MATHEMATICAL MODELLING DISSCUSSED BY MATHEMATICAL MODELLERS

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This paper examines and discusses how two mathematical modellers work and what aspects of mathematical modelling they emphasise. Based on interviews it was found that they operate differently in terms of work organisation and work tasks. They also emphasised different aspect of modelling, one stressing mathematical aspects and the other focusing on other aspects, like how models are being used in the society.

Introduction

The relevance of using mathematics in and for out-of-school activities, in particular in and for waged labour, is one main argument for teaching mathematics in education (Romberg, 1992). However, the synergy between mathematics used in different workplaces and mathematics taught and learned at school is not always straight forward, but one *major issue* described as an interface between mathematics and a workplace is mathematical modelling (Sträßer, Damlamian & Rodrigues, 2012). Mathematical modelling is described both in curricula and research literature as a link between education and workplace settings.

Take the Swedish government written curriculum for upper secondary school as an example. It emphasizes, in the section of the aim of the subject mathematics, the use of mathematics in relation to workplace situations and to use investigating activities in an environment close to practice (Skolverket, 2011). One investigation activity is mathematical modelling, which is described as one of the seven teaching goals, i.e. to develop students' ability to "interpret a realistic situation and design a mathematical model and to use and validate a model's properties and limitations" (ibid, p. 91, my translation). The descriptions from the Swedish curriculum above indicate the use of realistic modelling activities in the mathematics classroom with a relation to workplaces, at least if the modelling problem is chosen adequately.

One example of educational research literature addressing the issue of modelling and workplace mathematics is the proceedings from the EIMI-study (Educational Interfaces between Mathematics and Industry) conference (Araújo, Fernandes, Azevedo, & Rodrigues, 2010). It includes several papers often related to engineering and modelling. Other examples of research literature that include modelling at a workplace are focusing on what Skovsmose (2006) would call *operators*, like bankers (Noss & Hoyles, 1996), telecom technicians (Triantafillou & Potari, 2010) and operators in a chemical plant (William & Wake, 2007). *Operators* are employees that make their working decisions based on apparatus, technology, with input and output of numerical values, in contrast to *constructors* who develop the technology, and *consumers* who evaluate models used for decisions based on information gathered from reading, watching, and/or listening to statements (Skovsmose, 2006). A common finding related to *operators*' use of mathematics is illustrated by Noss and Hoyles (1996), who investigated bankers' use and understanding of models and modelling. They found that the bankers mainly used computer aided tools with input and output values and the bankers did not consider the underlying mathematical structure of the models they used. The understanding of the mathematical models was preserved to the *rocket scientist*, as the bankers called the *constructor(s)* of the models. However, Noss and Hoyles (1996) did not interview or discuss the situation with the *rocket scientist* themselves, at least not in that paper, which could have been one way to create a communication link between the *constructor* and the *operators*. Overall there seem not to be much attention in research literature focusing on those that are *constructors* in the field and who call themselves mathematical modellers. How do they work? What aspects of modelling do they emphasise? What mathematics do they use? What are their views about modelling? What challenges do they meet in their work? How do they communicate with *operators/ consumers*?

The questions stated above are explored in an ongoing research project, where this pilot-study is a part. The research project will contribute to the ongoing research in mathematics education about modelling and to the understanding and conceptualization of workplace mathematics in that answers may develop new insights into pedagogy and curricula, links between school and workplace, and how mathematical meanings are created in and out of school contexts. This paper will focus on the first two questions above, with aim to examine and discuss how two modellers work and what aspects of mathematical modelling they express as central during their work.

WORKPLACE MATHEMATICS

The goals within the research area of workplace mathematics are several, such as to explore what and how mathematics is used in specific professions (Noss & Hoyles, 1996), to identify discrepancies and similarities between what mathematics is taught in school and what mathematics is needed in the workplace (Triantafillou & Potari, 2010), to analyse communication between *operators* and *consumers* (William & Wake, 2007), and to find strategies that will improve a general curricula that better prepare students for future work (Wake, 2012). What seems to be accepted by educational researchers is that workplace mathematics is not identical to school mathematics. Workplace mathematics is situated dependent and more complex, including specific technologies, social, political and cultural dimensions that are not found in any educational settings (e.g. Noss & Hoyles, 1996; Wedege, 2010). For example the linguistic conventions of representing mathematical models (formula, graph, table) (Triantafillou & Potari, 2010; William & Wake, 2007) are different in mathematics education and in some vocations. Even though the models in a workplace are specific, they offer a potential together with metaphors and gestures to facilitate communication of mathematics between *operators* to *consumers* (William & Wake, 2007). To allow communication about development and validation of mathematical models are also described as "principals for strategic curriculum design" that support workplace mathematics (Wake, 2012,p. 1686). Other principals given by Wake (2012) are: to take mathematics in practice into account; facilitate activities that pay attention to technology; and, to let students criticise mathematics used by others.

METHOLODGY

According to Wedege (2010) a researcher investigating mathematics at workplaces should consider two closely linked approaches, a *subjective approach* and a *general approach*. A *subjective approach* focuses on the workers' abilities and their (subject-ive) needs in their specific workplace, whereas a *general approach* focuses on (gene-ral) demands from the labour market and the society for "formal" (school) mathe-matical competencies needed in a workplace. A heuristic theoretical model by Salling Olesen (2008), addressing both these approaches, is suggested as a helpful research tool for investigating the dynamics of workplace learning and especially for workplace learning in mathematics (Wedege, 2010). Workplace learning is described "as the process in which individual workers learn by participating in work as a specific activity" (Salling Olesen, 2008, p. 115). An investigation of workplace learning at a specific occasion could be seen as a snapshot of what skills, emotions, knowledge and commitments the worker(s) have developed up to that point in time.

The model is described by Salling Olesen (2008) with the use of a triangle, where each corner is the centre of a small circle (see the figure on p. 119), to illustrate a relation between the three components *the societal work process* (division of labour, type of tasks and work organisation), *the knowledge available* (discipline, craft, methods and skills used in a workplace), and *the subjective working experiences* (individual/collective life history and their subjectivities like values, norms, emotions, etc that appear to be profession specific). Inside the triangle the words experiences, practices, identification and defensive responses are written to illustrate "that learning in the workplace occurs in a specific interplay of experiences and practices, identification and defensive responses" (p. 118). For example, a mathe-matics teacher may say "this modelling task is useful as a class activity (learned by experience), but it doesn't fit into our school made tests (learned through practice)" and "to calculate the half-life of Caesium we do in mathematics (learned through identification), but to set up models for radioactivity belongs to physics (learned through a defensive response)". The suggested theoretical model is helpful for this paper, since "the model pays particular attention to the cultural nature of the know-ledge and skills with which a worker approaches a work task, whether they come from a scientific discipline, a craft, or just as the established knowledge in the field" (p. 118) and "we can also see general subjects and skill such as literacy and mathematical modelling in this perspective" (p. 124). Modelling used in a vocation may be seen as the craft and the discipline will refer to mathematics. In addition the three components (*the societal work process, the knowledge available* and *the* *subjective working experiences*) may indicate the origin of the given reasons why the modellers emphasise some aspects of modelling more than others.

One appropriate method to capture the complexity of workplace mathematics is to use observations in a workplace together with interviews (Wedege, 2010). For this paper, I have used and developed semi-structured interview questions that pay attention to Salling Olesen's (2008) model and the research aim. The main source used in the construction of the interview questions is the set of critical questions developed by Jablonka (1996) for analysing mathematical models. According to Jablonka, the key aspect when someone is working with mathematical modelling is to judge the quality of the mathematical model. The interview questions are stated in the appendix together with a description of their purpose (to describe **how they [the modellers] work** and examine the modelling **aspects emphasised**) and their relation to Salling Olesen's (2008) three components (*societal, subjectivity*, and *knowledge*).

This is a pilot study and both the participants Adam and Ben (fictitious names) were previously known to me. The interviews were conducted and audio taped in June 2012 and lasted about 40 minutes (Adam) and 90 minutes (Ben) and later transcribed, summarized and analysed based on the categories **how they work** and **aspects emphasised** together with the three components of Salling Olesen (2008).

RESULTS AND ANALYSIS

Case Adam

Adam got his PhD in numerical analysis working with solutions to partial differential equations. He held a post doc position for a year and a half and after that he has worked with climate modelling and aerodynamic problems. Recently, part of his position is situated at a university working to develop new methods to solve differential equations. Mathematical modelling is very central to him, he said. He gets his working problems, with an aim to describe/simulate a reality, from meteorological institute and aircraft manufacturers. Briefly, the problems consist of a set of differential equations, developed by some physicist, which Adam solves by constructing computer programs. The programs consider initial values and constraints and they are used to simulate and compare to real data. The division of labour at his company is constructed after individuals' different abilities (i.e. numerical analyst, meteorologist, geophysicist and computer scientist). They work collaboratively, often in pairs, to understand what the best way is to solve something, but the collaboration is also about everyday problems like how something is going to be delivered. The main tool used for communication is mathematics, "you cannot formulate anything without it [mathematics]", said Adam. Artefacts used are whiteboards, computers or anything that can illustrate and/or simplify the problem to find a solution. He expressed that "mathematical modelling means to translate physics to mathematics" and gave an example about the movement of a pendulum that can be modelled by a differential equation. In addition he expressed that he is not involved in all steps of the modelling work: his competence consists of "translating mathematics to a computer model that will emulate the 'real' mathematical model". Adam mentioned that there are several difficulties while solving the equations like how to represent the move from the continuous model to a discrete model, to make simulations that are both accurate and stable and produce a result that someone can trust. The programming languages used are Fortran, C or C++, but he also uses other ICT tools like Mat-lab and statistical toolboxes in the modelling work. He expressed that most of the programming he has learned in his vocation. Doing climate simulations one needs to know input values, such as how the climate is now, where on the earth you are, how the vegetation is, how fast the earth turns. In order to minimize problematic data they use many different measure series made by satellites, which measure thousands of things. However, measurement errors and techniques are not his field of expertise, Adam said. You can verify and control a model, because you know some expected values, but it is difficult in practice with computer codes to actually get these values. This part he expressed as a very central part of his vocation and a bit frustrating, because the computer programs he writes do not always do what they supposed to do. The results the modelling team produce are predictions, therefore it is not possible to know which solution is correct. The validity of the results is based on historical data and climate trends. Nevertheless, a critical point brought up is that these models are just predictions that the *operators/consumers* need to consider, and there is one unit at the workplace dealing with communication between *constructors* and *operators/ consumers*.

The mathematical modelling (the craft) in Adam's work situation seems to be originating from pure mathematics (discipline), in particular the solving of differential equations, and may be considered as intra-mathematical modelling. He has to reformulate the given task in the mathematical domain, select relevant data, translate the mathematical model to a computer model, solve the computer model and interpret and evaluate the result, and finally evaluate the validity of the computer model. Adam's interpretation of modelling work and what aspects he emphasised may be influenced from all three components from Salling Olesen's (2008) model. Maybe most important for his reasons to emphasise the mathematical aspects of modelling is the work task (*societal*) described as differential equations. Other reasons that he expressed were the work organisation (*societal*) with predefined division of labour (numerical analyst, meteorologist etc) and his experience of teaching mathematical modelling courses at the university focusing on differential equations (*societal*). There are things that he has learned through work, i.e. programming and other methods related to ICT, which he expressed as a central part of modelling (*knowledge*). More reasons for his emphasis on the mathematical aspects may be the way they communicate with the use of mathematics (*societal*) mediated by artefacts (whiteboards, computers etc) as a part of their work practice (*subjectivity*). In addition, according to Adam, his description of modelling was similar to those of his colleagues, which may have evolved through their practice (*subjectivity*).

Case Ben

Ben has a PhD in mathematics with a thesis on probability theory. His working experience, where he explicitly worked with mathematical modelling, is wide. He has experienced modelling from a variety of practices, such as a municipality, the military defence, consulting companies and he has worked at different universities. Some examples of work tasks are: constructing water conservation plans (constructing a reality), simulating the interplay between humans and their recourses (simulating/ creating reality), and to develop measurement instruments for identification and estimation problems (constructing tools). His modelling knowledge has not come from general education, he has learned through his vocation, especially programming (Fortran). He argued that one of the strengths of mathematics is that you have a notation that makes it possible to present research and findings in a compact way and to identify cause and effect. Ben described modelling by describing how he worked with modelling. The modelling tasks, he said, are given to him by supervisors or companies. When companies ask for help they have often thought through the problems and want to get help with the mathematical parts. However, he stressed, "as a mathematical modeller one must first make the complete problem clear to oneself, it is not enough with the last part /.../ this process to identifying the problem and formulate the problem is a very long and slow process". He continued to express that he does not necessarily always end up with the same problem as the one given to him. To identify what processes, what variables, and what quantities are needed is important, but most important is to know what type of data exist or can be developed. Also, the consumer's (the company's) purpose must be taken into consideration; otherwise it may be problems to put the paper product into action. Validating is also expressed as an essential part of the modelling work and described as difficult. Ben is a bit concerned that people often draw too far conclusions from their models, especially when models are built only on simulated data, because reality is something else and more complex, "the only positive one can get out of a simulation is if it doesn't work, than it won't work in real life either, but you cannot be sure of that either". However, if the problem is about economy or efficiency of something than it is possible to put the result back into practice and confirm whether it was a saving or not. A result is often one among other results (a maximum can be flat and several values may give almost identical results) and then the consumer has to consider the outcome. Ben expressed that the consumer often wants to have a yes or a no and he needs to explain that the world is not dichotomous. For communication it is useful if the consumers understand the mathematical model, but sometimes they do not understand the model, which may be problematic, especially if they like the result and can use the model. Other times the consumers do understand the model, but they are not interested to control the underlying reasons and assumptions, which may also cause problems. The "misuse" of mathematical models is frequent according to Ben and he gives an example, which he has read recently in a statistical journal and refers to the ad hoc and quasi methodology used in PISA. Ben expressed that he works individually quite often, but that there are regular meetings with the consumers to make reconciliations. A problem with these meetings is that the consumers have to build up a certain body of knowledge for the meetings to be constructive, which may be too much to ask for, he said. Ben also said, that he has become a bit skeptical towards mathematical models used in the society, “one doesn’t solve society problems with the use of mathematical models – they may be used in negotiations by one or the other part”. The best negotiator often wins and the best option is not always picked, but that is democracy. ”You cannot talk about any un-political neutral mathematical models”, he added.

In contrast to Adam’s description of modeling, with an emphasis mainly on the mathematical domain, Ben’s description is wider and including aspects related to non-mathematical issues. Ben stressed the following aspects: to identify and formulate the problem; to identify relevant processes, variables, quantities and existing or none existing data; and validating the model. He also expressed a concern how models are being developed and used in the society and in companies as well as emphasized that communication between *constructors, operators and consumers* about mathematical models is a factor for a healthy democracy. Much of his reasons can be analyzed from different components of Salling Olesen’s model (2008). The design of the working task (*societal*) seems to effect Ben’s expressions. His working tasks are quite general and he needs to clarify and formulate a problem for himself to be able to identify possible variables, and processes, which may be why he expressed these aspects as central. The division of labour (*societal*) plays a part. Ben often works individually (*subjectivity*), which means that he has learned through his practical experiences (working with these tasks), and through communication with consumers and employers, what aspects are valued as important in his community of practice. In line with Adam, Ben expressed that he had learned programming (*knowledge*) and that this was useful for his occupation and a part of the modelling work. The concern about how models are used in society and about the political commitment of modelling in society may stem from an individual (*subjectivity*) conviction based on experience from being a *constructor* (developing models used in the society), *operator* (used and tested colleagues’ models for society) and *consumer* (reading and listening to explanations of models, such as PISA).

DISSCUSSION AND IMPLICATIONS

Both Adam and Ben call themselves mathematical modellers but their descriptions of how they work with modelling at their respective workplace are quite different. Adam works in a modelling group where the members have different specific roles and Adam’s role is to solve well-defined problems (solve differential equations). The division of labour may be one *societal* reason why Adam mainly emphasises mathematical aspects of the modelling and he does not put too much attention to other aspects because it is not a part of his position. Ben on the other hand has a wide experience of working individually (*subjectivity*) with more open problems, which may be a reason for stressing aspects related to non-mathematical issues. An aspect he brought forward is his critical approach to how models are being developed and used in society where the best models according to the modeller is not always the model chosen in practice, since there are more stakeholders that come into play in society (i.e. politicians, companies, negotiators, etc) with their own purposes. Similar and other social aspects of modelling that one ought to consider in mathematics education are elaborated and reflected about by Jablonka (2010). However, there are also similarities identified, for example that both modellers have learned programming at the workplace which was seen as a typical *knowledge* for their vocation, both expressed the importance of qualitative data and validation, and both described communication about mathematical models between *constructor*, *operator* and *consumer* as an essential part of the mathematical modelling. This last similarity identified, i.e. that communication between different practitioners with use of mathematical models is important for mathematics education, is discussed by Wake (2012) and William & Wake (2007). As was discussed above, modelling can function as a link between school mathematics and workplace mathematics. Ben's expressions and the statements about modelling in the Swedish curriculum (see the introduction) highlight the importance of interpreting a realistic situation and to evaluate a model's properties and limitations in modelling work, which means that these aspects also should be emphasised in mathematics education. However, both modellers expressed that validation is difficult in their work, and according to Jablonka (2010) validation mostly is a missing part in classroom practice, because the result is almost never put back into action in out of school settings. Still and maybe more problematic for modelling to be the ideal interface between industry and mathematics education, is the difference in objectives. In industry mathematical modelling is "the gateway into the use of mathematics" (Sträßer et al., 2012, p. 7872) whereas in education modelling is a mathematical classroom activity either as an aim in itself (to develop modelling competencies) or as an aim to develop a broader mathematical ability (didactical tool to learn mathematics) (see e.g. Blum & Niss, 1991). To develop *a* modelling competence, will be difficult to pursue based on this study since the two modellers presented such different descriptions. However, only few students will end up as mathematical modellers and thus the other aim, to use modelling as didactical tool to develop a broader understanding of mathematics, might be more useful for students. Modelling as a didactical tool could be used in teaching about mathematics hidden in technology, which is one aspect of modelling emphasised both in research literature (e.g. Jablonka, 2010; Noss & Hoyles, 1996; Wake, 2012) and in this study as important. One example of activities is to critical analyse mathematical models developed from technology and are used in the society. Not just to develop students' mathematical understanding of technology and to gain knowledge about the importance of communication between *constructors* and *operators/consumers*, but also to develop a critical view of how mathematical models are used in the society, which is an important ability of a critical citizen in a democracy (Skovsmose, 2006).

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Appendix

Interview questions	***The main aim of the interview question is to find:***
1.What is your academic background?	Individual life history (*subjectivity*)
2.What are you working life experiences before you got here?	Individual life history (*subjectivity*)
3.What is your vocation and what role does mathematical modelling play in your vocation?	Individual life history (*subjectivity*) and the institution with its culture (*societal*)
4.What does mathematical modelling mean to you? Make a general description how you work with a modelling problem (from start to end).	Individual/ Institution's view of modelling, both **how they work** and **aspects emphasised** (*subjectivity, societal and knowledge*)
5.Have your view on modelling changed during the years? (If yes) How?	Individual life history, change in **aspects emphasized** and why (*subjectivity and societal*)
6.Who gives you the problems to work with? What are the aims with the problems you get?	Work tasks (*societal*), **aspects emphasized**.
7.How do you work with mathematical modelling in your vocation (by yourself, in groups) If it is group work how/what communication take place? What types of artefacts are used?	Work organisation and communication (*societal*) as well as methods used (knowledge), **how they work**.
8.What type of problems do you work with?	Work tasks (*societal*), **aspects emphasized**.
9.What kind of models do you develop (static/dynamic, deterministic/stochastic, discrete/continuous, analytic/simulations)?	What mathematics/ methods are used (*knowledge*), **aspects emphasised**
10.What are the connections between input and output?	Methods used (*knowledge*), **aspects emphasised**
11. How was the necessary measurement data obtained? Is there a way to control the quality and the origin of the data? Can you give example of values and quantity of the data?	Methods used (*knowledge*), **how they work and aspects emphasised**
12. What factors may have affected the investigated phenomena (measuring instrument or its use)?	Methods used (*knowledge*), **aspects emphasised**
13. Is it possible to control the result? What types of assumptions have been made according to the context? Who decide what assumptions are being important? What is the accuracy of the result?	Methods used (*knowledge*), individual/ institution's view on assumptions (*subjectivity and societal*), **aspects emphasised**
14. How does the solution contribute to understanding and action?	individual/ institution's view on the result (*subjectivity and societal*)**, aspects emphasised**
15. What is an acceptable solution? Who set the goal for the mathematical activity? Who defines the criteria? Are there other solutions?	Methods used (*knowledge*), individual/ institution's view on criteria used/defined (*subjectivity and societal*), **aspects emphasised** and **how they work**
16. Are there any risk to use the result? If so, how is that considered? Is ethical issues discussed?	Methods used (*knowledge*), individual/ institution's view on ethical issues (*subjectivity and societal*), **aspects emphasised** and **how they work**
17. Is mathematical modelling something that was a part of your education in school or something you learned in your vocation?	Individual work life experience (*subjectivity, societal and knowledge*)