BRAIN POTENTIALS DURING SOLVING AREA-RELATED PROBLEMS: EFFECTS OF GIFTEDNESS AND EXCELLENCE IN MATHEMATICS

Ilana Waisman, Mark Leikin, Shelley Shaul and Roza Leikin

Faculty of Education, RANGE Centre, University of Haifa, Israel

This event-related potentials (ERP) study investigates the impacts of general giftedness and mathematical excellence on behavioural performance and the cortical activity while solving geometric area-related tasks. We report on findings of comparative data analysis based on 74 right hand male-students. Effects of Giftedness and Excellence emerged at the behavioural and the neuro-cognitive level. We found that Giftedness is expressed in more efficient brain functioning. Excellence in mathematics didn't assure success in solving the problems and exerted a different effect on the cortical activity of gifted as compared to non-gifted. Giftedness can compensate the lack of Excellence in mathematics when performing area-related tasks

Keywords: Geometric reasoning, Area, Event related potentials, Giftedness, Neural efficiency

INTRODUCTION

A considerable body of research has been conducted towards understanding of the neural foundation of mathematical cognition (e.g., Santens et al., 2010). In addition there has been extensive neuroscience research on human intelligence including individual differences in general intelligence (e.g., Neubauer & Fink, 2009; Deary et al., 2010) as well on mathematical giftedness (O'Boyle, 2008). However, these studies have not gone beyond arithmetic, logic and spatial mental ability. That's why we focus our attention on brain activation (by means of Event Related Potentials – ERPs) associated with solving advanced mathematical tasks.

Additionally, to the best of our knowledge, differences between giftedness and expertise in mathematics were not addressed in brain research. This observation resulted in the integration of 4 groups of research population, divided according to general giftedness and mathematical expertise. Background

Studying areas of figures in school

Studying geometry in high school involves analyzing geometric structures characteristics and relationships (NCTM, 2000). Mental images of geometrical figures represent mental constructs possessing simultaneously conceptual and figural properties (Fischbein, 1993). Geometrical reasoning is usually associated with visual and logical components which are mutually related. Area of figures overlaps content areas of geometry and is considered to be a significant topic of school mathematics (NCTM, 2000). Mathematics researchers and educators have suggested that knowledge of the properties of the basic shapes; congruence, geometric motions and area measurement concepts are closely interrelated (Clements et al., 1997). Moreover the integration of geometric knowledge and area measurement can be important for conceptual understanding of area measurement (Huang & Witz, 2011). The problems involving comparison between the areas of two figures can be solved without using area formulae which enables to study student's conceptual understanding (Huang & Witz, 2011).

Mathematical abilities, cognitive skills and brain research

Literature review demonstrates quite consistent findings that connect different mental operations associated with mathematics and the location of brain activation. For example, research shows that attention control processes and general task difficulty (Delazer et al., 2003) are associated with the prefrontal cortex while mental rotation (Heil, M., 2002), and visuo-spatial strategies in mathematics (Sohn, et al., 2004) with the parietal cortex. The brains of the mathematically gifted show enhanced development and activation of the right hemisphere (Prescott et al., 2010) as well as enhanced brain connectivity and an ability to activate task-appropriate regions in both brain hemispheres in a well-orchestrated and coordinated manner (O'Boyle, 2008). There is strong evidence for special development of prefrontal and posterior parietal regions of the brain and their enhanced intra-hemispheric connectivity (e.g. Jung & Haier, 2007).

The goal of the current study was to investigate the impact of general intelligence as well as of excellence in mathematics on performance in area-related short geometric problems using electrophysiological measures.

MATERIALS AND METHODS

Participants

Seventy four right hand male high school students from the northern Israel (16-17 years old) participated in this study. The students were sampled as shown in Table 1.

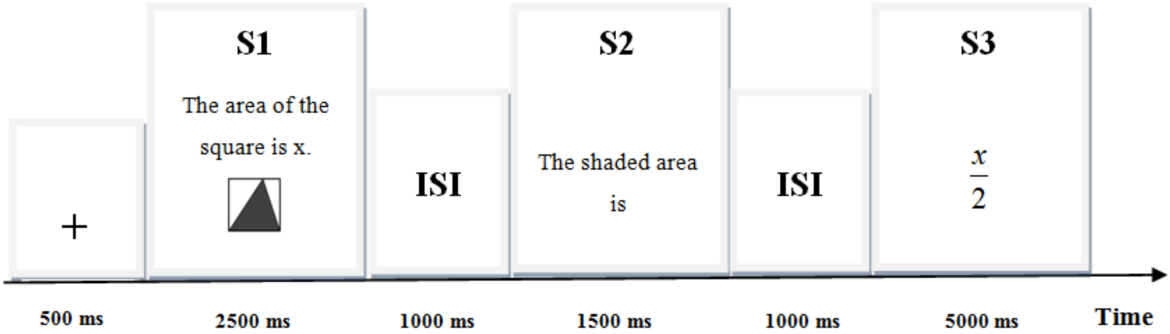
Table 1: Research population

	Gifted (G) IQ>135, Raven >28 of 30	**Non-Gifted (NG)** 100<IQ<130, Raven < 26 of 30	Total
Excelling in mathematics (E) SAT-M >26 of 35 or Math score > 92 in high level mathematics	20	17	37
Non-Excelling in mathematics (NE) SAT-M <21 of 30 or Normal level of mathematics instruction	20	17	37
Total	40	34	74

Stimuli and Procedure

A computerized geometry test was designed using E-Prime software (Schneider, Eschman, & Zuccolotto, 2002). The test included 60 tasks. Each task on each test was presented in three windows with different stimuli (S1 – Introducing a situation stage, S2 – Question presentation stage and S3 – Answer verification stage) that appeared consecutively. Figure 1 presents the sequence of events and example of the task. Time periods were determined by a pilot study with 15 participants.

Participants received a drawing of a geometric object. Part of this drawing was shaded. The participants had to determine what area of the drawing was shaded or what the area of the geometric object was in reference to the shaded part.



S1 – Introducing a situation; S2 – Question presentation; S3 – Answer verification;
+ – Fixation cross; ISI – Inter Stimulus Interval

Figure 1: The sequence of events and a task example

Behavioural data analysis

MANOVA was applied to reaction time for correct responses (RTc) and accuracy (number of correct responses - Acc) of the performance in order to examine effects of between-subjects factors (G factor and E factor).

ERP Recording and Analysis

Scalp EEG data was continuously recorded using a 64 channel BioSemi ActiveTwo system (BioSemi, Amsterdam, ND). [We do not present here technical details of data recording and analysis due to the space constrains of this paper]. ERPs (Event Related Potentials) are electrophysiological measures reflecting changes in electrical activity of the central nervous system related to external stimuli or cognitive processes occurring in the brain. The ERP waveforms were time-locked to the onset of S1, to the onset of S2 and to the onset of S3. The grand average waveforms (average of students waveforms) were calculated for each stage.

Early components (P1 and P2) and late potentials as well as the electrodes and time frames for statistical analysis were chosen based on the preliminary examination of grand average waveforms and ERP topographical maps (electrical voltage distribution) (Table 2).

Table 2: Electrodes chosen for statistical analysis

	Time frame (ms)	Selected electrodes
P1 component	S1, S3: 100 – 180; S2: 100 – 200	P3, Pz, P4, PO3, POz, PO4, O1, Oz, O2
P2 component (for S1 only)	180 – 250	AF3, AFz, AF4, F3, Fz, F4, FC3, FCz, FC4
Late potentials	250 – 500, 500 – 700 and 700 – 900	AF3, AFz, AF4, F3, Fz, F4, FC3, FCz, FC4, P3, Pz, P4, PO3, POz, PO4, O1, Oz, O2

We performed the following statistical analysis related to early components and late potentials.

(1) In order to examine early differences in amplitude and latency of early components associated with perceptual processing of each stage, we conducted for each stage repeated measures MANOVA using Laterality (3 levels: P, PO, O) as within subject factor, G factor (2 levels: G, NG) and E factor (2 levels: E, NE) as between subjects factors. **(2)** In order to examine the mean overall activity we performed repeated measures MANOVA on RMS (i.e., the square root of the mean of the squared potentials from each common referenced electrode) using Time (3 levels: 250-500, 500-700 and 700-900 ms) as within subjects factor, G factor (2 levels: G, NG) and E factor (2 levels: E, NE) as between subjects factors. **(3)** The mean amplitudes were averaged over six electrode sites (ELS) (PR - right posterior (P4, PO4, O2), PM - middle posterior (Pz, POz, Oz), PL - left posterior (P3, PO3, O1), AR - right anterior (AF4, F4, FC4), AM - middle anterior (AFz, Fz, FCz) and AL - left anterior (AF3, F3, FC3)). In order to examine the differences in electrical activity in the aforementioned electrode sites the repeated measures MANOVA was performed on the ERP mean amplitude considering the ELS (6 levels: 6 electrode sites – PL, PM, PR, AL, AM and AR) as within-subject factor, the G factor (2 levels: G, NG) and E factor (2 levels: E, NE) as between-subject factors. The measures were the mean amplitude in three aforementioned time frames. This was done for each stage of task's problem solving.

Pairwise comparisons were used for further investigation. *p*-values were corrected for deviation from sphericity according to Greenhouse Geisser method.

RESULTS

We report here only the main effects and significant interactions.

Behavioral data

Table 3 demonstrates reaction times for correct responses and accuracy (mean and SD) of the performance on geometric tasks found for the four groups of participants.

Table 3: RT for correct responses (RTc), Accuracy in different groups of participants

	Acc Mean (SD) in %			RTc Mean (SD) in ms		
	G	**NG**	**Total**	**G**	**NG**	**Total**
E	82.2 (7.2)	74.8 (9.1)	78.6 (8.9)	1184.8(271.8)	1363.4(414.7)	1271.7(355.5)
NE	81.3 (6.8)	73.3 (8.6)	77.7 (8.5)	1453.4(441.2)	1218.4(382.0)	1348.9(426.9)
Total	81.7 (6.9)	74.1 (8.8)	78.2 (8.7)	1322.5(388.4)	1295.2(400.4)	1309.8(391.5)

Acc – Accuracy; RT – Reaction time; RTc – Reaction time for correct responses

The MANOVA showed main effect for G factor (F (4, 66) = 5.447, p < .001). The follow up ANOVA analysis showed main effect of G factor on accuracy (F (1, 69) = 16.904, p < .001) and significant interaction G factor × E factor involving RTc (F (1, 69) = 5.274, p < .05). Figure 2 demonstrates these effect and interaction.

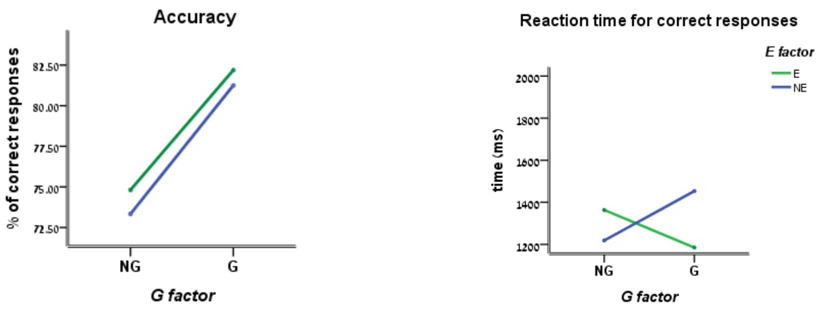


Figure 2: Accuracy and RT for correct responses in the four experimental groups

G participants were significantly more accurate than their NG counterparts. The accuracy of G-NE individuals was similar to the accuracy of G-E and much higher than those of NG-E. However, RTc of G-NE was the highest among four groups and significantly differed from RTc of G-E students (F (1, 69) = 4.783, p <.05).

Electrophysiological scalp data

The grand average waveforms for four experimental groups for each tasks' stage are displayed in Figure 3.

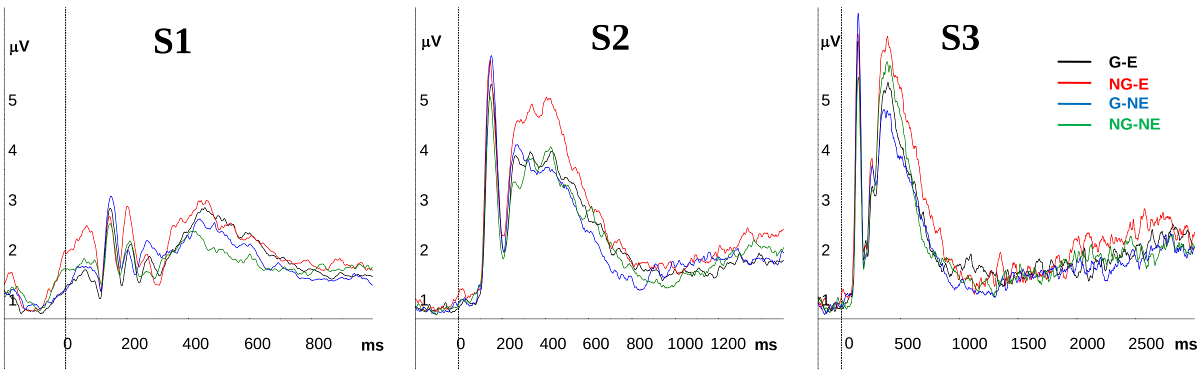


Figure 3: The grand average waveforms at each stage

Observing from the grand average waveforms in Figure 3, the posterior P1 is elicited in each of the three task's stages: S1 – Presentation of a situation, S2 – Question presentation and S3 – Answer verification. Anterior P2 is elicited only at Stage 1.

The significant results on latency and amplitude of P1 and P2 are shown in Table 4.

Table 4: Main effects and significant interactions on latency, amplitude of P1 and P2

Stage	Peak	Latency		Amplitude	
S1	P1	N.S.		Laterality × E factor at P electrodes	F (1.931,135.145) = 4.464*
	P2	Laterality × G factor at AF electrodes	F (1.929,135.038) = 3.464*	Laterality × G factor at F electrodes	F (1.730,121.088) = 4.918*
		G factor × E factor at AF electrodes	F (1,70) = 6.712*	Laterality × E factor at F electrodes	F (1.730,121.088) = 3.750*
				at FC electrodes	F (1.811,126.757) = 3.232*
		at F electrodes	F (1,70) = 5.285*	Laterality × G factor × E factor at AF electrodes	F (1.917,134.170) = 3.266*
S2	P1	Laterality × E factor at O electrodes	F (1.611,112.782) = 3.955*	N.S.	
		G factor at O electrodes	F (1,70) = 6.613*		
S3	P1	N.S.		G factor × E factor at O electrodes	F (1,70) = 4.852*

$^{*}p < .05$ (p-values with Bonferroni correction)

The aforementioned results suggest that different participants groups perform early processing of an object in different ways. These differences were expressed in latency and amplitudes of P1 and P2 as well as in laterality of these peaks. Space constrains of the paper do not allow us explaining all these finding in detail.

Following the P1 and P2 components, we analysed late potential components. Mental processing of the area-related task could be reflected in the late potentials.

Table 5 represents main effects and significant interactions followed by pair-wise comparisons.

Table 5: Main effects, significant interactions and pairwise comparisons for each stage

Stage	**RMS – root mean square**		**ELS – at electrode sites**		
S1	***E factor***	5.373*		250-500ms	10.029**
	500-700 ms	4.838*	***G factor × E factor***	500-700ms	8.621**
	700-900 ms	6.422*			
	for NG: RMS(E) >> RMS(NE)	8.162**	for NG: A(E) >> A(NE)	250-500ms	10.805**
				500-700ms	9.096**
	250-500 ms	4.409*	for NG at PM: A(E) >> A(NE)	250-500ms	4.911*
	500-700 ms	7.481**		500-700ms	8.886**
	700-900 ms	7.037**			
S2	***E factor***	5.298*	*N.S.*		
	for NG: RMS(E) >> RMS(NE)	5.132*			
	250-500 ms	5.093*			
S3	*N.S.*		***G factor***	500-700ms	4.911*

$^{*}p < .05$; $^{**}p < .01$ (p-values with Bonferroni correction); *d.f.* (1, 70)

We found the main effect of E factor at S1 and S2 stages for RMS, which is the measure of overall mean electrical activity. RMS of E was larger than the RMS of NE participants. The further investigation of this effect showed that significant difference between E and NE exists only for NG participants (for S1 stage see Figure 4). Interestingly, E and NE displayed similar RMS activity for G participants.

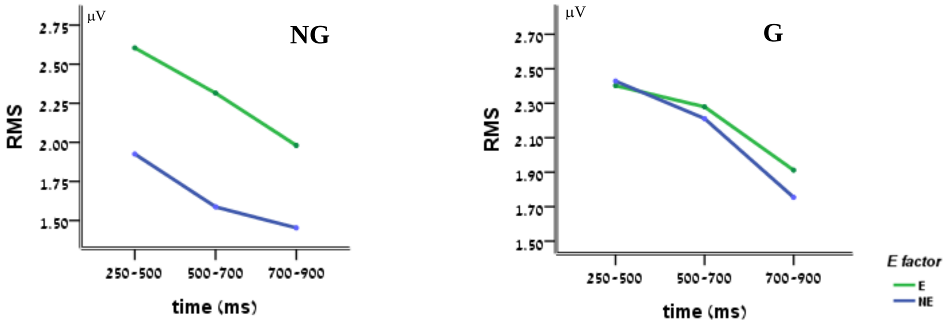


Figure 4: RMS activity at S1 stage for G and NG participants

To investigate more precise topographical distribution of the electric potential, we carried out an examination of the mean amplitude in the electrode sites defined above (AL, AM, AR, PL, PM and PR).

The statistical analysis found significant G × E interaction at S1 stage (250-500 and 500-700 ms time frames). At these sites the mean amplitude of NG-E participants was significantly higher than their NG-NE counterparts. However, the mean amplitude in G-NE students was only slightly different than in the G-E counterparts.

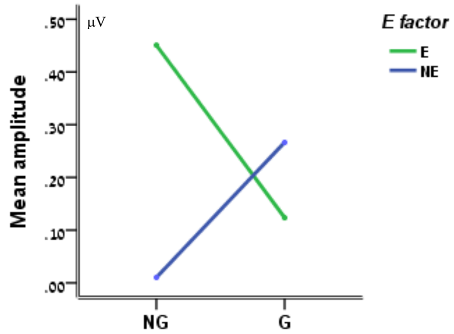


Figure 5: Mean amplitude at S1 stage (500-700ms) for the four groups

The most prominent difference between NG-E and NG-NE was at middle posterior (PM) electrode site (Figure 6).

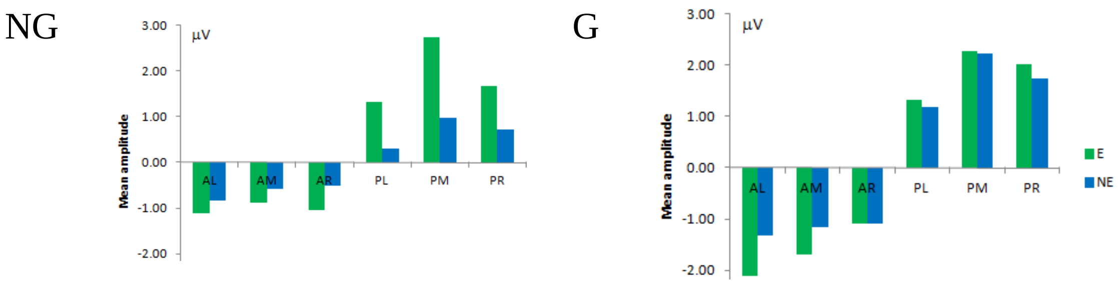


Figure 6: Mean amplitude at S1 stage (500-700ms) for G and NG students at six electrode sites

The statistical analysis revealed main effect of G factor at the S3 stage-answer verification. At 500-700 ms, G participants had lower mean amplitude in the predefined electrode sites compared to their NG counterparts (see Figure 7). Pairwise comparisons revealed marginally significant difference between G and NG participants only among E participants (F (1, 70) = 3.676, *p* = .059).

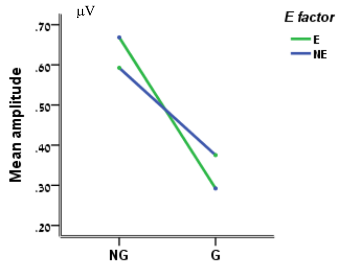


Figure 7: Mean amplitude of students in the four groups at S3 stage (500-700ms).

Figure 8 displays the scalp topography of participants in four groups at 500-700 ms as it is manifested in answer verification stage (S3).

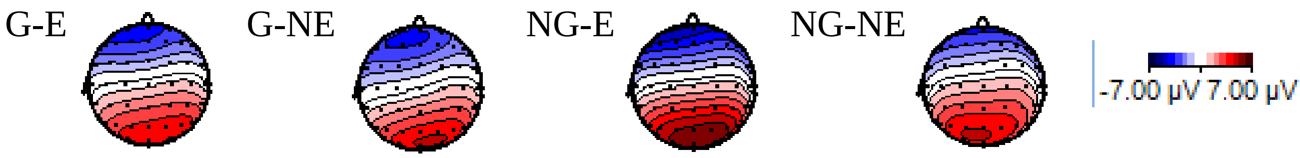


Figure 8: Scalp topography of participants in four experimental groups (500-700 ms).

From Figure 7 and Figure 8 we can notice that G-E participants have the lowest mean amplitude whereas the NG-E have the highest mean amplitude at the backside parts of the scalp. This suggests that G-E invested the least effort in solving area related tasks. On the other hand NG-E struggled to solve this kind of tasks.COnclusions

The present study concerns the differences in brain activity in Gifted versus Non Gifted and Excelling versus Non Excelling male adolescents while performing the area-related tasks involving transition from the mathematical object to its property.

Behavioural data of the study demonstrated that the accuracy of Gifted students was significantly higher than those of Non Gifted. The accuracy of G-NE individuals was similar to the accuracy of G-E but much higher than those of NG-E. Therefore we can conclude that Excellence doesn't affect the accuracy in solving geometry area problems among Gifted individuals. On contrary the RTc of G-NE was the highest among four groups and significantly differed from RTc of G-E. However, G-NE participants were slightly slower but more accurate than NG-E. Therefore, from the behavioural data it can be concluded that Giftedness exerts a strong impact on the performance in the NE participants. Thus Giftedness can compensate the lack of Excellence in mathematics in area-related problems.

Electrophysiological data revealed significant differences between participant groups as it was manifested in latency and amplitude of early components (P1 and P2) and their lateralization. Thus, the analysis of early components suggests that early processing of the mathematical object among participant groups was different.

The analysis of late potentials revealed that Excelling in mathematics participants had higher overall mean activity (RMS) than Non Excelling counterparts. However, further pairwise comparisons detected that this difference was significant only among NG participants. The RMS measure was similar between E and NE among G participants. Similar findings were obtained from analysis of electrical activity at six predefined electrode sites during the S1 stage.. At these sites the mean amplitude of NG-E participants was higher than their NG-NE counterparts. On the other hand, the mean amplitude of G-NE participants was slightly different than their G-E counterparts. The significant difference in mean amplitude between E and NE participants was found only for NG students. This difference was most prominent at middle posterior (PM) electrode site. Therefore we can conclude that G-NE and G-E participants process the stage of introducing a situation (geometric figure with shaded area) and the stage of question presentation in the same way. This leads us to the conclusion that in area-related problems Excellence loses its influence among the Gifted participants.

The electrophysiological results demonstrated that in time period 500-700 ms at answer verification stage (i.e., S3) Gifted students have lower overall mean amplitude for correct responses at six predefined electrode sites. That is, they seem to exhibit more efficient brain activation during this task (e.g; Neubauer & Fink, 2009). The lowest electrical activity at S3 stage was among G-E participants whereas the NG-E had the highest electrical activity. This indicates that that NG-E had invested a lot of cognitive resources in order to verify the given answer.

The present study provides evidence that different Giftedness levels as well as different Excellence in mathematics levels are reflected in the amount of cortical activation and in the behavioral measures. The electrophysiological data can provide a level of measurement and analysis that is difficult to approach by behavioral means. This may be relevant for conducting educational interventions for individuals with different abilities skills and evaluating their effects.

AKNOWLEDGEMENT

This study was made possible through the support of a grant from the John Templeton Foundation and the University of Haifa.

REFERENCES

Clements, D. H., Battista, M. T., Sarama, J., & Swaminsthan, S. (1997). Development of students' spatial thinking in a unit on geometric motions and area. *The Elementary School Journal*, 98, 171 – 186.

Deary, I. J., Penke, L., & Johnson, W. (2010). The neuroscience of human intelligence differences. Nature Reviews Neuroscience, 11, 201–211.

Delazer, M., Domahs, F., Bartha, L., Brenneis C., Lochy, A., Trieb, T., & Benke, T. (2003): Learning complex arithmetic-an fMRI study. *Cognitive Brain Research*, 18, 76–88.

Fischbein, E. (1993). The theory of figural concepts. *Educational studies in mathematics*, 24, 139 – 162.

Heil, M. (2002). The functional significance of ERP effects during mental rotation. *Psychophysiology*, 39, 535–545.

Huang, H-M. E., & Witz, K. G. (2011). Developing children's conceptual understanding of area measurement: A curriculum and teaching experiment. *Learning and Instruction*, 21, 1 – 13.

Jung, R. E., & Haier, R. J. (2007). The parieto-frontal integration theory (P-FIT) of intelligence: converging neuroimaging evidence. *Behavioral and Brain Sciences*, 30, 135–154.

National Council of Teachers of Mathematics, (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Neubauer, A. C., & Fink, A. (2009). Intelligence and neural efficiency: measures of brain activation versus measures of functional connectivity in the brain. *Intelligence*, 37, 223–229.

O'Boyle, M. W. (2008). Mathematically gifted children: Developmental brain characteristics and their prognosis for well-being. *Roeper Review*, 30, 181–186.

Prescott, J., Gavrilescu, M., Cunnington, R., O'Boyle, M. W., & Egan, G. F. (2010). Enhanced brain connectivity in math-gifted adolescents: an fMRI study using mental rotation. Cognitive Neuroscience. 1, 277–288.

Santens, S., Roggeman, C., Fias, W., & Verguts, T. (2010). Number processing pathways in human parietal cortex. *Cerebral Cortex*, 20, 77–88.

Schneider, W., Eschman, A., & Zuccolotto, A. (2002). *E-prime computer software (Version 1.0)*. Pittsburgh, PA: Psychology Software Tools.

Sohn, M.-H., Goode, A., Koedinger, K. R., Stenger, V. A, Carter, C. S., & Anderson, J. R. (2004). Behavioral equivalence does not necessarily imply neural equivalence: Evidence in mathematical problem solving. *Nature Neuroscience*, 7, 1193-1194.