THE CONNECTION BETWEEN MATHEMATICAL CREATIVITY AND HIGH ABILITY IN MATHEMATICS

Miri Lev and Roza Leikin

University of Haifa

This paper presents some findings from a large-scale study that explores the relationship between mathematical creativity, mathematical expertise and general giftedness, which is not obvious. We distinguish between relative and absolute creativity in order to evaluate mathematical creativity in school children. This paper demonstrates that general giftedness and excellence in mathematics has main effect on secondary students' creativity associated with production of multiple solutions to mathematical problems. However these effects are task-dependent. Thus, we conclude that different types of MSTs can be used for different research purposes.

Key words: Mathematical creativity, Multiple Solution Tasks (MST), General Giftedness, High Level of Mathematical Instruction.

RATIONALE

This study employs multiple solution tasks in order to explore students' creativity in mathematics. It continues series of studies that we directed at design and validation of the research tool used herein. (Leikin and Lev 2007, Leikin 2009, Leikin & Lev accepted, Levav-Waynberg and Leikin 2012, Guberman and Leikin 2012). The earlier studies also examined the relationship between mathematical creativity and the level of mathematical ability of the participants. All the studies led to several hypotheses that we are currently examining in this large-scale study: We hypothesized that (1) between-group differences are task dependent and (2) in the originality-fluency-flexibility triad, fluency and flexibility are of a dynamic nature, whereas originality is of the "gift" type.

BACKGROUND

Creativity

There is no single, authoritative perspective or definition of creativity (Mann 2006). Our study follows Torrance's (1974) definition of creativity with four components: *Fluency* refers to the continuity of ideas; *Flexibility* is associated with changing ideas; *Novelty* is characterized by a unique way of thinking; *Elaboration* refers to the ability to generalize ideas. Of these four components, novelty or originality is widely acknowledged because creativity is viewed as a process having to do with the generation of original ideas.

While drawing a connection between high abilities and creativity, researchers express a diversity of views. Some claim that creativity is a specific type of giftedness (e.g., Sternberg 2005), others feel that creativity is an essential component of giftedness (Renzulli 1978), and still other researchers suggest that they are two independent characteristics of human beings (Milgram and Hong 2009). Thus, analysis of the relationship between creativity and giftedness, with a specific focus on the various fields of mathematics, is important for better understanding of the nature of both mathematical giftedness and mathematical creativity.

Mathematical creativity

One of the complexities related to the relationship between mathematical giftedness and mathematical creativity is rooted in the contrast between viewing mathematical creativity as a property of the professional mathematician's mind (Ervynck 1991) and the opinion that mathematical creativity can and must be developed in all students (Sheffield 2009). Naturally, creativity in school mathematics differs from that of professional mathematicians. Mathematical creativity in school students is evaluated with reference to their previous experiences and to the performance of other students who have a similar educational history. Leikin (2009) suggested that viewing personal creativity as a characteristic that can be developed in schoolchildren requires a distinction between *relative* and *absolute* creativity. *Absolute creativity* is associated with discoveries at a global level. Our work deals with *relative creativity* which refers to mathematical creativity exhibited by school students when evaluated in relation to their previous experiences and to the performance of other students who have similar educational histories. The current study accepts relative perspective on creativity while evaluating originality of students' solutions.

THE MODEL FOR EVALUATION OF MATHEMATICAL CREATIVITY

Multiple solution tasks

A multiple solution task (MST) is an assignment in which a student is explicitly required to solve a mathematical problem in different ways. Solutions to the same problem are considered to be different if they are based on: (a) different representations of some mathematical concepts involved in the task, (b) different properties (definitions or theorems) of mathematical objects within a particular field, (c) different properties of a mathematical object in different fields.

Table 1 demonstrates an example of a multiple solution task (Jam problem) and depicts 10 different solutions to the problem.

Table 1: Jam problem-Multiple solution task

Jam Problem

Mali produces strawberry jam for several food shops. She uses big jars to deliver the jam to the shops. One time she distributed 80 litters of jam equally among the jars. She decided to save 4 jars and to distribute jam from these jars equally among the other jars. She realized that she had added exactly 1/4 of the previous amount to each of the jars. How many jars did she prepare at the start?

Group of solutions	Description			
A	System of equations -1	$\begin{cases} xy = 80 \\ 1.25x(y-4) = 80 \end{cases}$	System of equations -2	$\frac{x}{y} \Rightarrow \frac{x}{y-4} = \frac{5x}{4y}$
B	Equation -1 $\frac{4}{x} = \frac{1}{4}$	Equation -2 $\frac{4}{x-4} = \frac{1}{4}$	Equation -3	$1.25x = x + 4$
D	Equation in 2 variables	$4x = \frac{1}{4}x(y-4)$		
E	Diagram			
C	Insight: Fractions/ Percents	1/4 of initial amount is 1/5 of the new amount. 4 jars are 1/5 of all jars, thus there were 20 jars at the start.		
F	Insight Solution	4 jars equals one quarter of the remaining amount of jam → 20 jars in total		
G	Insight Solution	Jam from each of the 4 jars was distributed among 4 jars – overall all the jam from 4 jars went into 16 jars. Thus there are 20 jars in total		

The scoring scheme

The evaluation model was first introduced in Leikin (2009) and then employed in Levav-Waynberg and Leikin (2012) and in Guberman and Leikin (2012).

Table 2: Scoring scheme for evaluation of creativity (based on Leikin 2009)

	Fluency	Flexibility	Originality	Creativity
Scores per solution	1	$Flx_1 = 10$ - for the first solution $Flx_i = 10$ - solutions from a different group of strategies $Flx_i = 1$ - similar strategy but a different representation $Flx_i = 0.1$ - the same strategy, the same representation	$Or_i = 10$ $P < 15\%$ or for insight/ unconventional solution $Or_i = 1$ $15\% \le P < 40\%$ or for model-based/ partly unconventional solution $Or_i = 0.1$ $P \ge 40\%$ or for algorithm-based/ conventional solution	$Cr_i = Flx_i \times Or_i$
Total score	$Flu = n$	$Flx = \sum_{i=1}^{n} Flx_i$	$Or = \sum_{i=1}^{n} Or_i$	$Cr = \sum_{i=1}^{n} Flx_i \times Or_i$

n is the total number of appropriate solutions

$P = (m_j / n) \cdot 100\%$ where m_j *is the number of students who used strategy* j

Fluency (Flu) refers to the pace at which solving proceeds and the switches taking place between different solutions.

To evaluate *flexibility* (Flx), we established groups of solutions for the MSTs. Flexibility embedded in a problem is evaluated according to expert solution space. Two solutions belong to separate groups if they employ solution strategies based on different representations, properties or branches of mathematics.

Originality is evaluated by comparing individual solution spaces with the collective solution space of the reference group.

In the decimal basis we used in scoring, the total score indicates the originality and flexibility of the solutions in the individual solution space of a participant. For example, if the total flexibility score for a solution space is 21.3, we know that it includes 2 solutions that belong to different solution groups (based on different solution strategies), 1 solution that uses a solution strategy similar to a former solution but differs in some essential characteristics, and 3 solutions that repeat previous ones.

The *creativity* (Cr) of a particular solution is the product of the solution's originality and flexibility: $Cr_i = Flx_i \times Or_i$. The use of the product of flexibility and originality scores enables evaluation of the most creative solutions, with the highest score ($Cr_k = 100$) given for a flexible and original solution. This also addresses the fact that previously performed solutions cannot be considered as creative. The total creativity score on an MST is the sum of the creativity scores on each solution in the individual solution space of a problem: $Cr = \sum_{i=1}^{n} Flx_i \times Or_i$.

The model for evaluation of creativity applied with a particular set of MSTs constitutes the research instrument in this study.

THE STUDY

Research goals

There are two main interrelated goals in this study:

(1) To examine relationships between mathematical creativity, general giftedness, and mathematical excellence.
(2) To explore the power of different types of MSTs for the identification of between-group differences related to mathematical creativity as reflected in multiple solutions produced by the students.

The test

The problems included in the test differed with respect to:
(1) *Mathematics topic* to which the problem belongs in the school curriculum
(2) *Complexity*

(3) *Conventionality* of the problem and conventionality of the solutions, requiring insight in order to produce the solutions (following Ervynk 1991).

The test consisted of five problems (Leikin & Lev, accepted). We focus here on 2 of these problems (see Table 3) to discuss task dependency of the effects of G and EM factors on mathematical creativity (see the Population paragraph).

Correctness of the solution for a problem was evaluated according to the complete solution produced by the student to the problem. For a complete solution a student received 25 points. Creativity components were evaluated according to the scoring scheme (Table 2).

Table 3: Two problems in the test

Topic	Problem
Word problems 	**Jam problem:** Mali produces strawberry jam for several food shops. She uses big jars to deliver the jam to the shops. One time she distributed 80 litters of jam equally among the jars. She decided to save 4 jars and to distribute jam from these jars equally among the other jars. She realized that she had added exactly 1/4 of the previous amount to each of the jars. How many jars did she prepare at the start?
System of equations	$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$

Population

A sample of 191 students was chosen out of a population of 1200 10th -11th grade students (16-17 years old). The sampling procedure was directed towards investigating the effect of G and EM factors (see Table 4).

G factor: Students for G groups were mainly chosen from classes for gifted students (IQ>130). Additionally, the entire research population was examined using Raven's Advanced Progressive Matrix Test (RPMT) (Raven, Raven & Court, 2000) (see Table 4 for the sampling criteria).

EM factor: All 1200 students studied mathematics at high and regular levels (HL, RL). The level of instruction is determined by students' mathematical achievements in earlier grades. Instruction at HL differ from that at RL in terms of the depth of the learning material and the complexity of the mathematical problem-solving involved. Additionally, excellence in mathematics is examined using the SAT-M (Scholastic Assessment Test in Mathematics, adopted from Koichu, 2003). (see Table 4 for the sampling criteria).

After completion of this stage, 191 of the initial 1200 students were subdivided into four experimental groups, determining the research population according to varying combinations of the EM and G factors as presented in Table 4.

The fifth group of students - SG (super gifted) students included G-EM students who were members of mathematical Olympiads team or study mathematics in the university while learning in school. These students received recommendations from research mathematicians familiar with their achievements.

Table 4: Target population

	Gifted (G) IQ>130 Raven > 27/30	**Non-Gifted (NG)** Raven < 26/30	**Super Gifted (SG)**	**Total**
Excelling in math (EM) SAT-M >26 or HL in mathematics with math score > 92	G-EM N=38	NG-EM N=51	SG N=7	96
Non-excelling in math (NEM) SAT-M <22 and RL in mathematics with math score > 90 or HL in mathematics with math score < 80.	G-NEM N= 38	NG-NEM N=57		95
Total	76	108	7	191

Data analysis

A multivariate analysis of variance tests (MANOVAs) were used to compare the scores on each component of creativity that participants received in each problem.

Between-subjects differences were examined for each one of the problems and each one of the creativity components for G factor, EM factor and interactions between G and EM factors.

Within-subjects differences are examined for the performance on the different tasks.

FINDINGS

As mentioned earlier we present here findings related to the two problems. These two problems are regular curriculum-related problems which has unconventional (insight-based) solutions.

Table 5 presents percentage of students with different levels of fluency (the number of overall solutions produced by a students) and flexibility (the number of different solutions produced by a students). We learn from this data that students in all the groups were more successful, fluent and flexible in solving the system of equation. This may be due to the fact that in contrast to the system of equations, word problems similar to Jam problem are seldom for mathematics lessons in 10^{th}-11^{th} grades. From Table 5 we learn that though there is connection between fluency and flexibility in students problem solving performance they measure different mental ability. Production of multiple solutions does not mean production of different multiple solutions. Clearly students from G-EM group (including SG students) differed meaningfully in their flexibility when solving both problems. Statistical analysis – comparisons of column means – (that we do not present here due to space limitations) supports this observation: participants from the G-EM group differ from participants of all other groups in fluency and flexibility of their problem solving performance.

Table 5: Fluency and Flexibility

%		Jam problem						System of equations						
No. of solutions (Flu) / No. of groups of solutions (Flx)		0	1	2	3	4	6	0	1	2	3	4	5	6
G-EM (N=38)	Flu	5.3	16	**24**	**55**		**2.6**	0	0	**7.9**	**71**	**21**		
	Flx	5.3	66	**29**				0	61	**26**	**13**			
G-NEM (N=38)	Flu	29	32	**13**	**24**	**2.6**		5.3	0	**5.3**	**68**	**16**	**2.6**	**2.6**
	Flx	29	68	**2.6**				5.3	82	**13**				
NG-EM (N=51)	Flu	27	24	**22**	**27**			2	2	**9.8**	**78**	**7.8**		
	Flx	27	67	**5.9**				2	94	**3.9**				
NG-NEM (N=57)	Flu	51	21	**19**	**8.8**			1.8	14	**14**	**61**	**7**	**1.8**	
	Flx	51	44	**5.3**				1.8	91	**7**				
SG (N=7)	Flu			**14**	**86**						**43**	**57**		
	Flx		57	**14**	**29**					**29**	**57**	**14**		

Table 6 that presents Means and SD that we obtained for all the examined criteria on both problems provides additional support for the observation of the specific qualities of mathematical reasoning in G-EM students.

Table 6: Means and SD

		G-EM		G-NEM		NG-EM		NG-NEM	
	N	38		38		51		57	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
	Cor	23.68	5.657	16.58	11.455	18.14	11.268	11.4	12.165
	Flu	2.29	0.927	1.39	1.22	1.49	1.173	0.825	0.984
	Flx	12.95	5.489	7.67	5.218	8.09	5.527	5.593	6.128
	Or	1.43	3.061	0.803	2.229	0.625	1.948	0.54	1.859
	Cr	13.37	30.734	6.759	22.43	5.574	19.59	5.138	18.64
$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$	Cor	25	0	23.68	5.657	24.51	3.501	23.77	4.359
	Flu	3.13	0.529	3.13	1.212	2.88	0.653	2.63	0.938
	Flx	16.42	6.948	12.53	4.759	11.51	2.575	11.73	3.232
	Or	3.18	5.024	1.66	3.391	0.68	1.944	0.989	2.621
	Cr	30.09	50.727	14.54	34.18	5.03	19.602	8.294	25.74

MANOVAs demonstrate effects of EM and G factors on all the examined criteria (Table 7). Significant main effect of G and EM factors were found for Cor, Flu and Flx criteria on Jam problem while only G factor had main effect on Flu, Or and Cr criteria on the system of equation. The system of equations demonstrated that, when the problem is more familiar to students, EM factor influences flexibility only. We also found interaction between EM and G factors with respect to students' flexibility related to solving the system of equations: G factor strengthens effect of EM factor, that is excelling in mathematics students who are gifted in mathematics significantly more flexible that their non-gifted counterparts while no significant differences appear in flexibility of EM and NEM students among NG students.

Table 7: Effects of G and EM factors

Between-Subjects Effects			G -factor	EM-factor	G×EM
		F(3,180)	**F(1,180)**	**F(1,180)**	**F(1,180)**
	**Cor**	**10.311*****	**11.148****	**18.571*****	. 013
	Flu	**14.064*****	**17.914*****	**23.257*****	. 501
	Flx	**13.087*****	**16.761*****	**21.058*****	2.691
	Or	1.048	2.316	. 766	. 043
	Cr	. 886	1.835	.758	.078
$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$	**Cor**	1.043	. 114	2.955	.234
	Flu	**3.633***	**8.321****	. 932	.932
	Flx	**11.025*****	**18.492*****	**7.626****	**9.513****
	Or	**4.891****	**10.463****	1.501	3.469
	Cr	**4.830****	**10.084****	1.554	3.641

SUMMARY

This study examines relationships between mathematical creativity, general giftedness, and mathematical excellence. It also explores the power of different types of MSTs for the identification of between-group differences related to mathematical creativity as reflected in multiple solutions produced by the students. Five groups of 10^{th} grade to 11^{th} grade students who varied in the level of general giftedness and in the level of mathematical instruction participated in this study.

The study demonstrates that G students have higher scores on all the examined criteria than. Both EM and G factors has main effect on students fluency and flexibility associated with MSTs. Only G factor has main effect on originality. G and EM factors interact on flexibility criteria: EM factor has more significant effect in G students on flexibility and originality criteria.

The effects of EM and G factors are task dependent and are related to the level of insight embedded in the task and the familiarity of the conventional solution. Task dependency of the findings as well as different effects that were discovered in the study demonstrates that EM and G factors are interrelated, but different in nature.

AKNOWLEDGMENT

This project was made possible through the support of a grant from the John Templeton Foundation. The opinions expressed in this publication are those of the author(s) and do not necessarily reflect the views of the John Templeton Foundation. We are thankful to the Israeli Ministry of Education and the University of Haifa for their generous financial support to this project.

REFERENCES

Ervynck, G. (1991). Mathematical creativity. In D. Tall (Ed.), Advanced mathematical thinking (pp. 42–53). Dordrecht, Netherlands: Kluwer.

Guberman, R. and Leikin, R. (2012). Interest and difficulty: Changes in teachers' views of multiple solution tasks. Journal of Mathematics Teacher Education (on-line).

Koichu, B. (2003). Junior high school students' heuristic behaviors in mathematical problem solving. Unpublished Doctoral Dissertation, Technion, Haifa.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman and B. Koichu (Eds.), Creativity in mathematics and the education of gifted students. (pp. 129–145). Rotterdam, the Netherlands: Sense Publishers.

Leikin, R. and Lev, M. (2007). Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In J. H. Woo, H. C. Lew, K. S. Park and D. Y. Seo (Eds.), Proceedings of the 31st International Conference for the Psychology of Mathematics Education (Vol. 3, pp. 161–168). Korea: The Korea Society of Education Studies in Mathematics.

Leikin, R. & Lev, M. (accepted-2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM - The International Journal on Mathematics Education, 45(4).*

Levav-Waynberg, A. and Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. Journal of Mathematical Behavior, 31, 73–90.

Mann, E. L. (2006). Creativity: The essence of mathematics. Journal for the Education of the Gifted, 30(2), 236–262.

Milgram, R., and Hong, E. (2009). Talent loss in mathematics: Causes and solutions. In R. Leikin, A. Berman and B. Koichu (Eds.), Creativity in mathematics and the education of gifted students (pp. 149–163). Rotterdam, Netherlands: Sense Publishers.

Raven J., Raven J.C. & Court J.H. (2000). *Manual for Raven's progressive matrices and vocabulary scales.* Oxford: Oxford Psychologists.

Renzulli, J. (1978). What makes giftedness? Re-examining a definition. *Phi Delta Kappan*, 60, 180–184, 261.

Sheffield, L. J. (2009). Developing mathematical creativity – Questions may be the answer. In R. Leikin, A. Berman and B. Koichu (Eds.), Creativity in mathematics and the education of gifted students (pp. 87–100). Rotterdam, Netherlands: Sense Publishers.

Sternberg, R. J. (2005). The theory of successful intelligence. Interamerican Journal of Psychology, 39, 189–202.

Torrance, E. P. (1974). Torrance tests of creative thinking. Bensenville, IL: Scholastic Testing Service.