand how one develops mathematical ideas and how it is possible to build up a theory on the foundation of few basic ideas.

Summarising the beliefs of Mr. C concerning the teaching and learning of calculus, there exist several unambiguous examples of evidence for Mr. C´s formalist view. The high degree of coherence in different parts of the interview leads to the hypothesis that this formalist view is dominant and thus central in the belief system on calculus.

Regarding teachers´ beliefs on calculus, our study has so far revealed one particular aspect as some teachers stressed their need for formal and deductive nature of calculus which is characterised by accuracy, precision and a strongly formal and logical approach. However, in contrast to Mr. C, most of the teachers show a mixture of different views.

In particular, if teachers' hold beliefs that represent different views, we analyse relationships among the different views. We describe this analysis exemplarily by regarding Mr. A and Mr. B starting at the subjective definitions of their possibly central beliefs. In contrast to a (central) formalist view, these two teachers firstly delivered an insight on their views on applications:

Mr A.: „I quite agree with the emphasis on applications in the given example. That is certainly a way to motivate them (students), but nevertheless one should not reduce genuine calculus or the teaching of calculus to that topic. "

Mr. B.: „Examples for applications are quite suitable here, and with applications I always associate modelling of real data, […] increasingly introducing relevant applications into lessons may, for the students, succeed in a deeper insight into the concepts and ideas of calculus."

Both teachers have nearly completed their teacher training and have taught the first class of upper secondary level independently i.e. without being accompanied by a senior teacher. Mr A. supports the integration of applications as a principle of learning calculus at school for reasons of (student) motivation. In contrast Mr. B reckons that introducing real-world problems into calculus is a part of his system of aims and goals concerning his calculus teaching. The difference between the instructional goals of motivation on the one side, and solving real problems on the other, is stated by Förster (2011) concerning teachers who teach modelling, but, however, has not been reported in domain-specific research about teachers´ intended curricula so far. The aforementioned aspect seems to be relevant independent of the professional status of the teachers interviewed. Both views on applications can be found in all three groups of our sample.

In addition, the explicitly expressed goal of Mr. B. of integrating modelling tasks into his lessons is connected with another statement about dealing with the "formal logic" as a characteristic property particularly in calculus.

Mr B.: "…because I think that the formal derivation of integrals by limits is of no avail for secondary level students. It´s just too complex for most of them."

However, a general conclusion that applications are implicitly of primary importance than formality and logic cannot be drawn as the following quotation of Mr A. demonstrates:

Mr. A: "Calculus is more than just dealing with application-oriented tasks. Then, for example, one would not regard the precision and exactness of calculus and use applications as a means to an end."

Our hypothesis on basis of the present data is the following: If teachers hold a consistent formalist view on calculus, they do not mention any applications. The reversal conclusion, however, is not possible. Teachers who primarily favour applications in their calculus courses, e.g. Mr. A and Mr. B (see above), cannot be described as non-formalist. This example already demonstrates the abundance and need to differentiate the views of teachers on calculus.

As another aspect of relationships among different views or rather cluster of beliefs we describe in the following contradictory belief clusters that we call conflicts (of instructional goals). We reckon that the teachers' system of beliefs may have a quasi-logical structure and could involve contradictory clusters of beliefs with the paradigmatic example of Mr. E. Throughout the whole interview he speaks about the central role of logic in calculus lessons offering his perspective that exactness and logical rigour are necessary ingredients of secondary level calculus courses. Again, the degree of coherence of favouring formalist elements could provide an indication for a core belief. Yet, as he describes representative classroom situations, his subjective experience surfaces a conflict between his belief system about calculus and pedagogical processes in his calculus course.

Mr. E.: „ In my view it is quite important that there are formal definitions of concepts because you need them for proofs later on and it´s the tiny details that are particularly important…

In my class I clearly notice that students come to their limits concerning the degree of abstraction. [...] Remembering my own calculus course at school I can´t remember any bad experience with these formal aspects. So far I haven´t seen such a mismatch between teacher and students in maths."

Mr E. can be identified favouring a formalist view but probably will not enact his formalist view on calculus in the classroom in a predominant way because there is a conflict with the real situation he encounters in the classroom i.e. the students´ ability to understand the formal way of developing calculus ideas. Therefore this situation can be characterized as a conflict of objectives between his view on calculus and his teacher authority and responsibility. In particular when teachers are asked to reflect on representative examples of their actual teaching processes of specific elements of their calculus courses, the interview transcripts provide a deep and concrete insight of teachers´ subjective notions of their intended and enacted curricula and sometimes yield conflicts of a teacher's system of instructional goals. We hypothesise that a conflict of teaching objectives gives evidence for a central belief since peripheral beliefs might be superimposed if they show a conflict with central beliefs.

In addition to predominant beliefs that teachers show in different parts of the interview in a coherent way, the teachers also provide insights into some peripheral goals. These peripheral goals are neither coherent nor do they produce any conflicts between the intended and enacted curriculum. For example, the teachers indicate in some interviews the peripheral goal that calculus and the teaching of calculus is a collection of rules and procedures (i.e. toolbox) although their beliefs cannot be categorized globally as an instrumentalist view.

Mr. F.: The main goal of every student is to perform well in his final exams – therefore calculation rules and procedures have to be thoroughly practised in class. Especially the calculus part of final exam tasks are alike in some respect, so practising is a substantial guideline for my course.

Often these views are motivated by normative aspects such as final exam tasks, yet seem to have a considerable impact on their actual teaching of calculus.

It is apparent though that for all teachers in our sample the preparation of the final exam (so called "Abitur") does indeed play more than a subordinate role in their calculus beliefs as an inductive feature. The driving force of the written curriculum and a focus on student achievement scores has a particular influence on the realisation of learning processes and, thus, the enacted curriculum.

DISCUSSION

Certainly the scope of this paper could not present an exhaustive discussion on teachers´ beliefs on calculus. We discussed two views with the teachers' subjective definitions, the formalist view and the application view. Concerning all teachers one of these views seem to be central in the calculus teachers' intended curricula since other beliefs, in particular the instrumentalist view with respect to exams, have become apparent as peripheral goals. We further discussed two indications of central goals, i.e. the coherence of beliefs concerning different aspects of the teachers' teaching and the existence of conflicts of teaching objectives.

The results of this study of teachers´ (central) beliefs on calculus as one important mathematical domain at upper secondary level suggest that the assumption (occasionally stated, e.g. Tietze, 2000) of a biased instrumentalist orientation of calculus teachers is not valid.

The establishment of a causal relationship between different clusters of teachers´ beliefs on calculus is an effort to reconstruct the network structure. At the current state of our analysis, it is striking that a formalist view could be a key factor to understand the calculus teachers' intended curricula: Those teachers who favour a formalist view as a central goal neglect any application-oriented view. We hypothesise that the individual characteristic of the formalist view considerably forms the calculus teachers' intended curricula.

Having outlined that teachers´ beliefs on calculus and its teaching are inherently different from the domain-specific beliefs of stochastics or geometry teachers, comparisons to other domains have to be drawn carefully, since we did not investigate the teachers' beliefs concerning to all mentioned mathematical disciplines. From a pragmatic perspective this would not have been possible as a single interview on calculus took about two hours. On the basis of present evidence we assume that differences in belief systems in the aforementioned domains surface in every secondary teacher. However, regarding the larger research programme, it has been conclusively shown that there exist differences depending on the mathematical domain being looked at.

At this point, the analysis of the present data cannot yet give a reliable answer whether e.g. pre-service teachers show a higher preference of collaborative activities in their pedagogical practice than experienced teachers who are assumed to favour a more instructivist view of learning. The development of teachers´ belief systems depending on their degree of professional experience as well as some verification whether expressed beliefs are guiding factors in actual classroom practice will be considerately analysed after a further collection of data.

With regard to the previous argument this qualitative study will turn towards the question by means of subsequent interviews in what way pre-service teachers´ beliefs are subject to change in the course of their professional development and whether there will appear a cohesion or fraction of their intended or enacted curricula.

Furthermore an attempt will be made to quantify relevant aspects of our sample with respect to the results of the qualitative approach, which will be evaluated by relevant quantitative data interpretation.

However, up to the present stage, our studies already underline the importance of a differentiated investigation of teachers' beliefs referring to specific mathematical domains like calculus. The focus on a deep and valid understanding of intended and enacted curricula of calculus teachers might further facilitate to understand teachers' classroom practice and to identify approaches to change both teachers' intended curricula, and teachers' enacted curricula.

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