TEACHER BEHAVIOUR IN MODELLING CLASSES

Susanne Grünewald / Katrin Vorhölter /
Nadine Krosanke / Maria Beutel / Natalie Meyer

University of Hamburg

The paper deals with the question of teacher behaviour in modelling classes and focuses on teacher interventions. Different aspects of interventions are considered, such as triggers of interventions and teachers' purposes as well as effects and consequences of interventions. The presented results indicate that interventions which are based on the demand of explaining the state of work and teachers who act as moderator or facilitator of knowledge foster students' independent solving processes.

INTRODUCTION

Mathematical modelling has been one of the main research areas at the Working Group of Didactics of Mathematics at the University of Hamburg already for a long time. During this time various aspects of mathematical modelling have been investigated. In recent years, the focus has been on fostering students modelling competencies most effectively. At the moment, two different PhD theses are carried out. One of them compares the effectiveness of a holistic versus an atomistic approach (for detailed information see Grünewald, 2012). The other one focuses on the effect of different kinds of teacher interventions (for detailed information see Stender, 2012).Within recent research the question of teacher behaviour in modelling classes and the effects of teacher interventions were analysed. In this paper we will introduce selected results of these current research studies based on two master theses.

THEORY

Mathematical Modelling in the classroom

As mathematical modelling is the topic of this working group, we do not want to outline mathematical modelling in general. But as mathematical modelling in the classroom deals with different aims (see Kaiser & Sriraman, 2006 for an overview), we want to stress those points which are important for our understanding of modelling, which is reflected by the modelling tasks chosen for the studies. Departing from the realistic or applied perspective on modelling described by Kaiser & Sriraman (2006) the modelling tasks dealt with need to be complex in order to foster the pragmatic aims required by the realistic perspective on modelling. This means that the modelling process starts with a question to answer and some background information. Then students therefore have to find out the information they need to work on the task. Sometimes this information can be calculated by given photos, sometimes the students have to read them up and sometimes they have to make assumptions on the basis of their own experience. The grade of complexity depends on the students' experience with modelling tasks as well as on the time they have to work on the problem. Another characteristic is that the task is not implemented in a special content, so the students are completely unsure which mathematical techniques are useful for solving the given problem. Furthermore, we try to choose authentic tasks, i.e. tasks which are relevant at least for a special group of people, at the best for the students themselves.

Scaffolding in mathematical modelling classes

Teaching is a complex process. In order to support students effectively, teachers not only have to have strong knowledge about different contents, but also about different types of teaching methods and adequate assistance. One theoretical approach that deals with tailored and temporary support that teachers can offer students is scaffolding. Because scaffolding has been studied extensively in the last couple of decades, slightly different approaches exist[1]. However, the central goal of all scaffolding approaches is to enable students to solve problems on their own. For this purpose, students are supported in a very practical way when they are not able to solve given problems or when they get stuck. The support takes place on both the cognitive level (required strategies and concepts) and the meta-cognitive level (instructing self-regulated learning). The main principle is a consequent orientation of the students' individual learning process, which Van de Pol et al. (2010) calls *contingency*. A condition, therefore, is the willingness and the competency of teachers to be responsible for the demands of thinking and understanding of students. So the teacher should have content and diagnostic knowledge. In particular, when students work on complex modelling tasks and can choose mathematical techniques on their own, the teacher must be able to decide in a short period of time, if the students approach is expedient or not. Depending on how self-regulated students are in their working process, the teacher reduces the support, which is called *fading* in the sense of Van den Pol et al. (2010), because the teachers are *transferring the responsibility* to their students.

Within the framework of scaffolding, Hammond and Gibbons (2005) developed a model, which differentiates between scaffolding on a macro- and on a micro-level: on the one hand, teachers can provide or foster different didactical settings (e.g. group work) and consider different students' characteristics (e.g. thinking styles, beliefs). This kind of assistance on a macro-level is called *designed-in-scaffolding*. This can be planned before attending class and has to be based on pedagogical content knowledge as well as on didactical knowledge about the modelling process. On the other hand, teachers can intervene at special times while students are solving mathematical problems. This kind of assistance focuses on a micro-level and

[1] An overview on different approaches of scaffolding is given by Van de Pol et al. (2010).

Hammond and Gibbons call these interventions *interactional scaffolding*. Interactional scaffolding cannot be planned in detail. The single interventions have to be based on pedagogical content knowledge as well as on didactical knowledge about the modelling process. Whether these interventions are adequate or not depends on the circumstances in which they occur. Consequently, adequate intervention must be a consequence of a teachers' diagnosis of a students' difficulties while solving mathematical problems.

When talking about different kinds of interventions, one has to distinguish between the trigger, the level, and the effect of interventions. Concerning triggers, Leiß distinguishes between invasive and responsive interventions: every time teachers intervene on their own initiative, the intervention is called invasive. If students ask for help, the intervention is called responsive (Leiß, 2007: 105f.).

The most well-known distinction between different levels of interventions is the taxonomy of assistance according to Zech (2002). He differentiates motivational, feedback, strategic, content-oriented strategic and content-oriented assistance. The intensity of the intervention decreases gradually from motivational assistance to the content-oriented assistance. This classification has been used several times to describe possible assistance in modelling processes (see Leiß, 2007; Maaß, 2007). Based on this categorisation, Leiß created a descriptive analysis of adaptive teacher intervention in the modelling process. The analysed interventions were classified by their trigger, level, and intention (see Leiß, 2007). The main results of Leiß' study were, among others, that strategic interventions are included in the intervention-repertoire of the observed teachers only very marginally and that the teachers often choose indirect advice where students have to find only one step by themselves in order to get over their difficulty. Further studies like Link (2011) cannot confirm these results. This is significant because in these studies it was found that, in particular, strategic interventions lead to metacognitive activities of learners (see Link, 2011). The mentioned studies have one recommendation in common: Maria Montessories' principle "Help me to do it by myself".

Besides different triggers and levels of intervention and the knowledge of the modelling cycle, teachers should also be aware of the role of metacognition within mathematical modelling for a basis of possible interventions. In recent research on metacognition, a distinction between declarative metacognitive knowledge (i.e. learning strategies, person and task characteristics) and procedural metacognitive skills (i.e. controlling, monitoring and self-regulation) is made (Schneider & Artelt, 2010). Stillman, Galbraith, Brown & Edwards (2007) developed a theoretical framework for studying students' procedural metacognition while modelling which is based on studies of Goos (1998, 2002). Goos differs three types of so-called *red flag situations* in mathematical problem-solving processes, which occur in metacognitive barriers. These red-flag-situations are situations with a lack of progress, error detection and anomalous results. If these warnings are not recognized by the teacher or even by the students, this can lead to failure in the problem-solving process. Furthermore, Galbraith & Stillman (2006) point out that reflections should be related to mathematical content and the processing decisions for fostering the students' modelling competencies. Only in this way can students become better modellers and not just solvers of separate problems.

To sum up, in order to foster students modelling competencies teachers should see themselves as *moderators or facilitators of knowledge* rather than as *disseminators of information* (see Herget & Torres-Skoumal, 2007).

PROJECT SETTING AND SAMPLE

The research results we want to present in the following were achieved within different frameworks. For this reason the conditions of data collection and, of cause, the research questions were different in detail, although both projects were focused on teacher behaviour in modelling classes. In the following, we will briefly describe these different conditions as well as the samples and methods used to collect and analyse the data. An overview of the different project settings and samples of the two studies is given in table 1.

Beutel and Krosanke (2012) as well as Meyer (2012) analysed teacher behaviour in modelling activities of grade 9 students, but Beutel and Krosanke collected data during modelling days, while Meyer collected data from two of six modelling activities in double lessons that were integrated in normal math classes.

The modelling days are one important part of Peter Stenders PhD project (Stender, 2012). The modelling days are carried out once a year for all grade 9 students in one secondary school (Gymnasium). Over three days they are asked to work on one complex modelling task, which they can choose between four different tasks. The students work in small groups and are supervised by two university students who were prepared for this within a master seminar at university. So one can say that these students are experts concerning the special tasks and novices concerning teaching in general. In 2012, 160 students attended the modelling days and were looked after by 32 university students, so each small group contained about 10 students. All groups that had chosen one special task (“traffic lights or roundabout”) were videotaped.

During the other modelling project (ERMO, Grünewald 2012) different forms of arrangements (holistic and atomistic approach) were tested against each other in order to evaluate, which is more effective in fostering students’ modelling competencies[2]. For this purpose, around 20 classes of the 9th grade of six secondary and district schools were divided into two groups: group A tackled modelling problems according to the holistic approach and group B tackled modelling problems according to the

[2] A defintion of modelling competencies is for example given by Maaß (2006).

atomistic approach. The intervention period started in February 2012 with a teacher training. During the intervention period the classes performed six modelling activities. Before the first and after the fifth modelling activity the students wrote a modelling test. In addition to these tests, they filled in a learning questionnaire at the end of each modelling activity and the teachers filled in short questionnaires about the run of the modelling activity. During the modelling activities the students in the holistic group dealt with complete modelling problems with an increasing complexity of tasks. The students of the atomistic group dealt with sub-processes of mathematical modelling separately. The modelling activities of both groups were designed with an autonomously-orientated learning environment – such as small group work, the principle of minimal help and a demand for reflection.

Beutel and Krosanke chose one of the videotaped groups of students who worked on the task “traffic lights or roundabout” and analysed the interventions given by the university students. The reasons for the selection of this group were the quality of video and tone, the level of performance and – most importantly – the level of communication (in this particular group every aspect and every assumption were discussed, calculations were read aloud). This special group only consisted of four students who had very low experiences with modelling tasks and were looked after by one female and one male student. The videotaped students worked for ten hours on the question whether traffic lights or a roundabout is more effective for crossroads. Meyer watched the behaviour of two different teachers who were willing to be observed and interviewed from the holistic group during the first and the third modelling activity. Moreover, the two teachers were recorded, so Meyer was able to analyse the teachers’ verbal interaction with the students in detail. After each lesson she did an interview with both teachers.

Beutel and Krosanke, as well as Meyer, analysed their date using qualitative content analysis by Mayring (2010). Beutel and Krosanke reconstructed learners’ problems as well as the teachers’ behaviour, both in relation to the different steps of the modelling process. This proceeding allowed a more sophisticated look at the potential and the effects of teacher interventions during the modelling process. In contrast to other studies, Beutel and Krosanke analysed the effects of interventions in the context of the complete modelling process by short-term and long-term considerations connected with the solution process of the students. In addition, a more differentiated view was sought on the success of the teacher interventions. So the main aim of the study was to describe the effects of intervention at various levels and to analyse them according to their appropriateness. Meyer encoded all interventions following Leiß (2007), with regard to the three main categories: trigger, level, and intention. The main aim was to investigate teachers’ behaviour in modelling lessons, divided into introduction, group work and presentation phase. Furthermore, Meyer tried to relate the teachers’ behaviour to the concepts of the role of a teacher as a *disseminator of* *information* respectively a *moderator or facilitator of knowledge* (see Herget & Torres-Skoumal, 2007).

Table 1: Framework of the studies

	Beutel & Krosanke	**Meyer**
Students	One group of four students in grade 9, secondary school	Two classes in grade 9, divided into small groups, secondary and district schools
Teachers	University students, experts in modelling, novices in teaching	Experienced teachers, novices in modelling
Length of analysed solving process	2 days, each lasting 5 hours	2 lessons, each lasting 1.5 hours
Modelling tasks	One complex modelling task, students chose on their own	Two modelling tasks, all students working on the same task given by the teacher
Data collection	Videotaping of the whole solving process	Audiotaping of the solving processes, interviews with the teachers after each lesson
Data analysis	Qualitative content analysis	
Research aim	Analysis of teacher interventions and the effects and consequences on students behaviour	

RESULTS

Beutel and Krosanke, as well as Meyer, analysed teachers' behaviour in different kinds of modelling classes. In the following, we will outline the results of both studies referring to two aspects: different types of triggers of interventions and teachers' purpose of the intervention as well as effects and consequences of interventions.

Types of triggers and teachers' purpose of the intervention

The interventions identified by Beutel and Krosanke were mostly invasive. A detailed view reveals that these interventions were given after a period in which students were not working on the task. So Beutel and Krosanke conclude that teachers intervened not to guide students to solve the modelling task but obviously their aim was to help the students because they had recognized problems. The strategic intervention which occured most often was the request to present their state of work. As well as strategic interventions, feedback and content-related interventions occurred.

Meyer encoded the interventions of both teachers as mostly invasive, but while the interventions of teacher A mainly referred to organisational aspects of the modelling activities teacher B provided much more content-related help. The analysis of the observed math lessons shows clear differences in the teachers' intervention behaviour and suggests the typing of teacher A as *moderator or facilitator of knowledge* and teacher B as *disseminator of information*. Teacher A as *moderator or facilitator of knowledge* acted in a restrained manner during the different phases of the modelling classes. This, for example, can be seen in the fact that he did not immediately correct students' mistakes such as inadequate assumptions in the solving processes. Teacher B as *disseminator of information* intervened much stronger and more often during the modelling classes than teacher A. Teacher B, for example, corrected students' mistakes immediately and controlled the solving processes by setting the steps of solutions that he considered adequate. Instead of providing content-oriented assistance, teacher A's interventions were mostly motivational and strategic, for example the students were asked for the problem formulation and encouraged to make appropriate assumptions, as well as to use their own context knowledge.

Effects and consequences of interventions

Concerning the effects and consequences of interventions, Beutel and Krosanke reconstructed that the strategic intervention *presentation of state of work* had potential both for students and teachers. One consequence of this particular kind of intervention is the reflection and the structuring of present results and present action. As a result, Beutel and Krosanke could reconstruct students' ability to solve a partial problem after an intervention was given; the realization of the importance of obtained results and thus their incorporation into the solution process and the verbalization of previously remained intuitive insights. Another consequence, as mentioned before, was that Beutel and Krosanke were able to confirm this special kind of intervention as a diagnostic tool for teachers.

Looking at the effect of interventions, Beutel and Krosanke differentiate between short-term and long-term effects. However, a definition of effects was impossible because of the complexity of effects. But in trying to define effects, Beutel and Krosanke inevitably had to look at interventions which led to metacognitive processes. They were able to reconstruct effects on the declarative level as well as on the procedural level of metacognition. In most cases, strategic interventions are the trigger of such processes, but content-related interventions and feedback can also lead to metacognitive processes. Every time Beutel and Krosanke analysed feedback as a trigger of metacognitive processes, feedback was given in combination with a content-related intervention. They also point out, that metacognitive processes that were triggered by teacher interventions do not always lead to progress in the solution progress; sometimes the interventions did not influence the solving process at all.

As a result of the behaviour of teacher A as a *facilitator of knowledge*, Meyer observed the students as being encouraged to think and work for themselves. Due to the many teacher interventions of teacher B as *disseminator of information* during the solving processes the students were hardly able to work for themselves. Through numerous hints the students were directed to appropriate solutions.

While teacher B mainly focussed on the mathematics and the correctness of the real solutions, teacher A aimed at organizing and supporting the students' individual learning processes. Meyer assumed that the behaviour of the *moderator or facilitator of knowledge* can especially help students work independently on modelling tasks and promote the development of students' mathematical modelling competencies.

CONCLUSION

Both studies dealt with the question of teacher behaviour in modelling classes. In both studies the effects of teacher interventions were one analysed aspect. While the study by Beutel and Krosanke aimed at describing the effects of an intervention at various levels and analysing them according to their appropriateness, the study by Meyer focussed on describing the role of the two teachers as *moderator or facilitator of knowledge* respectively as *disseminator of information*.

Both studies describe the observed teacher interventions as mainly invasive, while different types of interventions could be reconstructed, for example motivational, strategic and content-oriented assistance. However, differences between the preferred types of intervention among the teachers could be identified. The study by Meyer shows exemplarily two different types of teacher behaviour in modelling classes, teacher A as moderator or *facilitator of knowledge* and teacher B as *disseminator of information*. While teacher A mainly used motivational and strategic assistance, teacher B provides more often content-oriented support. The study by Beutel and Krosanke describes the role of the teachers as moderator or facilitator of knowledge i.e. the teachers did not aim at guiding the students to solve the modelling task in a prescribed way. Concerning the effects and consequences of interventions, both studies reconstructed mainly strategic interventions.

The different results of these studies point out that invasive interventions are not to be rated as more appropriate than responsive interventions or the other way around: invasive interventions, which are of a organisational nature or which are carried out because teachers diagnosed a lack of progress or students' helplessness, seem to be valuable for the students' solving process.

A second point is that teacher B often expressed his own uncertainty in how to intervene in a strategic way. The university students who were acting as teachers in the other project were trained for this work over several weeks. According to this one can conclude that the usage of strategic interventions can be promoted by specific training activities within teacher education and that they are also accessible for young students.

A third point is that for all off the researchers it was difficult to find out which interventions were effective and have consequences. This is due to different aspects: the effectiveness of an intervention does not only depend on the intervention itself but also on the student to whom it is given. Thereby, one cannot generalize the effectiveness of special types of interventions. Another reason is the definition of effectiveness: can you classify an intervention as effective, if it enables the students to continue their work for only for a few minutes? What about long-term-effects? Meyer could classify the teachers she had observed as *moderator or facilitator of knowledge* (teacher A) and as *disseminator of information* (teacher B) and reconstruct that the behaviour of teacher A promotes students' independent solving processes much more than those of teacher B. If the results of this case study can be generalized, it would help us to answer the question of which interventions are adequate.

REFERENCES

Beutel, M. & Krosanke, N. (2012): *Rekonstruktion von Handlungsabläufen in komplexen Modellierungsprozessen – Schülerprobleme und Lehrerverhalten.* Masterarbeit im Studiengang Lehramt an Gymnasium, University of Hamburg (unpublished).

Galbraith, P., & Stillman, G. (2006). A Framework for Identifying Student Blaockages during Transitions in the Modelling Process. *ZDM, 38*(2), 143–162.

Goos, M. (1998). "I don't know if I'm doing it right or I'm doing it wrong!": Unresolved uncertainty in the collaborative learning of mathematics. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times* (pp. 225–232).

Goos, M. (2002). Understanding metacognitive failure. *Journal of Mathematical Behavior 21*, 283–302.

Grünewald, S. (2012, July). *Acquirement of modelling competencies – first results of an empirical comparison of the effectiveness of a holistic versus an atomistic approach to the development of (metacognitive) modelling competencies of students.* Paper presented at the ICME 12, Seoul, Korea.

Hammond, J., & Gibbons, P. (2005). Putting scaffolding to work: The contribution of scaffolding in articulating ESL education. *Prospect, 20*(1), 6–30.

Herget, W. & Torres-Skoumal, M. (2007). Picture (im)perfect mathematics. In G. Werner, L. Peter, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education.* The 14th ICMI Study (pp. 379-386). NY : Springer.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM, 38*(3), 302–310.

Leiss (2007). *Hilf mir es selbst zu tun - Lehrerinterventionen beim mathematischen Modellieren.* Hildesheim: Franzbecker.

Link, F. (2011). *Problemlöseprozesse selbstständigkeitsorientiert begleiten: Kontexte und Bedeutungen strategischer Lehrerinterventionen in der Sekundarstufe I.* Wiesbaden: Vieweg+Teubner Verlag.

Maaß, K. (2006). What are modelling competencies? *ZDM 38*(2), 113-142.

Maaß, K. (2007). *Mathematisches Modellieren. Aufgaben für die Sekundarstufe I.* Berlin: Cornelsen.

Mayring, P. (2010). *Qualitative Inhaltsanalyse: Grundlagen und Techniken* (11th ed.). Beltz Pädagogik. Weinheim: Beltz.

Meyer, N. (2012): *Verhaltensweisen von Lehrkräften im Mathematikunterricht: Eine Fallstudie im Rahmen von Modellierungsaktivitäten nach dem holistischen Ansatz.* Masterarbeit im Studiengang Lehramt an Gymnasium, University of Hamburg (unpublished).

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. In G. Stillman & Z. Mevarech (Eds.), *Metacognition Research in Mathematics Education* (pp. 149–161). Springer.

Stender, P. (2012, July). *Facilitating complex modelling activities – the role of the teacher*. Paper presented at the ICME 12, Seoul, Korea.

Stillman, G. (2011). Applying Metacognitive Knowledge and Strategies in Applications and Modelling Tasks at Secondary School. In G. Kaiser, W. Blum, R. Borromeo Ferri & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling: ICTMA14.* Dordrecht: Springer Netherlands.

Stillman, G., Galbraith, P. L., Brown, J., & Edwards, I. (2007). A framework for success in implementing mathematical modelling in the secondary classroom. In J. Watson & K. Beswick (Eds.), *Mathematics: essential research, essential practice. Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia,* (pp. 688–707).

van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in Teacher–Student Interaction: A Decade of Research. *Educational Psychology Review, 22*(3), 271–296.

Zech (2002). *Grundkurs Mathematikdidaktik. Theoretische und praktische Anleitungen für das Lehren und Lernen von Mathematik*. 10. Aufl. Weinheim: Beltz.