SOLUTION AIDS FOR MODELLING PROBLEMS

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One option to help students to process modelling problems is the use of solution plans. Some of these solution plans will be introduced. As part of a qualitative study a solution plan in connection with a modelling problem was used in Grade 6. The students were observed and interviewed during it. The assessment strongly shows differing work processes but comparable written solutions from the students dependent on the solution plan.

MODELLING AS COMPETENCY

Mathematical modelling is one of the six mathematical competencies that are accounted for in the German educational standards for mathematics. Students should acquire the skills, based on diverse mathematical content, to translate between reality and mathematics in both directions. Due to the high significance of this competency for classes, solution aids for students when working on modelling problems in math class will be discussed in this article. That is why the competency of modelling will be briefly introduced in the following.

The core of mathematical modelling was already described by Pollak (1977) as interplay between mathematics and the "rest of the world" (see Fig. 1).

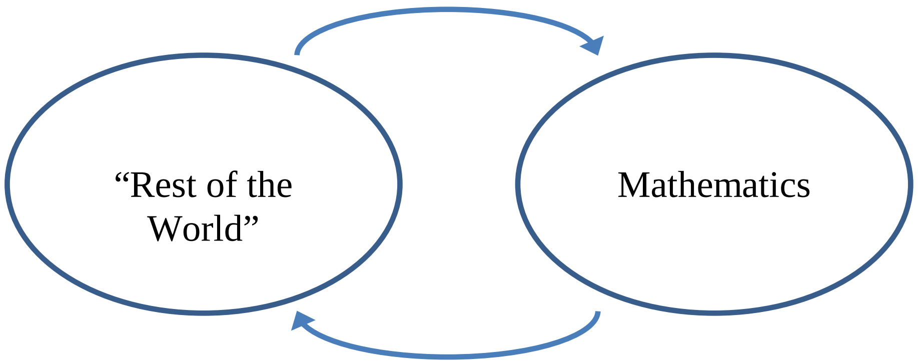

Fig. 1: Mathematics and the "rest of the world" (cf. Pollak 1977)

Modelling competency is described more precisely in Blum et al. (2007) as the ability to perform the respective required process steps while switching back and forth between reality and mathematics adequately in regard to the problem as well as to analyse given models or comparatively assess them. Modelling cycles (similar to that in Fig. 1) describe the different sub-processes of modelling activities in different detail and with different perspectives (Kaiser & Sriraman, 2006). A selection of these so-called partial competencies is listed in Table 1.

Partial competency	**Indicator**
Understanding	The student constructs her/his own mental model of a given problem situation and thereby understands the question.
Simplifying	The student separates essential and un-essential information of a real situation.
Using mathematics	The student translates appropriate simplified real situations in mathematical models (i.e. term, equation, figure, diagram, function).
Interpreting	The student relates the results obtained in the model to the real situation.
Validating	The student checks the results obtained in the model based on the real situation.

Table 1: Partial competencies of modelling (a selection)

The conscious division of modelling into partial processes is a possible way to reduce the complexity for those teaching and those learning and to set up suitable problems. Such a view of modelling especially makes it possible to train individual partial competencies in a targeted way and thus to construct extensive modelling competency. Also the view of the partial processes of modelling can be used to create solution plans for students and to thereby make solution aids for processing modelling problems available. Sjuts (2003) describes activities of planning, monitoring and checking, that are also initiated by solution plans, as *procedural meta-cognition*.

SOLUTION AIDS

Solution plans can aid the processing of modelling problems. Blum (2006), for example, developed a solution plan for students as part of the DISUM Project that is based on a simplified model building cycle (see Fig. 2).

This solution plan contains four steps called *understanding the problem, creating a model, using mathematics* and *explaining results.* Every step is explained to the student with a question and some clarifying points.

Blum’s solution plan belongs to the so-called indirect general strategic aids because although he does refer to general specialised modelling methods he does not give any concrete assistance that is based on the tasks in steps 2 and 3 of this solution plan and the commonality of the strategic aid is abandoned in favour of content-oriented pointers. Because of the pointers to equations and the Pythagorean Theorem it is a matter of a content-oriented strategic aid. This solution plan can be prepared for students. Its usage can also be practiced with the help of example problems.

1	Understanding the problem	What is given, what is sought?	Read the text precisely Imagine the situation exactly Make a sketch
2	Creating models	Which mathematical relationships can I establish?	Fill in missing entries, if required, i.e. set up equations or plot triangle
3	Use mathematics	How can I solve the problem mathematically?	I.e. work out the equation or use Pythagorean Theorem, write down the mathematical result
4	Explain the result	What is my end result? Is it logical?	Round off the result and relate it to the problem – possibly back to 1, write down answer

Fig. 2: Solution plan for modelling problems (Blum, 2006)

In a study by Schukajlow et al. (2010) as part of the DISUM project, significant differences in student achievement in modelling was verified using this solution plan in regard to the Pythagorean Theorem area of content. The class with the solution plan proved to be the more effective form of teaching and learning. In addition, the students in the solution plan group were also more aware of using cognitive strategies, in other words the solution plan.

A shorter solution plan is used by Zöttl & Reiss (2010) in the content area of geometry. This is reduced to three phases, namely

- Understanding the task,
- Calculating,
- Explaining results.

As part of the KOMMA Project, completed solution examples were used in addition to the solution plan above from Zöttl & Reiss (2008) that consisted of a problem and the description of the solution steps. In the area of mathematical justification and verification positive effects could already be ascertained when such solution examples were used (Reiss & Renkl 2002).

An alternative solution plan can be found in Greefrath & Leuders. In the set up of this solution plan the problem solving steps of Polya were taken more strongly into consideration. In his book *How to solve It* he developed a catalogue of heuristical questions that are supposed to help process problem solving tasks. Here the problem solving process is divided into the following sections (Polya, 1973): Understanding the problem, devising a plan, implementing the plan, review. Schoenfeld (1985) follows up on it and describes certain steps in more detail. At the end of the problem solving process he differentiates between verification and transition. The proposed solution plan for learners at the start of secondary school therefore contains five steps and can be used for modelling as well as for problem solving tasks:

- Understanding the problem: Formulate it in your own words.
- Choose the approach: Describe assumptions and plan the calculation method.
- Performing: Perform the calculation.
- Explain the result.
- Checking: The result, calculation and approach.

The solution plan is similar to problem solving activities in order to use it more frequently than a solution plan specifically for modelling problems. Because every modelling problem is also a problem, the strategies suitable for problem solving are also helpful for modelling.

Certain authors also use a simplified modelling cycle as solution aids for the students. As part of a qualitative study in Grades 7 and 8, Maass (2004) studied the modelling competence of students and by the end of the study could reconstruct proportionate meta-cognitive competencies in a large percentage of the students. A result she also describes is that the students sensed the knowledge of the modelling process and the depiction of the cycle as an orientation aid.

But disadvantages to such solution plans have also been named. Meyer and Voigt (2010) lead the way saying that a solution plan dependent on a modelling cycle with a structured formula for processing practical calculation problems from the 1960's and 1970's can be compared, where they were offered to students as alleged solution aids and turned out to be additional learning material.

Another option for a solution aid for processing modelling problems that is not dependent on a modelling cycle, is asking and answering indicative questions or simpler questions (Greefrath & Leuders). Here the students learn to ask and answer questions about the modelling problems. Two goals can be achieved hereby. Firstly, it is easier to recognise which information the text in the problem really supplies – possibly even information necessary to solve the problem – and which must be procured in a different way. Secondly, the modelling problem to be processed is disassembled into partial steps that can initially be processed individually reducing cognitive load (Sweller, 1988) before the partial results are then put together to solve the problem. Later the students can ask themselves such questions and decide whether they can be answered with help from the text.

STUDY DESIGN

As part of a qualitative study pairs of students from 6th Grade of an secondary chool were observed while working with the solution plan from Greefrath & Leuders and subsequently questioned. A qualitative study was chosen to get information about the processes while working with the solution plan. Of course the generalizability of such a qualitative Study is low, but the goal was to get information about the processes in detail. Up to now there are no emprical results on students working with this specific solution plan.

The students' activities while working on the task and the subsequent interviews were filmed. Until now three such interviews were evaluated. The students worked on two problems one after the other. The first problem served to understand the solution plan and to put the given steps into the right order. The second problem consisted of using the solution plan. For this the above solution plan from Greefrath & Leuders was presented together with the following problem:

> Work out the following problem according to this solution plan: What amount of liquid do I drink every week?

The videos of the observations and interviews were completely transcribed and evaluated following Grounded Theory (Strauss & Corbin, 1990). Here the transcripts were worked through line-by-line and the individual lines of text openly coded. In this way categories were developed which were then used to evaluate the results. The coding was done by two independent persons in order to achieve highest possible interrater reliability. (Hilmer, 2012)

RESULTS

After viewing the openly coded lines of text the following categories were developed that were then used to code the interviews:

- Orientation on the plan and example
- Fulfilling the requirements of the plan
- Difficulties in implementing the plan and example
- Incorrect or incomplete conversion

The fulfilment of the requirements of the plan will be observed in more detail in the following. Also the individual five steps of the plan were individually coded as part of this category and used for the following analysis. These are the first empirical results for this special solution plan.

Interview 1:

This tandem is strongly oriented on the given solution plan but they especially considered the concrete example of the first problem. The students mixed up the phases *Choose approach* and *Perform* and the test persons oriented themselves alternately on the question, example and plan. Although the plan did encourage meta-cognitive processes, it did not supply optimisation of problem processing. Figure 3 depicts the phases of processing of this pair of students.

PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Understand the problem	x					x									
Choose approach			x		x			x		x			x		
Perform		x		x					x			x			
Explain result											x			x	
Check							x								x

Fig. 3: Solution phases in Interview 1

The mix-up of the second and third steps of the solution plan becomes clear in figure 3. Even though the observation shows that both students could not orient themselves strongly on the given plan, they still answered the question as to whether the plan had helped them:

> "I thought it was good that I could always look there. If I had to do it by heart I think it would be more difficult. This way I had a kind of comparison"

Interview 2:

This pair of students oriented themselves clearly on the given solution plan. Difficulties only came up partially in the description of the corresponding phases. A content error occurred in the last processing step. Here the necessary plausibility observation does not succeed. Figure 4 shows the phases passed by this pair of students and shows a definite difference to Interview 1:

PHASE	1	2	3	4	5	6	7	8
Understand the problem	x							
Choose approach		x		x				
Perform			x		x		x	
Explain result						x		
Check								x

Fig. 4: Solution phases in Interview 2

This pair of students shows the sequence that should be given by the solution plan much clearer. Interestingly, one of the two students pointed out in the subsequent interview that he sees a connection with the known solution aid for written problems: Question, calculation, answer. Altogether the use of such a solution plan was seen as being very positive by the students. The written solutions of the pair of students clarified the clear sequence and the maintained structure of the solution plan.

Understand the problem: How much do you drink in one week?

Choose approach: I drink ca. 2 litres a day. I would calculate 2 x 7.

Perform: 2 l x 7 = 14 l

Explain result: I drink ca. 14 l a week and ca. 2 l a day.

Check: That would be 9 x 1,5 litre bottles and one 0,5 litre bottle.

Interview 3:

This tandem oriented themselves on the phases of the solution plan. All the phases are listed except for the last phase. But they oriented themselves more on the example given in the first problem than on the plan itself. This led to problems in some places with the abstraction of the example. The phases *Choose approach* and *Perform* were no problem. The following two phases however were mixed up and shortened. The overview shows the sequence of the phases:

PHASE	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Understand the problem	x					x			
Choose approach			x						
Perform		x		x			x		
Explain result					x			x	
Check									x

Fig. 5: Solution phases in Interview 3

This pair of students was very positive about the solution plan and finds the added explanations to be important:

Interviewer: Did the plan help you?

Student 1: I would say that if it wasn't there, the explanations, only understanding the problem, choosing the approach, I wouldn't have gotten it. Here it says to formulate in your own words, describe assumptions and plan the calculation method. That helped.

Interviewer: And if you had done the question without the plan? Would you have done it the same way?

Student 2: It would have been more difficult and would probably have taken a bit longer.

This group of students also noted down the written solutions according to the given plan structure. (Hilmer, 2012)

DISCUSSION

Of course this qualitative study with only three pairs of students is limited, but the three interviews assessed show that the student solutions were influenced by the given solution plan. Especially the written solutions receive a structure clearly adapted to the solution plan. Here it becomes clear that the choice of solution plan can have a great influence on the written solutions of the students.

However the actual solution path of the student pairs do not always follow according to the given plan. Even if the written, fixed solutions of the students all have the same structure the solution process differs significantly. This reminds one of the individual modelling routes that were described by Borromeo Ferri (2007). The decisive question in judging the effectiveness of these plans as solution aids is whether the solution process plays a role in the modelling process or only influences the result. Apparently there are students that have greater difficulties dealing with such a plan and others who only need a short introduction to work with the plan.

Regardless of whether the solution plan sustainably influenced the solution process, the six students interviewed made positive comments about the solution plan and felt supported by it. This highlights in a certain way the results of Schukajlow et al. (2010). Also the use of finished solution examples, like in our study in the first problem, seems to appeal to some students.

In the near future this study will be continued with additional cases and the observation of diverse solution plans. A detailed study of the solution processes seems indispensable since the difference were visible in the solution processes; however the written solutions of the students only exhibited few differences.

REFERENCES

Blum, W. (2006). Modellierungsaufgaben im Mathematikunterricht Herausforderung für Schüler und Lehrer. In A. Büchter, H. Humenberger, S. Hussmann, & S. Prediger (Hrsg.), *Realitätsnaher Mathematikunterricht vom Fach aus und für die Praxis* (p. 8-23). Hildesheim: Franzbecker.

Blum, W., Galbraith, P., Henn, H.-W., & Niss, M. (Hrsg.). (2007). *Modelling and Applications in Mathematics Education.* New York: Springer.

Borromeo Ferri, R. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*, (p. 2080-2089). Larnaca, Cyprus.

Greefrath, G., & Leuders, T. (in press). Verbrauch im Haushalt - Mit Schätzen und Überschlagen Probleme lösen. In B. Barzel, T. Leuders, S. Hussmann, & S. Prediger (Hrsg.), *mathewerkstatt.* Berlin: Cornelsen.

Hilmer, A. (2012). *Modellieren mithilfe eines Lösungsplans - Analyse der Bearbeitung eines Fermiproblems in der 6. Klasse.* Masterarbeit, Münster.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM, 38*(3), p. 302-310.

Maass, K. (2004). *Mathematisches Modellieren im Unterricht. Ergebnisse einer empirisches Studie.* Hildesheim: Franzbecker.

Meyer, M., & Voigt, J. (2010). Rationale Modellierungsprozesse. In B. Brandt, M. Fetzer, & M. Schütte (Hrsg.), *Auf den Spuren Interpretativer Unterrichtsforschung in der Mathematikdidaktik* (p. 117-148). Münster: Waxmann.

Pollak, H. O. (1977). The Interaction between Mathematics and Other School Subjects (Including Integrated Courses). In H. Athen, & H. Kunle (Hrsg.), *Proceedings of the Third International Congress on Mathematical Education* (p. 255 264). Karlsruhe.

Polya, G. (1973). *How to Solve It. A new Aspect of Mathematical Method.* Princeton: Princeton University Press.

Reiss, K., & Renkl, A. (2002). Learning to prove: The idea of heuristic examples. *Zentralblatt für Didaktik der Mathematik, 34*(1), p. 29-35.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando: Academic Press.

Schukajlow, S., Krämer, J., Blum, W., Besser, M., Brode, R., Leiss, D., et al. (2010). Lösungsplan in Schülerhand: zusätzliche Hürde oder Schlüssel zum Erfolg? *Beiträge zum Mathematikunterricht*, p. 771-774.

Sjuts, J. (2003). Metakognition per didaktisch-sozialem Vertrag. *Journal für Mathematikdidaktik, 24*(1), p. 18-40.

Sweller, J. (1988). Cognitive Load during Problem Solving. Cognitive Science 12, p. 257-285.

Strauss, A., & Corbin, J. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques.* Newbury Park, CA: Sage.

Zöttl, L., & Reiss, K. (2008). Modellierungskompetenz fördern mit heuristischen Lösungsbeispielen. *Beiträge zum Mathematikunterricht*, p. 189-192.