MERGING EDUCATIONAL AND APPLIED MATHEMATICS: THE EXAMPLE OF LOCATING BUS STOPS

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Abstract: *The reduction of complex mathematical models for solving real-world problems to a level applicable in secondary education is considered to be a very important goal to increase motivation and understanding. This is, for instance, witnessed by its inclusion in mathematics curricula of many countries and the OECD study PISA (OECD, 1999). The actual implementation of this goal of merging educational and applied mathematics (MEAM) is, however, still in its infancy and rather difficult. In this paper we present a case study in which we discuss our experiences with a two-day modelling project for German high-school students in the age group of 15-19 years. As real-world problem we introduce the problem of locating new bus-stops in public transportation systems. The mathematical optimization tools to tackle this problem reduce to questions solvable by basic geometry. The paper also includes an evaluation of this case study with regard to the motivation of students and our conclusion for the mathematics curricula of schools.*

Keywords: Applied mathematics, modelling, motivation, empirical research.

INTRODUCTION

The optimization problem of locating new bus stops in an urban area is just one of many mathematical real-world problems found in industry and society which are also suitable to be taught in schools. Research on how to merge educational and applied mathematics (MEAM) and how to insert it to the curricula of secondary education is still in the early stages. Integrating real-world problems to everyday school life is an aim we want to achieve to prepare and advance students for their academic studies and later employment, as well as motivating students of all capability levels for the subject mathematics (Blum et al., 2002). It is important that students learn and know the connection between mathematics and problems found in industry and everyday life as this can improve their understanding of mathematical issues as well as their motivation and enjoyment. The OECD study PISA (Programme for International Student Assessment) also emphasizes on bonding real-world and mathematical problems (OECD, 1999) which shows how important it is to integrate management mathematics to regular school lessons.

As applied mathematics and modelling real-world problems play a decisive role in industry and science the goal we pursue is to regularly integrate this to lessons in school, i.e. to merge educational and applied mathematics. The merging does not only have to take place by means of modelling problems, but can also mean to integrate these topics to school lessons in a more classical way. By emphasising specific curricular mathematical content, challenging applied mathematical topics can be used to repeat, combine or develop new and old mathematical subjects.

Since the 1990s handling modelling problems in regular school lessons or during extra-curricular projects of 2 to 5 days is a successful activity started by a group of industrial and management mathematicians at the University of Kaiserslautern (Bracke et al., 2011). Embedding real-world modelling into regular school lessons was discussed and evaluated by Bracke et al., combining the knowledge from selected topics of a whole school year using a modelling cycle as described in Figure 2 by Blum et al. (2005). This study came to the conclusion that real-world modelling tasks can be integrated into regular school lessons of a whole school year and that the attitude of students towards this method was very positive. Kaiser et al. (2006; 2010) studied authentic modelling problems with students during modelling weeks and describe problems which are mainly suitable for students of an upper secondary level in school. Their study also showed that most of the students taking place in the evaluation would appreciate if such authentic problems were included in school mathematics more often, since they liked the opportunity to apply mathematics in real life. Maaß et al. (2007; 2011) emphasise the insertion of modelling problems into regular school lessons on a lower secondary school level. Precise modelling tasks were developed and evaluated during the project STRATUM with the successful aim to improve modelling competencies of lower achieving students. Methods to teach authentic problems concerning combinatorial optimization were discussed by Lutz-Westphal (2006). The focus does not lie on modelling problems but the integration of combinatorial optimization problems to regular school lessons. A feedback questionary was given to participating students to evaluate what they liked or disliked about the topics. The feedback was on the average very positive. We can hence say that applied mathematical problems as well as modelling problems in general seem to enhance the joy and understanding for mathematical applications.

Management mathematical problems seem to be particularly suited for modelling activities, since these problems are often very close to the everyday-life experience of the students (Bunke et al., 2007; Kreußler et al., 2012). Operations research, including the area of mathematical optimization, is a field of study which implies a vast number of applications from industry and everyday life. Geometrical methods can often be used to solve optimization problems which provide an excellent opportunity to integrate management mathematics to lessons in schools while staying within the framework of given curricula at the same time. Several examples of this type have been developed in the research project MaMaEuSch: Management Mathematics in European Schools (Hamacher, 2001-05) or can be found in the book Hamacher et al. (2004).

In the subsequent sections we consider the optimization of bus stop locations and its integration to secondary mathematical education in schools. At first, we present the mathematical background and a solution strategy in order to highlight the advanced mathematics behind this topic. Afterwards, we discuss the difficulty of reducing the presented mathematics to a level suitable for schools and present a modelling project studying the motivation of students concerning management mathematical topics.

PLANNING BUS STOPS – A GUARANTEE OF ACCESSIBILITY

Due to the increasing growth of population in urban areas the optimal location of bus stops becomes a more and more important task to maximize acceptance and convenience of public transportation systems and to minimize pollution, noise and congestion. To achieve these goals the public transportation network of a city should be an efficient and worthwhile alternative to travelling by car. This includes better prices, small travelling times and short distances to the nearest bus stops. We should also take into account that construction costs and travel times both increase with each additional bus stop. Therefore, we need to find a compromise between short travel times and enough bus stops within reach for all customers. The distance a customer is willing to go to the nearest bus stop is called the *covering radius*. Research by Murray (2001) showed that approximately 92% of the already existing bus stops in Brisbane, Australia, were redundant, assuming a covering radius of 400m for bus stops. The reduction of bus stops could hence lead to an enormous gain in time and, as a result, to an increasing number of customers.

We now want to specify the exact problem statement. We are given an already existing public transportation network, i.e. the roads of a city, represented by a planar undirected graph $G = (V, E)$. The edges $e = (v_i, v_j) \in E$ represent possible bus routes between the nodes v_i and $v_j \in V$, whereas the set V consists of important breakpoints and junctions of the given transportation network. There is given a finite number of customer locations $P = \{ p_i \in \mathbb{R}^2, i = 1, \dots, m\}$ and a covering radius $r > 0$. The *Continuous Stop Location Problem (CSLP)* now wants to find a minimal number of bus stops that cover a given number of customers within the covering radius r while all points on graph G are possible locations for new bus stops (Schöbel et al., 2009). The set of all points on the edges of graph G can be defined as $T := \cup_{e \in E} e = \{ x \in \mathbb{R}^2 : x \in e, e \in E \} \subseteq \mathbb{R}^2$. Given a covering radius $r > 0$, a customer location $p \in P$ is *covered* by a point $s \in T$, if $d^p(p, s) \leq r$. Furthermore, we define the *unit ball* B_p of a customer location $p \in P$ as all points $x \in \mathbb{R}^2$ whose distance to p is less than or equal to 1. $B_p^r := p + rB_p = \{ x \in \mathbb{R}^2 : d^p(x, p) \leq r \}$ then defines all points $x \in \mathbb{R}^2$ whose distance from $p \in P$ is less than or equal to the covering radius r. To solve *CSLP* we use the ideas of Schöbel et al. (2009) who present a suitable algorithm for our problem. In the first step we need to check whether *CSLP* is solvable at all. This can easily be done, since *CSLP* is solvable as soon as each customer location $p \in P$ is covered by at least one point $s \in T$, i.e. if $B_p^r \cap T \neq \emptyset, \forall p \in P$. Since we are given a continuous set T of possible bus stop locations at the start, our goal is to reduce this to a finite dominating set $S_{cand} \subseteq T$ for which we know that it must contain at least one solution S^*. *CSLP* can then be stated as a well-known *Set Covering Problem* and hence be solved by any algorithm known to solve this, for example the greedy algorithm suggested by Chvatal (1979). The algorithm to solve $CSLP$ can be summarized as follows. We are drawing circles $B_{p_i}^r \; \forall \, p_i \in P, i = 1, \dots, m$ with a given covering radius r around all customer locations. The intersection points of these circles with the given road network as well as given junctions and breakpoints form the candidate set S_{cand} of possible bus stops. Assuming the same construction costs for all bus stops we can set the costs $c_j = 1 \; \forall \, j$ without loss of generality. We then sort these candidates in decreasing order of the number of customer locations covered by them. Using the greedy method of Chvatal (1979) we start choosing bus stop locations beginning with the candidate that covers the maximum number of customers, continuing by always choosing the candidate of highest coverage left. As soon as all customer locations are covered the optimal candidate set S^* is found.

A BUS STOP MODELLING PROBLEM

Since the optimal planning of bus stops in a city forms an important up-to-date subject it is a good example to demonstrate students the application and usefulness of mathematics outside school. To find out whether working on exercises dealing with management mathematics can foster the interest and pleasure of students for the subject mathematics a number of schools in Germany, more specific the State of Rheinland-Pfalz, were visited by mathematical staff of the University of Kaiserslautern. Two-day modelling projects were given to groups of three to five students currently attending 10th to 13th form in school, i.e. students at the age of 15-19 years. A variety of different projects were presented at the start such that students were able to choose a project matching their interests. One of the projects was called "Where should bus stops be positioned?" where the Continuous Stop Location Problem discussed above was a possible modelling tool. This project was presented and dealt with in two of the participating schools. The students were free to choose a project of their interest. In each school one or two groups of students worked on the bus stop modelling problem which came to a total of 14 students.

The students were given an exercise sheet explaining the significance and importance of optimal bus stops with respect to time and money. The data included a specific road network with information about the position and number of customers at the start (see Figure 1) together with the task to find a minimal number of bus stops for the given road network such that as many customers as possible are able to reach a bus stop within a covering radius of 400m. The students were then confronted to tackle the given task by running through a modelling cycle which is well-known in tackling real-world problems and has, for instance, been described in Blum & Leiß (2005) for educational mathematics (see Figure 2). The concept of a modelling cycle was explained to and discussed with all students at the beginning of the two-day modelling period in order to prepare them for the task lying ahead. While the students were working on the projects, teachers and university staff acted according to the "principle of minimal help" (Aebli, 1978): Students had the freedom to independently discuss and develop ideas; the modelling staff only interfered in small doses to avoid dead-end streets in the modelling process.

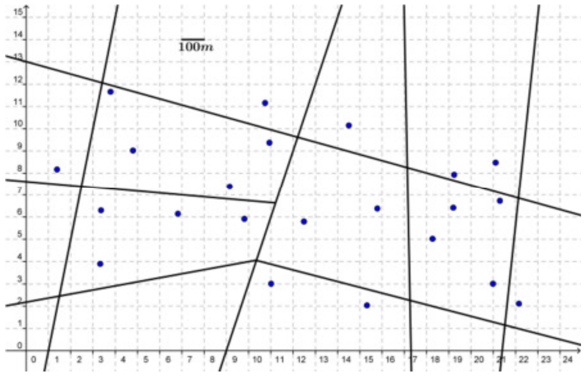

Figure 1: Road network given to students during modelling project (lines indicate streets, points the position of buildings, e.g. hospitals, schools, supermarkets, etc.)

One of the goals of the two-day modelling projects was to improve and strengthen the general mathematical competences (e.g. arguing mathematically, mathematical modelling, communicating) required by the curricula of the German school system (MBWJK, 2007). Additionally, management mathematical topics like locating bus stops were used to motivate students with projects concerning relevant real-world problems as well as improving their understanding of the application of mathematics outside school. Finally, as the previous example shows, these subjects automatically combine and repeat many individual basic contents of school mathematics.

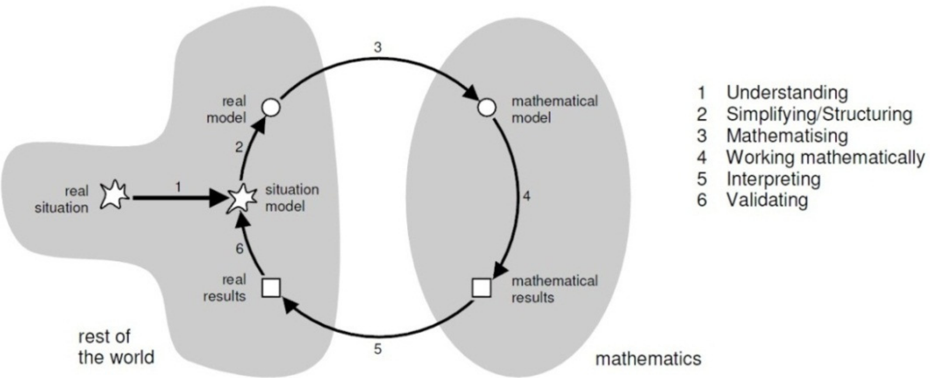

Figure 2: Modelling cycle according to Blum & Leiß (2005)

After having developed a strategy to tackle the problem, the students were asked to generalize their approach such that it could be applied to any other city. In addition, the students examined the city centre of their own city and were able to present optimal bus stops often matching the in reality existing stops, confirming the efficiency of their strategy just developed.

Student Results

The outcome of the two-day modelling project was, in general, very impressive. On their own account the students developed strategies very similar to the algorithm solving *CSLP* that we have presented above. This can, for example, be seen in the following student solutions (Figure 3). Since an Euclidean distance measure was assumed, circles with the given radius of 400m were drawn around each of the marked customer locations. Afterwards, the students sought for those areas where as many circles as possible intersected and which were then marked red (see left picture of Figure 3). The points where those areas intersected with the transportation network were then defined to be possible places for new bus stops. Choosing points from the highest to a lower number of intersections the number of required bus stops was minimized, exactly as done in the greedy algorithm discussed earlier.

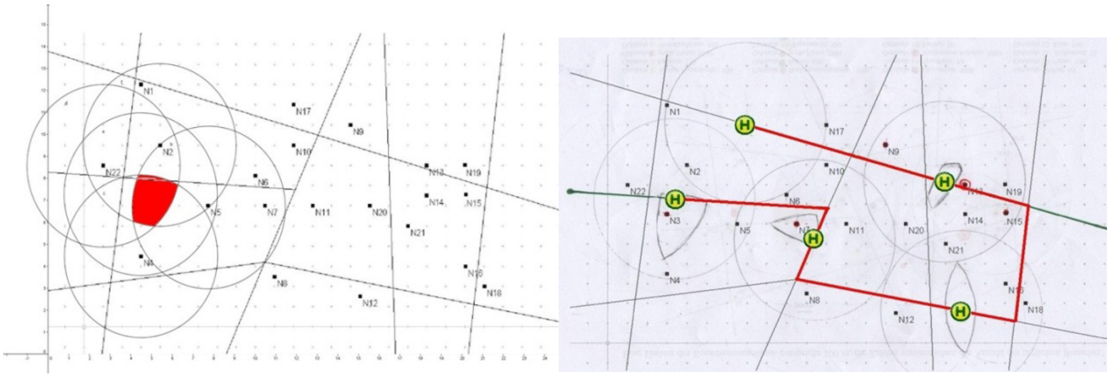

Figure 3: One solution by students of 11th form

Another group of students of 11th form improved this method even more by including the weights, i.e. the number of people at a customer location. The size of the radii drawn around each location was then adapted with respect to the importance of the place considered. The more customers likely to travel from a certain location, the more important it became and, hence, had a smaller radius drawn around its location. The maximum radius allowed remained to be 400m. Other students not only presented solutions for an optimal route but also thought about a most customer-friendly or economic solution, ensuring that a certain percentage of customers was being covered. Some students observing the city centre of their own town developed a grid of triangles placed over the map such that each pedestrian would be able to use a bus stop within reach of 400m. The corner points were then adjusted to the streets as well as taking important places of the city into account.

Evaluation of Survey

At the end of each modelling project the students were asked to fill in an evaluation sheet asking multiple-choice as well as free-answer questions. They were, for example, asked about their overall impression, how they liked the choice of their topic and if they could identify and understand the connection between mathematics and business as well as its usefulness outside school. This empirical survey was used as a preliminary stage for further research planned in this area. It gives a first impression of the usefulness and effect that the integration of management mathematical topics can have on students in school and justifies its embedding into everyday lessons. Altogether, the feedback was very positive. The multiple-choice questions could be answered on a scale from 1 to 6, where 1 means "very good/much”, decreasing to 6 standing for “very bad/little”. The following table lists some questions with their corresponding average and median student answer, demonstrating the results of the 14 students working on the project “Where should bus stops be positioned?”

Question	Average	Median
Overall impression	1,9	2
Choice of topic	2,3	2
Were you able to identify the connection between mathematics and business?	2,0	2
Did you learn what mathematics is useful for outside school?	1,9	2
Did you enjoy working on the project?	1,8	2

These answers show that the students enjoyed the MEAM approach and found the topics of management mathematics on the average “good”. This was also confirmed by the free-answers. For instance, the question “What did you like most?” was answered with statements like “Using classic school mathematics to solve real-world problems.”, “Getting to know applications of mathematics in practice.” or “The material to work on.” In addition to this, observing the students was also very meaningful. It was obvious that the students were working very enthusiastically and enjoyed the teamwork with their schoolmates. Often, students continued working on their projects in the evenings at home, presenting further ideas and solutions the following day. Even students who, at first, seemed a little uninterested or usually displayed a low competence level in mathematics were inspired by the topics and worked very hard and enthusiastic in the end. Since all the students who were tested were very motivated to work on topics concerning management mathematics we draw the conclusion that the inclusion of applied mathematics to the curricula of schools will increase the motivation and understanding of the subject mathematics in schools.

EMBEDDING INTO THE CURRICULA OF SCHOOLS

Optimizing bus stops in a city is a topic covering a lot of different subject-matters specified in the curricula of secondary schools. By didactically reducing and adapting the contents to the abilities of the students in question this topic could also be discussed with students in lower grades. Especially the required geometrical competences to solve the problem can already be achieved in 5th and 6th grade. The curriculum for 5th and 6th grade of the German school system in the State of Rheinland-Pfalz (MBWJK, 2007) lists general mathematical competences like “arguing mathematically”, “mathematical modelling” or “communicating”, as well as content-related competences that should be learned by students of this age. These topics include drawing circles and identifying center point, radius and diameter, getting to know the difference between a straight line and a line segment, using the Euclidean distance to describe the shortest distance between points and learning how to use dynamic geometry software. These subject-matters already form a good background to study the Continuous Stop Location Problem on a visual, geometrical basis. The curriculum for 7^{th} and 8^{th} grade introduces the absolute value, while the theoretical calculation of intersection points as well as solving systems of linear equations using matrices is not covered until 11^{th} or even 13^{th} grade (MBWJK, 1998).

The difficulty in reducing the topic of optimizing bus stops to the competence level of a group of students lies with the different backgrounds and knowledge the students might have gained so far. It might be necessary to introduce the theory of graphs in one class while another group of students might have already studied and used these in other subjects, e.g. computer science. Management mathematical topics are, hence, also very suitable for interdisciplinary education and can show students the important connections of their subjects in school.

There is a variety of possibilities how to insert the topic of optimizing bus stops into everyday lessons at school. This could, of course, be done by means of a modelling problem as shown above, usually taking at least 3 to 4 lessons of 45min each. In doing so, the teacher should not be expecting a specific solution but remain open for all kind of solutions presented by the students. Most of the teachers may find this very hard as they are used to pose questions to the class where they know the exact answer beforehand. For this reason, continuing education training courses for interested teachers with the aim to learn how to organize and teach modelling projects in school are regularly offered at the University of Kaiserslautern and find great acceptance among teachers. As we have pointed out, modelling projects dealing with subjects of management mathematics are very attractive to all students tested and are a great possibility to not only combine different topics learned in school but to also improve the capability of working in a team, discussing mathematical matter as well as improving modelling competences, which are all part of the mathematical competences required to be learned according to the German curricula for secondary schools. Another method would be to include this topic into the standard lessons, either using it as a motivating example or to strengthen already learned tools. This could, for example, be done at the beginning of the topic “Linear Algebra & Analytical Geometry” in 11^{th} – 13^{th} grade (MBWJK, 1998) driving towards the exact calculations of geometric intersection points. Motivated by this interesting example and its usefulness, students could at first develop geometric ideas guided by their teacher, learning afterwards how to implement these ideas theoretically.

SUMMARY AND CONCLUSIONS

The optimal planning of bus stops in urban areas is a management mathematical topic suitable to be taught in schools. Dealing with up-to-date mathematical problems can lead to greater motivation and understanding of the subject mathematics and its application outside school. This was tested in a preliminary survey during two-day modelling projects with students of various schools in Germany and showed how enthusiastic and interested students of all competence levels can be when dealing with management mathematical topics. Integrating real-world problems to lessons in school can not only increase motivation but also prepare students for further studies and their working life after school. This can, for example, be done by introducing modelling problems. Their solution procedure combines different topics learned in school and also enhances mathematical competences like "mathematical modelling" and "communicating". Management mathematical topics are also a good opportunity to be included into the curricula of secondary schools while being compatible with the guidelines given therein. Examples like the optimal planning of bus stops can be used to motivate at the beginning of a new topic as well as strengthen already learned facts. Hence, we conclude that management mathematics should be included in the lessons in schools more often as this increases motivation, understanding and pleasure for the subject mathematics.

REFERENCES

Aebli, H. (1978). *Grundformen des Lehrens – Eine Allgemeine Didaktik auf kognitionspsychologischer Grundlage* [Basic forms of teaching – Universal didactics based on cognitive psychology]. Klett-Cotta Verlag

Blum, W. et al. (2002). ICMI study 14: Application and Modelling in Mathematics Education – Discussion Document. *Educational Studies in Mathematics 51,* pp. 149-171

Blum, W., & Leiß, D. (2005). *"Filling Up" – The Problem of Independence – Preserving Teacher Interventions in Lessons with Demanding Modelling Tasks.* Paper presented at the Fourth Congress of European Research in Mathematics Education (CERME 4), Sant Feliu de Guixols, Spain

Bracke, M., & Geiger, A. (2011). Real-World Modelling in Regular Lessons: A Long-Term Experiment. In Kaiser, G., Blum, W., Borromeo-Ferri, R., & Stillman, G. *Trends in Teaching and Learning of Mathematical Modelling, ICTMA 14,* Springer Verlag

Bunke, F., & Hamacher, H.W. (2007). Wirtschaftsmathematik gehört in die Schule!? [Management mathematics belongs into school!?]. *Beiträge zum Mathematikunterricht 2007,* pp. 338-339. Verlag Franzbecker. Berlin 2007

Chvatal, V. (1979). A Greedy Heuristic for the Set-Covering Problem. *Mathematics of Operations Research, Vol. 4, No. 3,* pp. 233-235

Hamacher, H.W. (Project Coordinator), (2001-2005), *Management Mathematics in European Schools (MaMaEuSch)*, Research project funded by the European Union

Hamacher, H.W., Korn, E., Korn R., & Schwarze, S. (2004). *Mathe & Ökonomie: Neue Ideen für den praxisnahen Unterricht"*[Maths & economy: New ideas for an application oriented education]. Universum Verlag

Kaiser, G., & Schwarz, B. (2010). Authentic Modelling Problems in Mathematics Education – Examples and Experiences. *Journal für Mathematikdidaktik 31 (1)*, pp. 51-76

Kaiser, G., & Sriraman, B. (2006). A Global Survey of International Perspectives on Modelling in Mathematics Education. *Zentralblatt für Didaktik der Mathematik 38,* pp. 302-310

Kreußler, J., Bunke, F., & Hamacher, H. (2012). Motivationssteigerung im Geometrieunterricht anhand von Modellierung kompetitiver Standortplanung [Increasing motivation in geometry by means of modelling competitive location planning]. *Beiträge zum Mathematikunterricht 2012, Band 1,* pp. 477-480. WTM – Verlag für wissenschaftliche Texte und Medien. Münster 2012

Lutz-Westphal, B. (2006). *Kombinatorische Optimierung – Inhalte und Methoden für einen authentischen Mathematikunterricht* [Combinatorial optimization – Contents and methods for an authentic maths class]. Dissertation Technische Universität Berlin

Maaß, K. (2007). *Mathematisches Modellieren – Aufgaben für die Sekundarstufe 1* [Mathematical modelling – Exercises for secondary education]. Cornelsen Verlag Scriptor

Maaß, K., & Mischo, C. (2011). Implementing Modelling into Day-to-Day Teaching Practice – The Project STRATUM and its Framework. *Journal for Didactics of Mathematics 32*, pp. 103-131

Ministerium für Bildung, Wissenschaft, Jugend und Kultur Rheinland-Pfalz [MBWJK] (ed.): 2007. *Rahmenlehrplan Mathematik (Klassenstufen 5 - 9/10)* [Core curriculum mathematics (Grades 5 - 9/10)]

Ministerium für Bildung, Wissenschaft, Jugend und Kultur Rheinland-Pfalz [MBWJK] (ed.): 1998. *Lehrplan Mathematik. Grund- und Leistungsfach Jahrgangsstufen 11 bis 13 der gymnasialen Oberstufe (Mainzer Studienstufe)* [Curriculum mathematics. Higher and lower level courses for grades 11 - 13]

Murray, A.T. (2001). Strategic Analysis of Public Transport Coverage. *Socio-Economic Planning Sciences, Vol. 35, Issue 3,* pp. 175-188

OECD (ed.): 1999. *Measuring Student Knowledge and Skills – A New Framework for Assessment.* OECD, Paris

Schöbel, A., Hamacher, H.W., Liebers, A., & Wagner, D. (2009). The Continuous Stop Location Problem in Public Transportation Networks. *Asia-Pacific Journal of Operational Research, Vol. 26, No. 1,* pp. 13-30