MODELLING IN SPANISH SECONDARY EDUCATION: DESCRIPTION OF A PRACTICAL EXPERIENCE

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The objective of this work is to present preliminary results of a study about the situation of modelling in Spanish secondary education and a proposal of action. Within the framework of anthropological theory we will show, through the analysis of the official syllabus, that modelling is knowledge to be taught. Despite this, after an analysis of official texts, we have observed that didactic formation of secondary teachers is very poor. Through an experience, developed and implemented in classroom by one of the authors, we will show a different way to work modelling tasks in secondary education.

INTRODUCTION AND THEORETICAL FRAMEWORK

During the last thirty years several authors have described mathematical modelling process -see for instance the works of Blum and Niss (1991), Lesh and Harel (2003) or Zbiek and Conner (2006)-. The main purpose of making use of a mathematical model is to tackle problems of the "real world" (or "extra mathematical world"). The modelling process can be divided in several steps: it begins with the interpretation and simplification of a problematic situation (real world problem). If appropriate, real data can be collected to provide more information about the problematic situation. These data lead to a mathematical problem (mathematical model). Thus, mathematical methods are used to get a mathematical result of the problem. This mathematical result has to be interpreted in relation to the extra mathematical context of the real world problem, thus the solver must validate the model checking if the mathematical solution is reasonable or can be improved. Finally, the solver has to communicate and justify the solution.

In the first part of our paper we analyse the state of modelling in secondary education in Spain using the lenses of Anthropological Theory of Didactics (ATD) to map the current social and political educational context. The method we have used in this first part is a careful documental analysis of official documents. This work is complemented with the work presented in Cabassut and Ferrando, (2012). In the second part, as a result of the previous analysis, we offer a proposal to promote the introduction of modelling activities in secondary classes. This second part is not strictly related with ATD but it is closely linked with the ideas of LEMA (Learning

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and Education in and through Modelling and Applications) project. Here we will present a teaching experiment through a quality-descriptive approach.

We have choose to focus on secondary school since last PISA results have shown that Spanish students from grades 9 and 10 have some difficulties to use mathematical knowledge to solve real world problems (OECD, 2010, p.133).

We assume that institutions have a great importance in teaching and learning process, that is why we have chosen to use two tools provided by the Anthropological Theory of Didactics (ATD): the didactic transposition and the determination levels.

The notion of didactic transposition was developed by Yves Chevallard at the early eighties. It results to consider that what is being taught at school is, in some sense, an exogenous production that is transposed to school. That is, a knowledge that will be taught in the school necessary goes through a process of transformation. The interest of this notion is that "considering the restriction bearing on educational institutions contributes to explain, in a more comprehensive way, what teachers and students do when they teach, study and learn mathematics", Bosch and Gascón (2006). A distinction is established between different kinds of "knowledge" appearing in the transposition process: the "scholarly" mathematical knowledge is the original one produced by institutions, the "knowledge to be taught" corresponds to the mathematical contents appearing in the official curricula, it doesn't necessary coincide with the mathematical knowledge as it is actually taught by teachers in the classrooms and, of course, this last one differs from the learned knowledge. The following scheme reproduces this process:

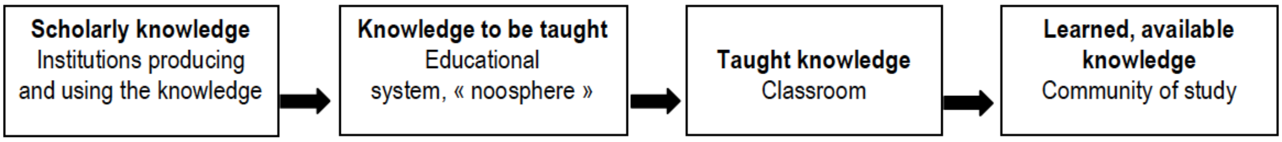

Figure 1: didactic transposition scheme (Bosch and Gascón, 2006, p. 56)

Chevallard (2002) proposes a way of structuring the conditions and restrictions that limit classroom work in a hierarchy of levels of didactic determination. Determination levels can be represented as follows:

Figure 2: determination levels

In (Chevallard, 2002) the reader can find a detailed explanation of the determination levels illustrated with several examples. Further information about the ATD, the theory of transposition and determination levels can be found in the work of Bosch and Gascón (2006).

MAPPING THE STATE OF MODELLING IN SPANISH SECONDARY SCHOOL

As we have mention in the introduction, the aim of this section is to analyze the state of modelling is in Spanish secondary education. In the article by Cabassut and the Ferrando (2012) the authors make a compared analysis of the current French and Spanish syllabus. After analysis of the program they conclude that, indeed, modelling is designated as knowledge to be taught. In what follows we discuss about the modelling from the point of view of the didactic determination levels of Chevallard. To achieve this goal we will refer to the official national syllabus that regulate secondary education studies and the training of teachers.

The structure of secondary education in Spain, regulated by the Education Law published in 2006 (BOE nº 106, 2006), is divided into two stages: compulsory secondary education (grades 7 to 10) and high school (grades 10 and 11) from which, after passing a screening test, students can enter university. As we argued in the introduction, we focus our work on compulsory education, from grade 7 to grade 10.

Let us analyse the status of the modelling through the didactic determination levels of Chevallard (see Fig. 2). As Cabassut and Ferrando (2012) point out modelling appears explicitly both in the content of the mathematics program (that corresponds to *discipline level*), in the methodology to be followed and program guidelines for secondary education (that corresponds to *pedagogic level*). To see what happens at *school level* (the upper determination level considered in this paper), we will analyze the training of future and in service teachers. Therefore, we will check whether it is feasible, as indicated by the curriculum, the use of modelling as a teaching/learning tool. For doing that, we will focus on official documents regulating access of teachers to the secondary school and regulating specialized courses that have existed for decades. According to the official document which regulates the access to the secondary teachers staff (BOE nº155, 1993), the first requirement is to have a bachelor's degree (4 or 5 years of higher education) that does not have (necessarily) to coincide with the speciality they will teach (a degree in biology or even in history can apply to a math position). It is also essential to have passed either the Teachers Aptitude Course (CAP, whose contents were defined in BOE nº192, 1971) or the Master Degree to be teacher in secondary school (that replaces the former CAP, BOE nº. 312, 2007). The later corresponds to a professional master degree and it has a higher teaching load. Those graduates who meet the requirements must pass an exam that consists, at least, of two parts: a theoretical part in which the applicant develops one of the items of the list of topics (see for instance BOE nº.226, 1993, p.27414) and a practical part in which the applicant should expose a specific teaching unit. Unfortunately, at this moment, there are still no active workers teaching in classrooms that have followed the master degree, therefore we cannot consider it yet. Then, we will focus on the Teacher Aptitude Course to elucidate which is the actual training of teaching secondary school teachers in Spain. As already mentioned, we must go back to 1971 to find the bases governing these courses that have a duration of 300 hours divided into two cycles of equal length. The first cycle is fundamentally devoted to theory and the second cycle is a practical cycle that is performed under the supervision of one or more teachers in a public school (the tasks in this second cycle are not fixed by the official document). Clearly, when we compare the training in their respective disciplines with pedagogical training of secondary school teachers (see for example the list of topics of the exam, BOE nº.226, 1993, p.27414), we conclude that the latter is exceedingly poor, particularly from the point of view of mathematical modelling.

Regarding in service teacher training it is also important to consider the role of ongoing formation courses. In Spain there is no centralized system of teacher training, so it is not easy to obtain global responses. Far as we know, the training courses for in service teachers are, except in special cases, of short duration and only the assistance is considered. In general they do not provide practical tools for its direct use in classroom, often by the logistical difficulties that this would represent for administrations managing courses. In Andalusia, a teacher training center has implemented a teachers training course oriented to modelling during the academic year 2008/09 based on the resources provided by LEMA project. Currently there is a group of teachers who is trying to implement a similar course in Valencia. In view of this lack of training for teachers, we understand that for most of the secondary school teachers it is difficult to work along the paths marking the official curriculum that, in terms of methodology, are not enough specific.

DESCRIPTION OF A EXPERIENCE

In this section we introduce the experience based on modelling activities carried out in courses of compulsory secondary education in the region of Comunidad Valenciana in Spain (grades 8 and 9). Mathematical subject of functions is on the basis of all the real problems presented to the students for modelling. Functions are present in all courses of compulsory secondary education. During grades 7 and 8, we start with the construction of tables of values and the interpretation of data and graphs, and in grades 9 and 10 students deal with the study of nonlinear functions and their interpretation in the real world. We begin with a general description of the experience and we will also provide the details of an instance of the material used (modelling practice).

The experience

A modelling practice is an activity that is conducted in small groups of two or three students. This allows the collaboration between them to deal with the problem, that is, our experience fosters the cooperative work (E.F. Barkley and others, 2007). This way of working gives them the option to discuss different points of view, compare their results and draw conclusions together, thereby facilitating the integration of students with different academic and cultural levels. John Dewey (1938) says, "you learn by doing." Through these modelling tasks students become the protagonists of their own learning. The groups are formed freely, depending on the own choice of students. Nevertheless the teacher can guide students can guide students for promoting that the groups are heterogeneous and the choice of members is governed by realistic criteria: schedules of all members of the group should be compatible, the geographical location of the homes should make possible the work together at home, and responsibility towards work or cultural and socioeconomic background could also be considered. This could lead to a culturally enriching activity for pupils attending at this kind of classes.

For the development of the activity, we have designed the following material:

- A teacher’s guide where it is described how to implement the specific practice, for example the one we will explain in the following section, “Footprints and mathematics”. The reason for giving this teacher’s guide is based on the conclusions stated in the first part of the paper, the preparation to teach mathematical modelling is very poor. This teacher’s guide is similar to the material used for traditional lessons.
- A student’s guide, where a “zero model” is described. The role of this model will be explained later.

Once the groups have been formed, the teacher explains what the work consists in, introducing different modelling practices that the work groups can choose freely, depending exclusively on their own interest. This presentation is made as attractive as possible for students in order to catch their attention. All the modelling practices are based on real situations and complementary materials are obtained through resources that students are used such, Internet, TV, Radio, advertisements, etc.

A whole session in classroom is devoted to the formation of working groups and the presentation of practices. During subsequent sessions every group will work on the chosen practice. Once every group have chosen a practice, the teacher gives to them the corresponding practice’s guide. This is a document where the work they have to develop is described, they can find the necessary material to start the job and it is included the development of a "Zero Model" or first stage model. With this material, they have to get familiar with the mathematical language, understand the information contained and try to deduce how it has been gathered. In summary in this first phase, they have to reproduce and interpret data and results contained in the guide. This will help them to understand a simple example. Then we ask them to create a new model, either applying the one showed in the practice’s guide, considering new data or observations or even creating another one totally different but inspired by the previous one.

The completion of the task concludes with a presentation of a document that is a report about the model and an oral presentation by the group members where they show to the rest of students their model and the achieved results. For the development of these practices, students know how the activity is going to be evaluated since we give them a table where the items considered are listed. The objective pursued is help students organize their work, always knowing what is asked from them. Students are not used to work in this way, and even less in the mathematics class, so they need a lot of help and guidance.

Modelling practices are organized throughout four different types of sessions: collective sessions, tutorial sessions, group sessions and oral presentations. The collective sessions are held during school time, every week during two months, and all students are present. In these sessions, the teacher answer general questions that arise in one group but can be of interest to the rest of the class -otherwise, the question will be answered only to this group-. One of the goals is to promote the collaboration and the debate. The objective is also to create a weekly routine, that is, every week the students have to devote time to the modelling practice and also they have a common space with the teacher and their fellows to solve any problems that could arise. The whole sessions take place either in the classroom or in the computer room, where students can look for information and also ask for questions that arise when they are using different computer programs (Word, Excel, etc). This activity makes the modelling task interdisciplinary, involving other subjects as Computing, Technology, etc.

Tutorial sessions take also place during school time, where every group works together with the teacher that acts only as a supervisor if help is required. At these sessions each group must show the state of work and can ask about the doubts or about of which is the approach they could give to the problem. Thus, if the group is stuck, the teacher can help them to consider possible ways to solve the problem.

Group sessions are held after school, that is, each group meets when they want and in some place chosen by them. The work developed by students in these sessions is difficult to be traced but, thanks to the use of the diary of the group (see next section), the teacher can know how many times they have met, who was present in each session and some details about the subjects treated, the produced ideas, the debate inside the group, etc. The delivery of this diary is compulsory for all groups.

The oral presentation of the practices is organized at the end of the activity and plays an important role in our modelling practices. The main goal is give the students the opportunity to show to the rest the work they have done. This session is carried out in the computer lab in any classroom with media. The teacher and the rest of the students are present. Each group has about ten minutes to make their presentation. The rest of the students can ask questions at the end of every presentation and the members of the team have to answer them. The teacher acts as a chairman. All presentations are recorded in video that represent a valuable tool for the further analysis of the productions of the students. The main goal of these sessions is to give the opportunity of students of arguing the conclusions and defending the results.

As a summary, with this kind of work we want to show to the students how, through mathematics, they can solve problems of daily life. Working mathematics in a different way than ordinary (lectures) is intended to arouse motivation and interest (Aravena and J. M. Giménez, 2002).

Practical example

We show below the description of one of the modelling practices proposed by one the authors to their students. This practice, entitled "Footprints and mathematics" has been carried out on students of degrees 8 and 9 and it is inspired by a task proposed in a PISA test (item 1 in 2009 test).

The starting point of the task is the question “what do you think it is the length of a human step?” and students have to work contents related to linear functions (concept of variable, linear dependency, parameters, etc). This is an open task in the sense that, from the starting point, students should consider other issues and learn how to solve them using the tools they consider necessaries.

The task begins with some open questions to arouse curiosity of the students. Some of the questions are, for example: What is the length of your footstep? How many paces can you walk in a minute? How fast are you? These questions are often difficult to answer a priori. That is why, before beginning the modelling task, some ideas and specific easier questions are presented to them. The following table presents real data. It is important to note that the table shows different variables: weight, height, gender and number of steps but students have to decide which of them are relevant.

	Weight (kg)	Height (m)	Age (years)	Number of paces in 10 meters				
Woman	65	1,67	36	13	14	13	13	13
Man	75,7	1,82	35	14	15	13	15	13
Child	26,6	1,28	6	16	17	15	15	16

Table 2: Data extracted from a real experience realized by one of the authors with her own family

Step by step the teacher explains in this “Zero model” how to obtain the average length of the step of the woman whose data appear in the table. Then, the pupils have just to apply the same reasoning in order to obtain the lengths of the steps of the man and child. After this short introduction some questions arise: How far can walk women in 10 steps, and in 15 steps, and in 20 steps? At this time students have to distinguish two variables: one independent (the number of steps) and one dependent (distance travelled) and also one parameter: the length of one step. To visualize the relationship between the variables and the parameter, the teacher asks them to build up a table of values and a graph. Once they have understood the calculations, the table and the graph, students are able to reproduce these calculations with the data provided in the case of the man and the child. After completing this section based on the table other questions arise. For example: “What kind of function have you obtained in each case?”, “Give a mathematical expression that relates the number of steps to the distance travelled.”, “Who is the fastest?”, “Why?”

Once they have understood the example, the process of developing the modelling task starts. At the beginning, in general, all groups try to repeat the above process using data they have collected by themselves. In order to arouse them to investigate other ways to find the length of a step, the address of several web sites are provided. Therefore they can observe that there is not only a unique valid method for this calculation, in fact they can try to create their own method. Once each group has obtained the step length of each group member, the teacher suggests them to use these data to calculate both distances and speeds. At this point it is important to compare both the estimate and the real measured distances so that they can see how small errors are amplified.

The end of the work corresponds to some proposals that the workgroups have to choose, for example: to make a study about the pedometers (how they work, what variables are introduced, etc) or conduct research on the relationships that may exist between the length of the footstep and the length of the leg or arm. This section is clearly the most open, since the students have to decide what issues arise from the initial problem.

The task ends whenever each group make an oral presentation of the work to their classmates and teacher but the final mark is based both in daily work, contained in the student's diary, in a written work and in the oral presentation.

As an example we introduce below some ideas and comments of the pupils. From the method shown in the practice students were able to create or find new methods to obtain the length of a footstep, as the following:

- With the footstep length obtained in the 10m and 20m they obtain the average of the lengths.
- They have found other ways to compute the length in Internet. For example they use the fact that the pace's length of a person corresponds to half the distance of the eyes to the soles of the feet.
- One of the students participating in the experience tried to adapt the methodology learned in the work to other physical activities like running or swimming.

We also want emphasise the reactions of the students after finish this task, these are some examples of the comments of the pupils:

- Ana: "I loved the task because I discovered interesting things and it taught us to apply mathematics to everyday things as simple as walking. The work was also too heavy because it required many hours of effort we suffered a lack of time."
- María: "This is an interesting task since we learned things and we learned how to apply mathematics to simple act of walking or swimming. I had fun despite the conflicts that have occurred in this work both in the classroom and in the group. Despite this I liked."

CONCLUSIONS

We have found that mathematical modelling is included as knowledge to be taught in the official programs that establish the contents and the methodology to work in secondary education. However, the poor training in didactics of teachers implies that, indeed, modelling is not a taught knowledge in Spanish secondary education. One way to bridge this gap between the programs and the real classroom praxis is to propose materials and methods that are easily implementable in relation to the training of teachers.

Our approach is different from other similar (as activities from LEMA project) in the sense that, besides proposing real life problems, it provides students a previous knowledge (Zero Model) that they can adapt, mimic, extend and use to test their own hypotheses. Our methodology tries to mimic the activity of research teams in science as the natural way for the discovery and research, in the sense that the starting point is the knowledge achieved previously by other groups and researchers. This methodology, in our opinion, does not detract the student's initiative and creativity, moreover, it is inclusive in two aspects: first, regarding student's diversity (social, cultural, knowledge, etc.) and second, in the sense that it connects more effectively with classical methodologies (lectures, problem sessions, etc) which are customarily followed in the classroom.

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