EXCEL MODELLING IN UPPER SECONDARY MATHEMATICS – A FEW TIPS FOR LEARNING FUNCTIONS AND CALCULUS

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The paper brings a few simple but powerful tips to using spreadsheets for teaching and learning functions and calculus. As the teaching practice of the authors has shown, the skills are unfamiliar to most of graduates of secondary schools and even upper secondary teachers. Some of the presented applications can be implemented in a very short time provided the students are basically skilled in spreadsheets.

INTRODUCTION

Mathematical modelling and applications has been in the focus of research in mathematics education for several decades (see Blum & Niss, 1991; Beare, 1993; Blum, 2002; Borromeo Ferri & Blum, 2011; Lingefjärd, 2011). Connection of mathematics to other sciences, its relevance to the outside world, learning concepts in context and connecting them through applications, teaching conceptually through helping students construct their own meanings grounded in real-life experiences – these are some of the basic ideas that underpinned this new vision of mathematics education at the pre-college level (Abramovich, 2003).

A powerful tool that enables easy access to ideas and concepts through a computational experiment are spreadsheets. The tool represents a most popular general-purpose software used by educators to promote the spirit of exploration and discovery by integrating experiment in teaching.

The first spreadsheet, VisiCalc, was created in 1979 as a tool to perform repetitive business calculations (Baker & Sugden, 2003). Since about 1995, Excel® has governed the spreadsheet market. It disposes of attractive and functional interface, number of functions, charting facilities, and tools such as Goal Seek, Solver, etc.

Educators discovered the educational potential of spreadsheet in mathematics and sciences in the early 1980s. Spreadsheets allow using problem-solving and heuristic methods that are close to the talented pupils. Only applications developed in special programming environments offer such ability of analyzing scientific problems.

"Spreadsheets … have a number of very significant benefits many of which should now be apparent. Firstly they facilitate a variety of learning styles which can be characterized by the terms: open-ended, problem-oriented, constructivist, investigative, discovery oriented, active and student-centred. In addition they offer the following additional benefits: they are interactive; they give immediate feedback to changing data; they enable data, formulae and graphical output to be available on the screen at once; they give students a large measure of control and ownership over their learning; and they can solve complex problems and handle large amounts of data without any need for programming" (Beare, 1993).

Baker and Sugden (2003) gave a detailed account on spreadsheets in education from 1979 till 2003. They prove on a wide range of research papers about spreadsheets in teaching mathematics, physics and computer science that

"… there is no longer a need to question the potential for spreadsheets to enhance the quality and experience of learning that is offered to students. Traditional barriers (…) need to be removed, either by ensuring that access to computers is improved or by changing assessment methods. Further expansion is needed of the types of topics that can be effectively covered by spreadsheet examples ..." (also see (Sugden, 2007)).

The paper was written on the occasion of launching the electronic journal Spreadsheets in Education. The goal of the journal is to create a forum for scholarly research into the use of spreadsheets at all levels of education, in which ideas on the use of spreadsheets can be exposed, explored, and reported to a practising audience to enable to adopt a technology for life to which all students should be exposed. Many articles has been published in the journal on spreadsheets in upper secondary mathematics (Abramovich, 2003, 2012), (Sugden, 2005), physics (Benacka, 2009), informatics (Lovászová & Hvorecký, 2003), (Benacka, 2008), etc. There are many articles in other journals and books on spreadsheet modelling in upper secondary mathematics and sciences (Neuwirth & Arganbright, 2004), (Benacka, 2011).

While developing the spreadsheet application, students project their mathematical knowledge into a tangible form and gain another view of mathematics. Spreadsheets offer to the students the power of dynamical discovering, which is an important element of inquiry based learning (IBL) in mathematics and science. Spreadsheets enable the students to develop applications that satisfy his/her specific requirements (the cells can be formatted conditionally; the parameters can be controlled by scrollbars and other components, etc.). These all are powerful motivating factors. When the application is ready, it can be saved and used again; if necessary, it can be modified or improved. A great advantage of Excel$^{®}$ is that it is a common equipment of computers at school and home; it is accessible almost everywhere.

In this paper, we bring a few simple but powerful tips to using spreadsheets for teaching and learning functions and calculus. As the authors' teaching practice has shown, the skills used are unfamiliar to most of graduates of secondary schools and even upper secondary teachers. The first author started developing such applications with his students during his 14 year teaching practice at gymnasium (grammar school; age 15–19) where he taught mathematics, physics, and informatics in ordinary classes as well as classes specialized in mathematics and informatics. Some of the presented applications can be implemented in a short time provided the user is basically skilled in spreadsheets. The authors practice developing such applications with gymnasium students within research, with preservice mathematics and informatics teachers in such subjects as Didactics, Methods of solving mathematical tasks, and Introduction to modelling and simulation, and also with inservice mathematics and informatics teachers within courses of professional development.

GRAPHING FUNCTIONS

Functions are a significant part of mathematic curriculum at upper secondary school. Students study the properties of the graphs, the relations between the parameters and the graph, etc. The knowledge is essential for studying STEM (see Michelsen, 2006). Computer programs are helpful when studying the topics. Among them, spreadsheets play especial role as they enable the students to develop their applications in the main idea of constructionism "learning by doing".

The application on the left side of Fig. 1 graphs functions if the definition domain is R. The graph is of xy line type made over 100 points in range B17:C117. Cell C13 contains the formula =(C12-C11)/100. Cell B17 contains =C11, Cell B18 contains the formula =B17+$C$13, which is copied down as far as row 117. Cell C17 contains =$C$5*(B17-$C$6)^2+$C$7, which is copied down as far as row 117. The application on the right side of Fig. 1 graphs functions if the definition domain is not R. The graph is of xy point type made over 10000 points in range B17:B10017, M17:M10017. Cell C13 contains the formula =(C12-C11)/10000. Cell C17 contains the formula =($C$5*B17+$C$6)/($C$7*B17+$C$8) while cell M17 contains =IF(ISERROR(C17),NA(),C17); both formulas are copied down as far as row 10017. The points that are out of the definition domain are skipped due to function NA().
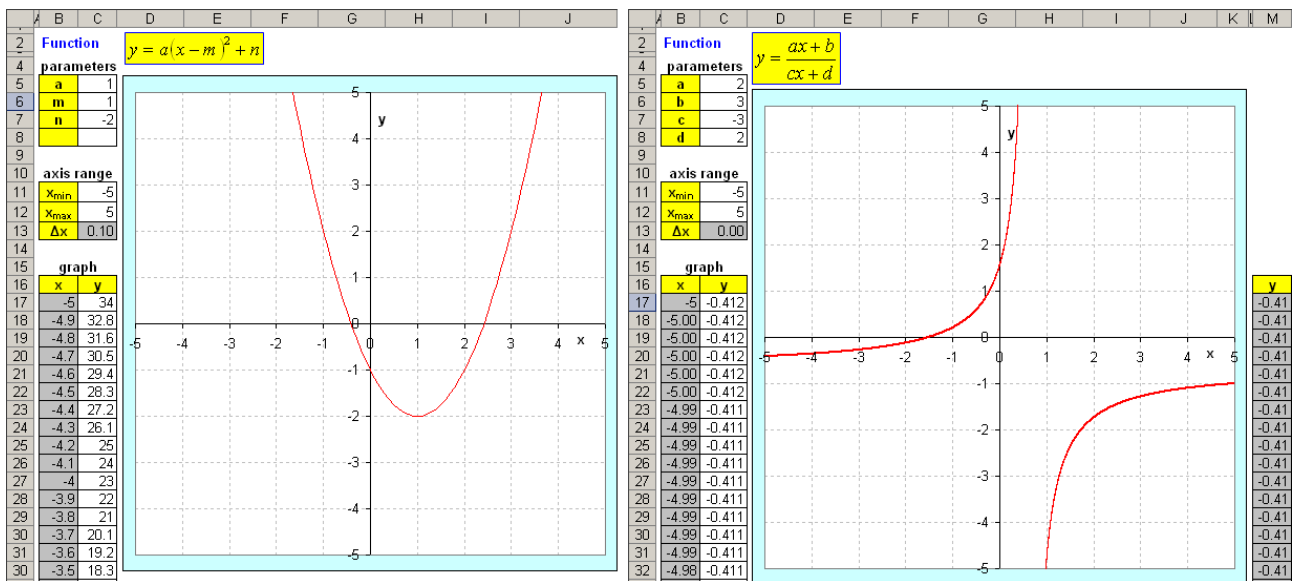
Figure 1. Graph of a function with D = R (left) and D ≠ R (right)

The trajectory of a projectile moving in a vacuum is depicted in Fig. 2, right side as a practical application of graphing functions. The projectile moves in accordance with the physics formulas. It is put in motion by clicking and holding down the spinbutton next to cell Q4. No programming is necessary. The user can change the elevation angle α, initial speed v_0 and acceleration due to gravity g, and verify the formulas derived in the lesson about the horizontal and vertical range, etc. The details can be found in Benacka (2009).The topic is taught at Slovak grammar school in the

optional subject Physics seminar. The application is an example of modelling in interdisciplinary teaching (see Maass & Mikelskis-Seifert, 2011).
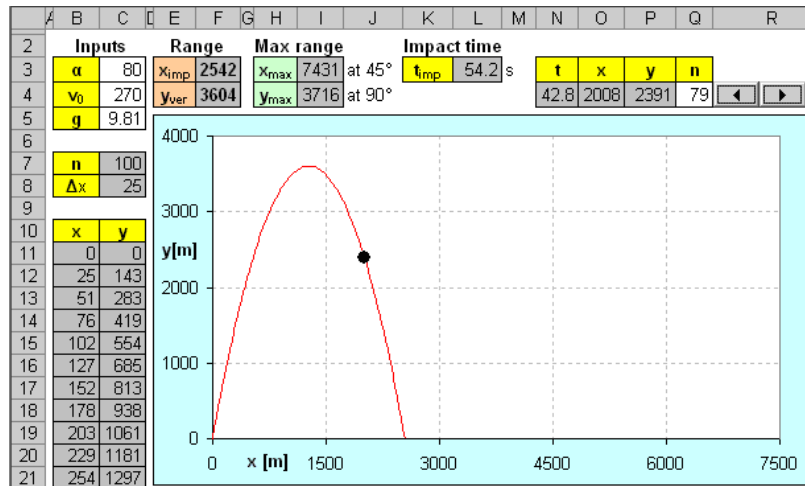

Figure 2. Projectile moving in a vacuum

FINDING EXTREMES

Extremal task are traditionally solved within calculus. However, they can be solved in Excel® without using derivatives. The problem of finding the maximum volume of a box made of a square sheet of 1m side length is solved in Fig. 4. After inputting the function $V(x) = x(1 - 2x)^2$ in range C17:C117 and adjusting manually the ranges of the axes, the maximum clearly appears on the graph. It can be found by enlarging the axis ranges (Fig. 4, right side) or by using the Solver tool. The latter is made in range J4:K4. The function is copied in cell K4, which is the Solver "Set target cell". From the Solver section "Equal To", the option "Max" is clicked. A value close to the stationary point (e.g. 0.2, see Fig.4, middle) is put in cell J4, which is the Solver input "By Changing Cells". After clicking OK, the maximum, appears in cell K4.
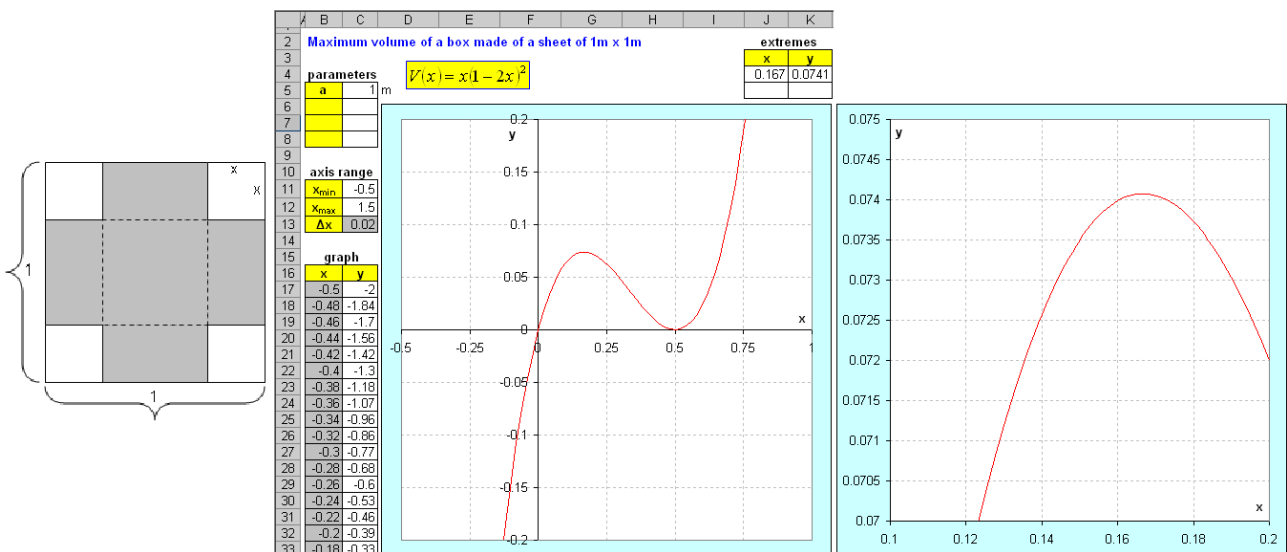

Figure 3. Maximum volume of a box made of a square sheet

SOLVING SYSTEMS OF LINEAR EQUATIONS

Another possibility of using Solver is to solve systems of linear equations. The problem of finding the parabola that is going through 3 given non-collinear points is solved in Fig. 5. Point A, B, C are given in range L6:M8 and added in blue into the chart. The system is solved in range L12:P15. Cells L12, M12 and N12 contain the formulas =L6^2, =L6 and =1. They are copied down into rows 13 and 14. Cell O12 contains the formula =M6, which is copied down, alike. Cells L15, M15 and N15 contain the estimated values of *a*, *b*, and *c;* it is enough to input 1, 1, 1 at the beginning. Cell P12 contains the formula =L12*$L$15+M12*$M$15+N12*$N$15, which is copied down into rows 13 and 14. Cells O14 and P14 contain the formulas =SUM(O12:O14) and =SUM(P12:P14). They are necessary for Solver. Now, we can start Solver. The Solver "Set target cell" is P15. From the Solver section "Equal To", the option "Value of" is clicked. Number 7 is inputted in the box next to it; it is the value from cell O15. The Solver input "By Changing Cells" is $L$15,$M$15,$N$15, that is, the cells that the estimate is in. After clicking button "Add", the constrains $P$12 = $O$12, $P$13 = $O$13 and $P$14 = $O$14 have to be added. After clicking button "Solve", the values of *a*, *b* and *c* appear in cells L15, M15 and N15. Then, the formulas =L15, =M15 and =N15 are inserted in cells C5, C6 and C7. The points of the parabola are calculated in range B17:C117 (see section Graphing functions). The parabola is going through the given three points. If the coordinates of points A, B and C are changed, then it is enough to start Solver and click "Solve".
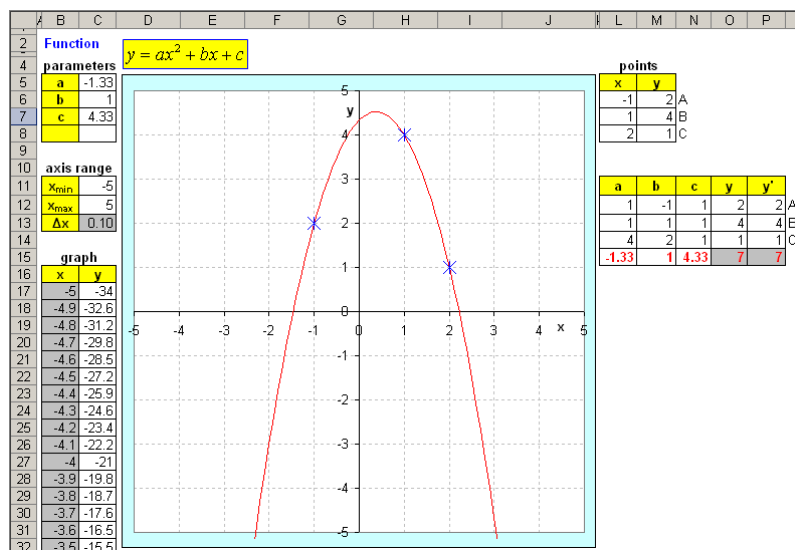

Figure 4. Finding the parabola that is going through 3 given non-collinear points

CALCULATING AREA – THE RIEMANN INTEGRAL

The application is in Fig. 6. The maximum number of subintervals is 1000. After creating the graph, the bounds are inputted in cells N4, N5. The formula =0 is inserted in cell L10. Cell L11 contains =L10+1, which is copied down as far as cell L20. Cell N6 contains =COUNTA(L11:L1010). Cell N8 contains =(N5-N4)/N6. Cell

M10 contains =N4. Cell M11 contains =M10+$N$8, which is copied down as far as cell M20. The equation of the function is copied from cell C15 into cell N10. Cell N10 is copied down as far as cell N20. Cell P11 contains =IF(N10<N11,N10,N11), cell Q11 contains =IF(N10>=N11,N10,N11); the formulas are copied down as far as row 20. Cell P10 and Q10 contain =SUM(P11:P1010)*N8 and =SUM(Q11:Q1010)*N8. Cell Q4 contains =(P10+Q10)/2, which gives the result 2.335 as the average of the lower and upper integral sum. Cell Q5 contains =(Q10-P10)/2, which is the maximum possible error. To get a more accurate result, it is enough to select range L20:Q20 and pull down, e.g. as far as row 20; the result is 2.334 (Fig. 6, middle). To reduce the number of subintervals, e.g. to 5, it is enough to select the redundant range L16:Q30 and delete. The result is 2.340 (Fig. 6, right).
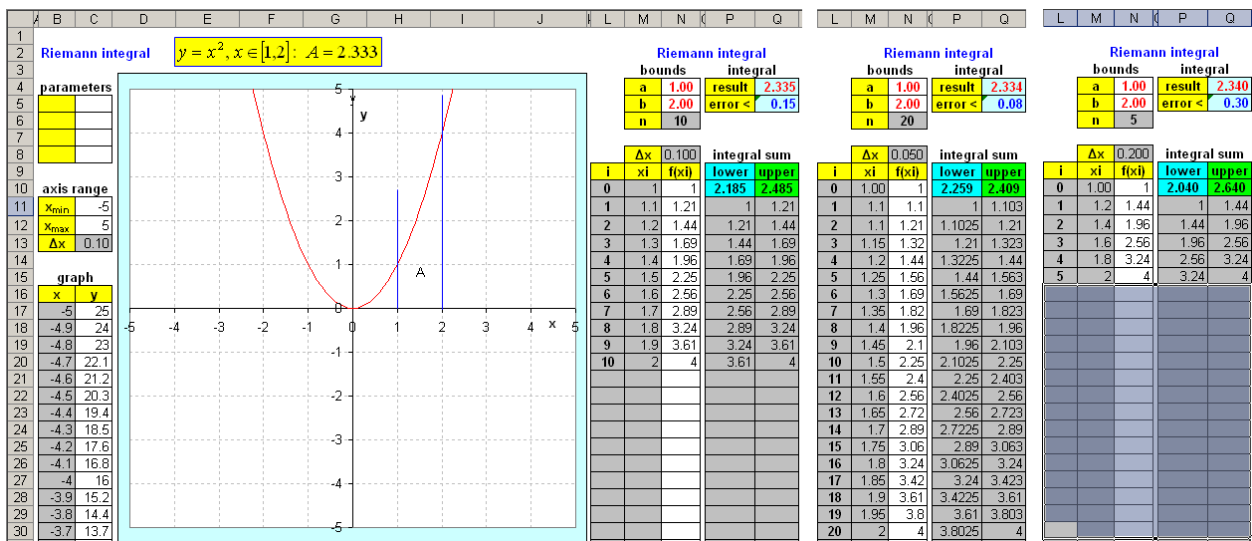

Figure 5. Calculating area by the Riemann integral

CALCULATING AREA - THE MONTE CARLO METHOD

The application is in Fig. 7. The data for the graph are in columns H and I (the blue curve in the chart). After creating the graph, bounds a and b are inputted in cells C3, C4. Maximum M of the function on the interval is inserted in cell C5. The area S of the rectangle between a, b, M and axis x is computed in cell C6 by the formula =(C4-C3)*C5. Cell F2 contains =1000, which is the number of the generated numbers. Cell C9 contains =$C$3+($C$4-$C$3)*RAND(). Cell D9 contains =1/C9. Cell E9 contains =$C$5*RAND(). Cell F9 contains =IF(E9<=D9,1,0). It gives 1 if y_i is on the graph (the blue curve) or below it, otherwise it gives 0. The formulas from range C9:F9 are copied down as far as row 1008. The (x_i, y_i) pairs from ranges C9:C1008, E9:E1008 are graphed as xy poin graph (the red points in the chart). Cell F3 contains =SUM(F9:F1008), which gives the number of the points on the graph or below, which are the "good" ones. Cell F4 contains =(C4-C3)*C5*F3/F2, which gives the result. The calculation is based on the fact that the ratio of the number of the "good" and "all" points equals the ratio of the area below the graph and the rectangle that

comprises it. Cell C2 contains =LN(2). Cells F5 and F6 contain =C2-F4 and =F5/C2*100. The calculation can easily be adjusted to 10000 points if =10000 is inputted in cell F2 and the formulas in range C9: F9 are copied as far as row 10008. To show the 10000 points (ranges C9:C10008, E9:E10008), their data have to be added into the chart (Fig. 7, right side). If 10000 points are generated instead of 1000, then the error is smaller about ten times.
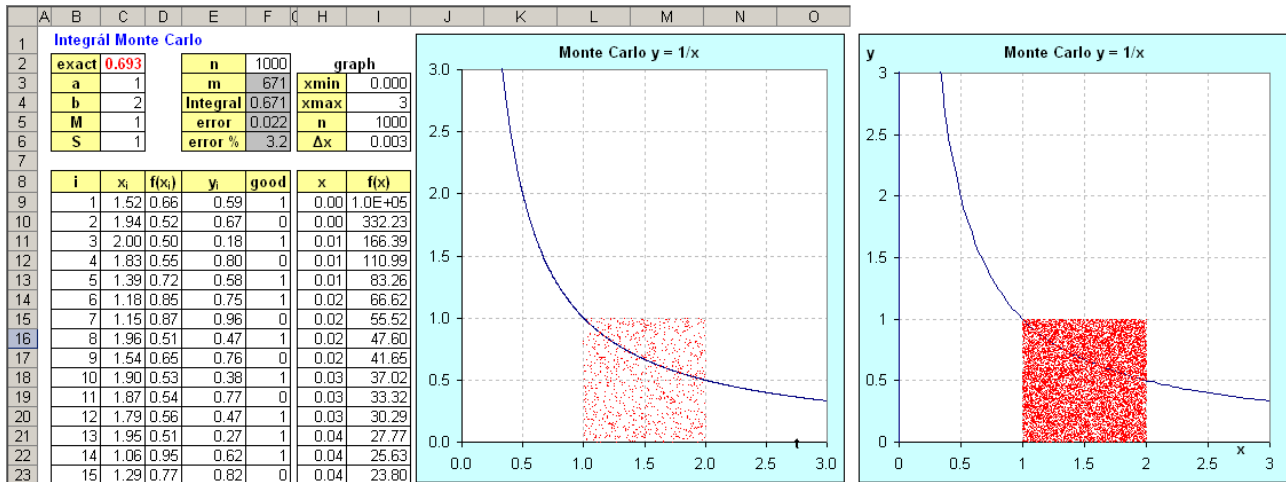

Figure 6. Calculating area by Monte Carlo method; 1000 points (a), 10000 points (b)

PEDAGOGICAL REMARKS

Applications identical or similar to the presented once were developed recently with four groups of gymnasium students. They were familiar with writing formulas, absolute addressing and making bar and pie graphs. They had never graphed a function, heard of Goal Seek and Solver, and used Active X or Form components.

Group 1: Applications identical to those in Fig. 1 were developed as an introduction to graphing functions in Delphi with eight students of age 17 – 19 at Gymnazium in Nove Zamky in a 90 minute lesson within Computer Modelling Club that the first author runs at the school. It was emphasized at the end that "Now, you are able to graph any function given by a formula", however, no real background was given to the functions graphed, e.g., what problem they origin from, etc. Then, the students were asked to answer the following three most subjective questions: 1) The lessons were (1:very; 2:quite; 3: little; 4:not) interesting; 2) I understood (1:all; 2:majority; 3:minority; 4:nothing) of the mathematics; 3) I learned (1:very much; 2:quite much; 3: little; 4:no) new skills in Excel. The average answers are in Tab. 1 in column NZ.

Group 2: Applications similar to that in Fig. 2 were developed with ten students of class 2A (age 16 – 17) of Gymnazium Parovska in Nitra in two 45 minute lessons with 20 min break. They simulated free fall and projectile motion in a vacuum. The theory was presented, which was new to the students. Despite it was emphasized that the nearest place in which the theory holds is the Moon due to the vacuum, the students were much engaged – putting the ball in motion was clearly a hit. The

students were asked the same questions as Group 1 just the word "mathematics" was replaced with "physics" in question 2. The answers are in Tab. 1 in column 2A.

Group 3: Application similar to those in Figs. 4 and 5 were developed by nine students of class 2B of Gymnazium Parovska in Nitra in two 45 minute straight lessons (10 min break). The first one was introduced as follows: "You are the owner of a small factory that produces pins. You pack the pins in boxes that you make from square sheets of 100 cm sides that the paper factory delivers to you. The method is shown in the figure. The bigger is the volume of the box, the more money you save on the costs. Find the x at which the volume of the box is maximum". To make the solution closer to everyday life, side a was inputted in centimetres and the volume was calculated and graphed in litres. It was found that x can only go from 0 to $a/2$. The maximum $V_{max} = 74.1\,\ell$ at $x = 16.7\,cm$ was found by Solver. Then, the problem was solved of finding the x at which the volume is 50ℓ, first by Solver and then by Goal Seek. After showing finding the first root ($x = 6.65\,cm$), the students quickly found the second one ($x = 29.4\,cm$). The second application was introduced as follows: "Imagine a battle going on the Moon. Cannon shell was fired by the enemy. We can monitor its motion by radar. Three positions are given by points A, B and C in cells L6:M8. In a vacuum, the trajectory of a projectile is a parabola. Find the points from which the shell was fired and in which it will hit the ground. Find the maximum altitude reached". The application can be solved faster if y' refers to cells C5:C7, Solver constrains refer to cells M6:M8, and no matrix is created. Showing the points with negative y coordinate has no sense in this case. The students were asked at the end of the second lesson the same questions as Group 1. The answers are in Tab. 1 in column 2B.

Group 4: Applications similar to those in Figs. 6 and 7 were developed with seven students of class 2C of Gymnazium Parovska in Nitra in two 45 minute straight lessons (10 min break). It was emphasized at the end that "Now, you are able to calculate any area bounded by the x axis and the graph of a given function", however, no real background was given to the tasks. The students were asked the same questions as Group 1. The answers are in Tab. 1 in column 2C.

NZ	2A	2B	2C
1.6	1.1	1.1	2.0
1.3	1.5	1.8	2.0
1.9	1.5	1.6	1.9

Table 1. Average answers to the questions

The answer most important to the authors was the first one. It can be seen in Tab. 1 that even if there was no real background to the task, the students found the lessons quite interesting; if there was a more or less real background, they found the lessons very interesting. There is the question if the samples were representative enough. While the students in Group 1 are supposed to have positive attitude to maths and sciences, the students in the other groups were chosen randomly. The classes

comprise 30 students at most. They are divided to three groups, which are the thirds of the alphabetical list, to fit the number 10 of the computers in the computer rooms.

Final remark: Actually, four questions were given to the students in Group 2. The fourth one was if they would like to model fall and projectile motion in the air. They all answered "yes". The question is whether they really wanted to make the model or just kill another lesson of traditional teaching. Both answers are positive from the aspect of making teaching and learning more attractive. However, when the first author was teaching next day in the other class, two of the students came in the room when they spotted him and said that they would come even after the school to develop the models in the air. That was a hit!

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