TEACHING HIGHLY ABLE STUDENTS IN A COMMON CLASS: CHALLENGES AND LIMITS OF A CASE-STUDY

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Abstract. The case-study presented in this paper is built around a middle school teacher's beliefs, a professional who teaches in a mixed ability class. We focused on identifying links between self-efficacy beliefs, content knowledge, classroom practices and mathematical beliefs. The data come from a semi-structured interview, group discussions, and individual work on a problem-posing task. The results revealed a mismatch between the teacher's self-efficacy beliefs and his specialized content knowledge. The teacher was found to posses most of the expert features except theory and schema change – a capacity to change mental frames due to an inquiry process. This finding might explain both his success in training high ability students and failure to address adequate tasks for orchestrating students' understanding.

Keywords: Self-efficacy beliefs; Specialized content knowledge; Mental frames; Mixed ability class; Expert thinking

INTRODUCTION

Teacher instructional practices are strongly influenced by their beliefs. On the one hand, we consider beliefs about mathematics, its teaching and learning and, on the other hand, affects and beliefs regarding personal efficacy. The study of beliefs continues to represent a strong line of research, as many working groups and publications can witness (for example, Maasz & Schloeglmann, 2006). It has been shown that teachers who regard mathematics as a static body of knowledge, with procedures and rules to apply, also consider, as goal of mathematics learning, the skillful application of these and will focus much less on aspects of understanding the origin of the procedures or that of the concepts (Thompson, 1992). This "traditional view" of mathematics will manifest in classroom practices as the direct transmission of the rules, solving problems as illustrations to the rule and practicing them on a new, but structurally identical problem (Stigler & Hiebert, 1997). Such findings of strong interdependence between beliefs and classroom practices were confirmed by more recent studies, too (for example, Stipek, Givvin, Salmon, & MacGyvers, 2001).

Teacher self-efficacy belief has been defined as teacher's belief in his/her abilities to organize and execute courses of action to bring about desired results (Tschannen-Moran, Woolfolk-Hoy, & Hoy, 1998) and has been related to student motivation, achievement, student's self-efficacy beliefs and, also, to teacher's classroom practices (for a review, see Tschannen-Moran and Woolfolk-Hoy, 2001). Teachers with high self-efficacy beliefs might think of themselves as highly effective teachers – a thought that, in turn, influences their need to reflect and adjust their classroom practices. A way to inspect the concordance of such belief with reality is to focus on teachers' specialized content knowledge (SCK) as part of the mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). The construct of mathematical knowledge for teaching refers to the knowledge teachers need in their practice. It has several components: common content knowledge, specialized content knowledge, knowledge of content and teaching, knowledge of content and students, and knowledge of curriculum. SCK represents a body of mathematical knowledge beyond the one taught to students and different from mathematical knowledge used by research mathematicians or applied in professions where mathematics is a tool. It is mathematical knowledge that is specifically needed in teaching. It is needed in carrying out tasks such as: "recognizing what is involved in a particular representation, finding an example to make a mathematical point, modifying tasks to be either easier or harder" (Ball, Thames, & Phelps, 2008).

The interplay between beliefs construct, the mathematical knowledge for teaching and the classroom practices, in all its complexity, has not been studied, as far as we know. However, a huge body of research indicates that the interactions are not one-way, simple causal relationships, but rather they are characterized by continuous feedbacks, often pulling in opposite directions.

In our article we look at the case of a middle-school teacher, habituated in training high-ability students in regular settings. Our interest is on identifying links between his self-efficacy beliefs, his SCK, his classroom practices and mathematical beliefs, both on concordant and discordant cords, and to look at the ways in which his practices might impact the students' learning and understanding of mathematics.

METHOD

Design and procedure

The paper presents a case study. The data was collected through participant observation, semi-structured interview and documentation (as defined by Yin, 1994). In the following we describe the circumstances of data collection.

The authors organized an interactive seminar, of 3 days, around the topic of problem posing. In all, 43 mathematics teachers, at different grade levels, participated in that seminar. The seminar proceeded as follows:

In an initial session, teachers were shown several examples of problems and a discussion was initiated about the formulation of the problem. A qualitative analysis was undertaken and techniques for modification/posing of problems were presented.

During the next phase of the seminar, the participants had the task to pose, individually or in groups of 2-3 people, multiple-choice problems. A special request was imposed on defining the distracters: they should reflect, as much as possible, typical mistakes – including erroneous strategies, misconceptions, computing errors etc. Each posed problem had to include an indication/hint for its solving and a brief comment addressed to the student regarding the choice of each of the distracters.

On the last day of the seminar, every participant presented a problem, posed by the person or selected from those posed by his/her group. These problems were analyzed and critically evaluated during interactive discussions between all the participants at the seminar. Several criteria were taken into consideration: the level of difficulty, the quality of formulation, solution hints, etc.

The present study idea came from the experiences within this seminar. One of the participants (referred from now on as Mr. T.) got remarked for his perspective on mathematical knowledge and teaching strategies. We concentrated on his case: we analyzed his interventions during the group discussions, we invited him to an interview, and we documented on his personal history as a teacher.

Intervention during the seminar

We start by listing below the problem that Mr. T. presented for the seminar discussions, along with his given solution. This problem was selected by Mr. T. as being the most relevant for the posed problem activity of him and his team. Some other of his interventions will be presented during the discussion section.

Problem 1. *The number of integer solutions for the following equation* $\big|x - |3x-2|\big| = 8$ *is:*

A) 1; B) 2; C) 3; D) 4; E) 0.

We include the idea of the solution of this problem as Mr. T. presented it:

Equation $\big|x - |3x-2|\big| = 8$ *is equivalent with* $x - |3x-2| = \pm 8$.

$$x - |3x-2| = +8 \Rightarrow |3x-2| = x-8 \geq 0 \begin{cases} 3x-2 = x-8 \Rightarrow x = -3 \in \boldsymbol{Z}, but -3 < 8 \\ 3x-2 = -x+8 \Rightarrow x = \frac{5}{2} \notin \boldsymbol{Z} \end{cases}$$

$$x - |3x-2| = -8 \Rightarrow |3x-2| = x+8 \geq 0 \begin{cases} 3x-2 = x+8 \Rightarrow x = 5 \in \boldsymbol{Z}, and\ 5 \geq -8 \\ 3x-2 = -x-8 \Rightarrow x = \frac{-3}{2} \notin \boldsymbol{Z} \end{cases}$$

In conclusion: x = 5.

Interview

The starting point in the interview was the following problem:

Problem 2. In the sequence 1, 4, 7, 10, 13, ... the numbers are counted by three's, meanwhile in the sequence 4, 9, 14, 19, ... the numbers are counted by fives. Which are the first four common terms in the two sequences?

The interviewers firstly addressed questions related to teaching: For what grade would you consider this problem? For what chapter /content would it be appropriate? What goal could it serve? What helping/bootstrapping questions would you address to your students? He was also asked to modify Problem 2 and, then, to explain the criterion used in modification.

The interview was coded using codes to refer to different beliefs: a. What is a “good class”; b. Who is a “good student” (how should he/she behave, what should he/she do); c. What does it mean “solving a math problem”; d. What alternate ways of solving might be; e. How should a solution be communicated and noted down; f. PP techniques; g. Criteria for judging the posed problem; h. Explicitly state the purpose of problem modifications actions. We considered the codes a-e as relevant to the teacher’s mathematical beliefs, meanwhile the codes f-h as being relevant to his SCK.

Documentation

Given our interest in his personal history as a teacher, we collected information from a diversity of sources: mass-media; autobiographic file, and discussions with a school district responsible who supervises Mr. T.

Mr. T. is a middle school teacher, from a small town in Romania. He has been teaching for 33 years. Lately, one of his former students (now college student) had exceptional results at national and international contests, being a medalist at the International Mathematical Olympiad (IMO) and at Balkan Mathematical Olympiad (BMO). The student asserted, in interviews given to mass-media that his middle school teacher had an important role in his formation as mathematician.

Mr. T. considers himself a successful teacher in preparing students for mathematical competitions, as he revealed it in the interview. He said about his student: „I’m helping him to understand mathematics. He is a really good student.”

RESULTS AND DISCUSSION

We focused on the following research questions: Is there any relationship between high personal efficacy belief and the specialized content knowledge (SCK)? In what ways mathematical beliefs and personal efficacy beliefs relate to classroom practices? To answer these questions, we analyze the data gathered from Mr. T’s interventions during the seminar, the interview, and other sources of documentation.

Beliefs on teaching and learning

During the interview, we asked Mr. T. about the level of difficulty he would associate to problem 2 if using it in class. Mr. T. insisted that this problem has an above average level of difficulty, proper for classes with highly able students and earliest possible to be tackled is in grade 5. He justified this assertion as follows:

> I consider it is an above average difficulty problem, because it asks the student to write a general expression of a term…because, like this, step by step, you have given 4 elements, but I could give 444 [*common elements*].

For Mr. T., a problem needs to hide a challenge, and in this case, the problem is seen as consistent only if we take into account its potential for generalization. By having this perspective, he thinks from the very beginning to "take away" the intuitive support. Mr. T. gave instantly an algebraic solution. Indeed, in the presence of an algebraic solution, the numerical exploration of the problem might become uninteresting. The question is: does exploration have value for children's learning? As expressed in the interview, Mr. T. thinks that such exploratory initiative should be reprised in students. As such, at our question about the possibility that some students might first observe the regularity in the succession of common terms and then extrapolate that in order to obtain the 444 common terms, Mr. T. made a refusal gesture and said:

> We don't tell stories at mathematics. The student must have the capacity for synthesis, to write just what is strictly needed, nothing more. Along with the required arguments!

Mr. T. perceives the above problem as difficult because the only solution method accepted by him is the formal, algebraic solution. Consequently, for his classroom practice, essential is to transmit the mechanisms of formal solutions, which, it is, of course, difficult, for 5^{th} graders. By his attitude of refusal of explorations, Mr. T. gives the impression that he focuses only on the high performing students, ignoring the others. Moreover, this exclusion seems conceptually automatized in him: he was unable to come up spontaneously with a simpler problem of the same type even at the request of the interviewers.

Returning to problem 1, proposed by Mr. T., we assume that his goal was to force a case-analysis on the student's behalf to identify possible variants and exclude the impossible ones. It seems that Mr. T. defined the numbers in the problem carefully, so that the solution phases allow highlighting possible errors.

The solution algorithm given by him is, of course, correct, but the learning of the algorithm is not necessarily leading to the understanding of the reasons for the unique solution. From a didactical point of view, the solving might not be a proper one. From mathematical perspective, the exercise can be identified as one of finding the solution of a polynomial equation. In such problems, establishing the domain of the involved functions is essential and conditions the solution of the equations.

We observe that Mr. T. evaluates the problem based on an algorithm that is correct from mathematical point of view, but looses the profound justification for which certain conditions appear. This justification is linked, in fact, to the understanding and explicit use of the concept of function.

During the seminar discussion, it was suggested to solve the problem through a graphical representation, which would better reflect the involved mathematical phenomena (see Figure 1). Moreover, if the representation is done on a grid, the integer solutions could be visualized. Such visualization would help to avoid errors that could occur during a formal solution.

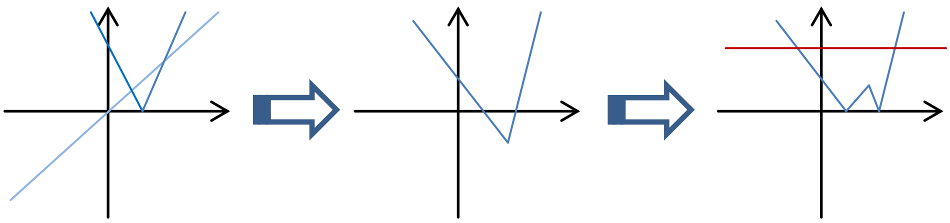

Fig. 1: A graphical solution of the equation proposed by Mr. T.: We compare x to $|3x-2|$, draw the graph of $x \to x - |3x - 2|$, then visualize the equation $|x - |3x - 2|| = 8$.

Mr. T. demonstrated no interest for the graphical solving: his answer was that the other approach (algebraic, presented by him) is more interesting. We hypothesize that his reaction was determined by the fact that a graphical solution would eliminate the traps hidden in the "algebraic" solution.

Specialized content knowledge

Further, we analyze the solutions proposed by Mr. T. to problems 1 and 2 in more detail, in order to reveal his underlying knowledge and understanding. For the first problem, the solution consists of: an algorithm of repeated steps in which the critical elements are: checking the non-negativity conditions and checking if the partial results belong to the set requested in the problem. We compared this solution with the ones, given to the same problem, by five mathematicians, teaching at the university level. We noticed that the ideas mathematicians used for solving problem 1 were the same as Mr. T. used, eventually containing some not-so important shortcuts. We can assert that in solving problem 1, Mr. T. behaves as an expert mathematician.

In solving problem 2, Mr. T. writes the generic terms of the two arithmetical progressions, and then he arrives to a Diophantine equation and identifies the set of solutions. It is clear that Mr. T. knows the general solution mechanism of Diophantine equations and he can particularize that one for the current problem. Therefore, when he is asked to formulate a more difficult problem, he modifies the question by asking for 10 common terms of the given sequences that, besides being common terms, are also divisible by 13. He asserts that the solution strategy will not change, because we will still arrive to a Diophantine equation. Again, his behavior exhibits a level of mathematics expertise. According to Glaser (1997), an expert possesses: structured, principled knowledge (the expert rapidly accesses the underlying meaningful patterns) and principles stored in coherent chunks of information and proceduralized and goal-oriented knowledge **(**recalling a principle or a rule and efficient implementation of it). In addition, an expert makes use of an effective problem representation, which allows a qualitative assessment of the nature of the problem and the development of a mental model that reduces and organizes the problem space. Relevant is also automaticity to reduce attention demands. Another dimension of expert behavior refers to self-regulatory skills (Glaser, 1988, 1997). Experts develop metacognitive skills that control their performance in particular areas of knowledge, they learn to monitor their problem solving by predicting the difficulty of problems, allocating time appropriately, noting their errors or failures to comprehend, and checking questionable solutions.

The solution strategies of Mr. T. for the presented problems bring evidence for the above features identified by Glaser for an expert. Mr. T. spontaneously framed the problems into more general classes. This assertion is reinforced by one of his seminar interventions we present bellow.

During the initial session of the seminar, the following problem, given at the admission exam to the Faculty of Mathematics, was analyzed:

> Let $G = \{(a, b) \mid a, b \in \mathbf{R}, a \neq 0\}$, and the algebraic operation defined by $(a, b) * (c, d) = (ac, ad + b)$. Show that G is a group that is non-abelian.

Some of the students enrolled to this exam proved that the operation * is not commutative, without verifying that (G,*) was indeed a group, and consequently have lost most of the score for this subject. The majority of the participants at the seminar agreed that the solver’s omission was generated by the use of the word “that”, which, in this case, filters the correct decoding of the problem. Several modified formulations were proposed, considered as more accessible/ recognizable by students, like: “G is a non-abelian group”; or in two steps: a) “G is a group; b) G is not an abelian group”. Mr. T. had a different opinion. He sustained that the wording is mathematically correct and that the discussions about reformulation are a waste of time, since a “serious” student, in the context of such problem, will automatically prove both.

Mr. T. has the conviction that a student who created routines of solving problems of such class (verification of the properties of an operation) will prove first that the structure exists and, then, will prove certain properties of it. Such a student will be not deviated by the preposition used in the problem text: he will correctly solve even those problems with fuzzy formulations, because he knows that type of problem. In other words, he expects students to behave also as experts without considering the process for becoming so – a quality indispensable for classroom teaching.

Creativity training versus competition training

Mr. T. is, in certain interpretation, an efficient and successful teacher, if we focus on some of his students’ performance at competitions. We saw, in the above discussion, that Mr. T. focuses his classroom practices to promote a formal understanding of mathematical concepts, refusing the idea of explorations /investigations. The refusal of a gradual approach is expressed by a classroom practice strongly related to his views on mathematics and knowledge of mathematics. His inability to simplify a problem and use it as starting point for learning illustrates that his SCK is at imbalance with his self-efficacy beliefs.

From his conceptual thought it seems to be missing what Glaser calls *theory and schema change* (Glaser, 1988) – that is: a dimension of expert thinking that reflects a process of interrogation, confrontation, conflict, and discovery. Mr. T. is reluctant to explorations in problem solving but also in changing his personal approaches of various issues. Therefore, we ask, what explains then his success in the creativity and/ or competition training? In order to answer, we analyze the manner in which Mr. T. changes a given problem. For example, at the request to modify problem 2, he proposed the following:

> I could keep one of the sequences, and for the second we consider one with a bigger ratio. I'm thinking to still ask for the first 4 common terms or the first 10, to extend them more and to force them to…for example, 2, 22, 42, 62 …[with a ratio of 20].

Mr. T. keeps one sequence unchanged. With regard to the problem question, for him, requesting the first 4 common terms, or the first 10 is the same thing because, due to the low number of terms, the students could still proceed by trial and error. The fact that he chooses the second sequence with a higher ratio has the purpose to farther the terms from the origin of the axis such that the numbers with which the students should work be larger. Such situation would force the student to take an algebraic approach and (eventually) to accept the necessity of an algorithm in solving the problem. Moreover, Mr. T. chooses the ratio as 20 (multiple of 5). Why 10 (number of terms) and 20 (ratio)? We suppose that he is governed, subconsciously, by a certain mathematical aesthetic, a sense of beauty that rises from the fundamental nature of the base 10 that leads to "beautiful", "elegant", "round" numbers.

In a previous study (Voica & Singer, 2012), we discussed the relationship between problem posing and mathematical creativity in terms of cognitive flexibility. We concluded that, in problem posing, high performing students express a functional type strategy – they vary one single element of the starting problem, to control the quality of the newly obtained statements. Mr. T. proceeds similarly, because he keeps unmodified a part of the data and varies a single parameter – the ratio of the second sequence. It is possible that this "mono-dimensional" cognitive flexibility of the teacher, similar to the one of high performing students, allows him to do an efficient training of creative students. This behavior properly fits a way of thinking that focuses on efficiency in problem solving, an aspect requested within mathematics Olympiads.

CONCLUSIONS

Mr. T. has strong positive self-efficacy beliefs. He proves to have a mathematical content knowledge similar to an expert mathematician; however his SCK mismatches his perception of efficacy. As solver and trainer for International Olympiad style competitions, Mr. T. is interested in efficiency. From this point of view, Mr. T.'s objective is the acquisition of solving algorithms for classes of problems. More complex these are, more are the chances that the student turns into a highly performing one. However, from a teaching perspective, a practice focused on algorithms leads to a failure on profound understanding of mathematical ideas and concepts. The algorithms targeted by Mr. T. are of high level of abstraction and complexity. He seems to have "cut off" SCK and kept only those mathematically "delicate areas" that he teaches in a systematic manner to highly performing students. These students are trained, for example, in avoiding "traps" such as the ones concerning checking initial constraints over the data.

However, a fundamental problem might be his "traditional" view of mathematics: mathematics is a set of formalized knowledge where performance is essential. This view explains his belief that knowing a complex algorithm means knowing mathematics – there is nothing more to look into there. Such beliefs lead to specific classroom practices, where the children should be the ones to "keep the rhythm" (as some performing students do). To be a good student means, for him, to work a lot after you have been taught the algorithm. However, such an approach can have a negative impact on students in a mixed ability class and, maybe, even on high performing children.

Problematic issues derive from the fact that he has no way of knowing if the students really "understood" (because the question does not address this target) and if the students really develop their own creativity (because the lack of opportunities for exploration could lead to inhibition and misunderstandings). He perceives learning as a product, not as a process: Mr. T. does not accept that a child can be coached towards the solution with helping hints that deepen understanding – it is a one-chance approach for him: "either you get it from the first moment, or you don't". Such view can have a counteractive impact on students' self-efficacy perceptions, as the feedback they receive is a negative one. Besides the existence, in general, of a traditional view of mathematics, the source of such beliefs can be linked to the vision the general public (including parents and students) share about what it means to be good at mathematics: for most, the only criteria is the performance at competitions.

In the same time, high-ability students can have additional sources of information or interest in individual learning: in general they receive training from multiple sources. They can overcome certain deficiencies in understanding by being exposed to diverse training practices. Even the former student of Mr. T. admitted that, besides his middle-school teacher, he had also other trainers, including parents.

Contrary to the case of the students with high potential and interest in mathematical performance, the other students are usually ignored by such teacher. Breaking the vicious circle asks for much more than highlighting ways to increase teacher's SCK; there is a need to change teachers' mentalities and beliefs. Beyond Olympic preparation, all the students, gifted or less gifted, should be prepared for a successful professional life and optimum social inclusion; in today's dynamic world, training on problems typologies and complex algorithms is far from being enough.

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