CREATING NECESSARY CONDITIONS FOR MATHEMATICAL MODELLING AT UNIVERSITY LEVEL

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This paper focuses on the study of the 'ecology' of mathematical modelling in the teaching of mathematics at university level. Using the framework of the Anthropological Theory of the Didactic, we introduce the notion of study and research courses as an 'ideal' didactic device for integrating mathematical modelling into current educational systems. We explore some of their essential characteristics or principles that can help their design, testing and analysis in order to face and overcome some of the constraints that hinder the development of mathematical modelling activities.

1. RESEARCH BACKGROUND: THE ECOLOGICAL DIMENSION IN THE MATHEMATICAL MODELLING PROBLEM

The starting point of our research is the problem of teaching mathematics as a modelling tool in first-year university courses. More specifically, we focus on studying the 'ecology' of mathematical modelling practices in these institutions, that is, the set of *conditions* that favour and the *constraints* that hinder (or prevent) its large-scale development as normalised activities into current educational institutions (Barquero, Bosch & Gascón 2008 and 2010, Artaud 2007). We postulate that the integration of modelling activity needs an in-depth analysis of this ecology.

We use the framework of the Anthropological Theory of the Didactic (ATD) and its conception of mathematical modelling. The ATD uses the notion of mathematical praxeology (MP) as a fundamental tool to describe and analyse mathematical activity. In accordance with García et al. (2006), we consider modelling as a *process of reconstruction and articulation of MP of increasing complexity*. This process can start with the study of a question arising in an extra-mathematical situation, that we call the 'initial system', and leads to the construction of a MP that can act as a 'model' of the considered initial system. This work usually creates new questions, which require the construction of new models, the previous model thus acting as a 'system' of this new modelling process (Serrano, Bosch & Gascón 2010). From this perspective, intra-mathematical modelling—that is, the process of modelling a mathematical system—appears as a particular case of mathematical modelling and allows considering it not only as a way to make the functionality of mathematics visible, but also as a key tool for the construction and connection of mathematical contents. Thus, *mathematical modelling cannot be considered only as an aspect or modality of mathematical activity but has to be placed at the core of it*. This integration constitutes an essential aspect of our research problem. We will refer to it as the *ecological problem* of mathematical modelling:

What *limitations* and *constraints* in our current educational systems prevent mathematical modelling from being widely incorporated in daily classroom activities? What kind of *conditions* could help a large-scale integration of mathematical modelling at school? What kind of *didactic devices* would make large-scale integration of mathematical modelling possible?

In order to face this problem, from the ATD we propose to use the notion of *study and research courses* (SRC), introduced by Yves Chevallard (2006), as a didactic device to facilitate the inclusion of mathematical modelling in educational systems, and, more importantly, to explicitly situate mathematical modelling problems in the centre of teaching and learning processes (Barquero et al. 2008). In this paper, we will introduce some of what we consider the main traits of SRC. They can be understood as working principles (or assumptions) that can be taken into account for a design, implementation and analysis of a SRC. We postulate that they are necessary to create appropriate *conditions* to face some of the most important *constraints* to the 'normal' life of mathematical modelling at university institutions (Barquero et al 2012).

2. TOWARDS A NEW DIDACTIC PARADIGM: RESEARCH AND STUDY COURSES

Chevallard introduced the notion of SRC (Chevallard 2006) as a general model for designing and analyzing study processes. Its main purpose came from the need to introduce a new epistemology to replace the still dominant 'monumentalistic' epistemology (where mathematical contents appear as monuments to visit) for one which could (re)establish the 'raison d'être' and the functionality of mathematics at school. In Chevallard's words, a change of paradigm at school is completely necessary: from the paradigm of *visiting works and its shortcomings* towards the new didactic *paradigm of questioning the world* (Chevallard 2012).

2.1. General conditions and research methodology for the testing of the SRC

During the academic year 2006/07, our research group started implementing SRC with first-year university students of business and administration degree (4-year programme) in IQS School of Management of Universitat Ramon Llull in Barcelona (Spain). Since then, they have been implemented year after year with some variations and improvements. A special device, called the 'mathematical modelling workshop', was introduced in the general organisation of the mathematical course. It consist in 90-minutes weekly sessions covering one third of classroom time for students, and more than half of their personal work outside of the classroom. The instructor of the course is also the responsible of the workshop sessions. These ran in parallel to the three-hour weekly lecture sessions, which included problem-solving activities. Its attendance was mandatory for the students and it would provide forty per cent of the final grade.

In the general organisation of a workshop, students work in teams of 3 or 4 members, under the supervision of the instructor responsible of the course and, if possible, of a researcher who acts as an observer. In most of its implementations, the workshop focuses on a single initial problematic question *Q*, to which students have to provide a complete response during the entire academic year. It can also consist in three linked questions, one for each term. Once the initial question is presented, two kinds of workshop sessions are combined every week: teamwork and presentations. In the first ones, each team has to look for 'temporary' responses to partial questions derived from *Q* and prepare a 'partial' report with these responses. Then, the reports are orally defended on the subsequent sessions by some selected working teams. A discussion follows to state what progress has been made, and to agree on how to continue the study process. During the presentation sessions, one member of the class (named the 'secretary') prepares a report containing the main points in the discussion and the new questions proposed to be studied in the following sessions. At the end of a term, each student has to individually write a final report of the entire study (evolution of problematic questions, work in and with different models, relationship between them, etc.).

At this stage of the process, the collected data of the implemented SRC comprises the students' team and individual reports, the teacher's written description of the work carried out during each session, the worksheets given to the students and a brief questionnaire to the students at the end of the each term. It constitutes the empirical base upon which the *analysis a posteriori* of the SRC rests.

2.2. Development of the SRC implementation: How does the population of users of a social network evolve over the time?

We focus here on the most recent experimentation of the SRC, during the academic year 2010/11. In this occasion, the implementation of the SRC focused on the generative question (Q_0) about *the evolution of the number of users in a social network* called *Lunatic World* (see figure 1). The initial question led to consider different kinds of mathematical models depending on the assumptions made on the initial system.

This initial question was divided into three sub-questions, which were approached in one term each. The division was made based on the necessary tools for their resolution. For instance, the initial and generative problematic question Q_0 was partially approached using discrete models and assuming independent generations of users during the first term of the course. It was then approached using functional models, so as to fit the best continuous function to real data during the second term. The third branch, developed during the third term, came from the use of discrete models, assuming that all users belonged to different groups. It led to reformulate Q_0 as Q_3 (see figure 2). We will refer to this case in the following section.

www.LunaticWorld.com

What is a social network? A social network is a social structure made up of a set of actors (such as individuals or organizations) and the dyadic ties between these actors. The study of these structures uses social network analysis to identify local and global patterns, locate influential entities, and examine network dynamics.

We will focus on studying the evolution of the number of user of a social network, which is called *LunaticWorld*. It was created in 2004 by 18 users. One of its main characteristics is that a person can become part of it only if another user invites him/her. In the following table you can find information about the evolution of the number of users of this network over some years. With this information, it is proposed to look for responses to the following initial question (Q_0):

Year	Number of users
2004	18
2005	56
2006	151
2007	447
2008	1034
2009	3143

Given the size of population over some time period,

- Can we predict its size after n periods? Is it always possible to predict the long-term behaviour of the population size?
- What sort of assumptions on the population and its surroundings should be made?
- How can one create forecasts and test them?

Figure 1. Introductory worksheet to the workshop:

Presentation of the social network and of the initial question Q_0

MATHEMATICAL MODELLING WORKSHOP – 3rd term

The social network *Lunatic World* has recently introduced important changes. From now on, all users will be distributed in three different groups: ***Basic***, ***Medium*** and ***Premium***. After one month, a user can remain in the same group, be promoted to another group or leave the network. Moreover, the only type of users that are allowed to invite other users to join the following month are 'Medium' and 'Premium' groups.

CASE 1 [One of the 16 cases that were distributed to the different teams]:

- *15% of 'Basic' users remains as 'Basic' one month later and 75% of 'Basic' changes to 'Medium';*
- *25% of 'Medium' changes to 'Premium' the following month and 50% remains as 'Medium'. Each 'Medium' user invites on average 4 new users that enter as 'Basic' users the following month;*
- *90% of 'Premium' remains in the same group. Each 'Premium' user invites on average 3 new 'Basic' users.*

Figure 2. Introductory worksheet to the third branch of the SRC:
Discrete models with users distributed in different groups

The different phases of the design, application and analysis of this SRC were similar to those developed in a SRC on population dynamics (Barquero at al. 2008 and 2009)[1]. After the implementation of SRC with first-year university students for more that six academic years, we can talk more concretely about the main traits of SRC that seem important to create appropriate *conditions* to a 'real' development of mathematical modelling at university institutions. As mentioned earlier, they can be understood as working principles (or assumptions) for the design and carrying out of SRC.

[1] For more details see: http://webprofesores.iese.edu/valbeniz/bbarquero/BarqueroBoschGascon_app.pdf

3. Creating conditions for mathematical modelling

3.1. A generative question is the starting point of functional study processes

The starting point of a SRC should be a 'lively' question with real interest for the community of study. We call it the *generative question* of the study process, and denote it by Q_0. It should not be a question imposed by the instructor to cover some didactic needs fixed a priori. That is, obtaining answers to Q_0 has to become the main purpose and an end in itself. The study of Q_0, together with the derived questions that can appear along the study, is the *origin*, *engine* and *'raison d'être'* of all the study process. In this sense, Q_0 should be present during the entire study process and acts as its articulating axis.

The case of the SRC on '*How does the population of users of a social network evolve over the time?'* provides a good example of the power of its generative question. With its implementation, we verified how the sequence of questions arising from Q_0 led the students and the teacher to consider most of the main contents of the entire mathematics course. In each term, different aspects that revolve around the initial situation were analyzed. They required the mobilization of various types of mathematical models: forecasting the number of users in the short and long term, considering time as a discrete variable (first-order sequences models, *1st term*), the same forecast considering time as a continuous variable (differential equations, *2nd term*), and the forecast in discrete time distinguishing three user groups with different privileges (models based on matrix algebra, *3rd term*). However, during the SRC, these contents appeared in a very different structure from the traditional organisation. Instead of the classical 'logic of mathematical concepts', the workshop was more guided by the 'logic of the problematic questions' and 'types of models' that progressively appeared.

Another important outcome consists in the necessity to break the rigidity of the classical structure 'lectures - problem sessions - exams', based on the sequence 'introducing new contents - applying the contents'. It can be considered as an important constraint to the integration of mathematical modelling. But it was still important to ensure that both lectures and problem sessions were taken into account during the workshop. As a result, there had to be a bidirectional relationship between all these didactic devices. On the one hand, 'lectures and problem sessions' are used to provide students with some of the necessary tools to be able to follow with the workshop. And, vice versa, the workshop motivates and to shows the functionality of the main content of the course.

3.2. SRC have a tree structure, as a consequence of the search of responses to Q_0

During a SRC, the study of the generative question Q_0 evolves and opens many other 'derived questions': Q_1, Q_2,…, Q_n. One must constantly question whether these derived questions are relevant. The fundamental criterion to decide whether they are indeed relevant is to ensure that they are capable of providing responses R_i that are helpful in elaborating a final response $R^{\heartsuit}$ to Q_0.

As a result, the study of Q_0, and of its derived questions Q_i, leads to successive temporary responses R_i which would be tracing out the possible 'routes' to be followed in the effective experimentation of the SRC. We claim that the work of production or construction of $R^\heartsuit$ can be described as a tree of questions Q_i and temporary answers ($R_i = \text{MP}_i$) related to each other during a modelling process that is both *progressive* and *recursive*. For instance, we can see in the following diagram, in terms of questions and their successive responses, the structure of the 3rd branch of the SRC about 'Discrete models with users distributed in different groups'. Its study led to the consideration of two MP: the fist one based on the construction of models based on Leslie matrices and its use for the short-, medium- and long-term forecast of users' distribution and the second one focusing on the study of powers of matrices, with Leslie matrices being a particular case.

$$Q_0 \Rightarrow H_1^{(3)} \text{ and } Q_1^{(3)} \rightarrow \begin{cases} \left(Q_{1.1}^{(3)}, R_{1.1}^{(3)}\right) \\ \left(Q_{1.2}^{(3)}, R_{3.2}^{(3)}\right) \\ \left(Q_{1.3}^{(3)}, R_{1.3}^{(3)}\right) \end{cases} \Rightarrow Q_2^{(3)} \rightarrow \begin{cases} Q_{2.1}^{(3)} \rightarrow R_{2.1}^{(3)} \\ Q_{2.2}^{(3)} \begin{cases} \left(Q_{2.2.1}^{(3)}, R_{2.2.1}^{(3)}\right) \rightarrow \left(Q_{2.3}^{(3)}, R_{2.3}^{(3)}\right) \\ \left(Q_{2.2.2}^{(3)}, R_{2.2.2}^{(3)}\right) \rightarrow \left(Q_{2.3}^{(3)}, R_{2.3}^{(3)}\right) \end{cases} \end{cases}$$

MP Leslie matrices

MP Study of $\{M^n\}_{n\in\mathbb{N}}$

Some examples on the formulation of questions that appear in the structure of the third branch of the SRC:

$Q_{1.1}^{(3)}$: How can we describe the evolution of the distribution of users in groups under the new conditions of *Lunatic World network* (see figure 2)?

$Q_{1.2}^{(3)}$: Will it be always possible to forecast the future distribution of the users after some periods of time? Which will be the long-term distribution of users? **[…]**

$Q_{2.2.2}^{(3)}$: In the case of Leslie matrices (L) of 2nd order, what are the main properties of L^n? What can we say about $\lim_{n\to\infty}\{L^n\}_{n\in\mathbb{N}}$? Can we generalise their properties to the case of a Leslie matrix of *n-order?*

Let us stress the importance and utility for students/teacher/researchers of what we have called the *mathematic a priori design* of the SRC to guarantee that the generative question is sufficiently 'fertile' to lead to many other derived questions. The design also gives a detailed description of the possible evolution of the study of Q_0 in terms of potential derived questions and their successive responses (Q_i, R_i), which would be tracing out the possible routes to be followed in the effective experimentation of the SRC.

3.3. Promoting the role of the study community: The dialectics between individual and the community

A 'real' integration of mathematical modelling needs to *promote the role of the study community* along with that of the *director* of study. This study community has to be in charge of 'collectively' studying Q_0 and producing an appropriate response $R^\heartsuit$. In contrast with the 'dominant pedagogy' where there is a dominance of 'individual' work under the orders of teacher, the group of students and their director of study have to share the set of tasks and negotiate the responsibilities that each of them has to assume.

This displacement going from the individual to the community has many important consequences to make the existence of mathematical modelling possible. On the one hand, the collective study of questions provides the opportunity of *defending responses* produced by the community, instead of accepting the *imposition of the official answers*. On the other hand, this work required that students took a lot of new responsibilities that the 'traditional didactic contract' assigns exclusively to the teacher, for instance: addressing new questions, creating hypotheses, searching and discussing different ways of looking for an answer, comparing experimental data and reality, choosing the relevant mathematical tools, criticizing the scope of the models constructed, writing and defending reports with partial or final answers, etc. The teacher thus had to assume a new role of acting like the leader of the study process, instead of lecturing the students. And it soon appeared that the teaching culture at university level does not offer a variety of teaching strategies for this purpose.

3.4. The dialectics of questions and responses as engine of the SRC

An important dialectic that is integrated in the SRC is the task of posing questions and that of the search for responses. In the 'traditional' didactic contract, the responsibility of posing questions generally falls on the teacher, while students only comes up with doubts or questions that the teacher can answer quickly.

As we saw in the experimentation of the SRC, the mathematical modelling process required the entire community to focus on the study of a single question for a long period of time (the whole year!). This question had to remain 'alive' and 'open' session after session. Furthermore, the relevance of the derived questions and the opportunity of its consideration must appear as one more gesture of the study process. It had to be negotiated between the teacher and the students.

This situation is rarely seen under the 'dominant' pedagogy. For instance, it only attributes to the teacher the ability to 'teach' certain contents, the value of which nobody argues. In order to overcome the constraints that appeared during the experimentation of the SRC (students' passiveness, their request for a close supervision by the teacher, etc.), the teacher introduced some relatively new didactic devices. For example, the teacher asked to the students that, with their weekly team report, they had to pose at least one new question that arose from the work carried out. Moreover, at the beginning of the following session these new questions were brought together and students–under the teacher's watchful eye–agreed on the way to continue. It was an excellent way to compare and discuss the work done during all the process, and particularly, a way for the study community to formalise all the questions approached and their successive temporary answers.

3.5. The dialectics of the diffusion and reception of responses

Against the temptation of imposing some answers that are acceptable within the educational institution, the group of students needs to be invited to *defend* the successive answers R_i they provide, although they may still be of a temporary nature.

In the case of our experimentation, as we have mentioned before, we introduced a device named 'Report of results', relatively foreign to the mathematical teaching culture. Each week, in groups, the students had to elaborate a written text in which they gathered both the documents provided by the teacher, and the partial results of the work done in the workshop session. They complemented it with their personal comments and the information on the subject they would have been able to gather. They had to hand the report in to the teacher. These dossiers thus contained the answers that each group would defend in class at the beginning of each session. At the end of the workshop, each student had to hand in their own 'Final report' that no longer contained the chronicle of the study process but focussed on presenting and defending a final answer to the question initially posed. Undoubtedly, the students did not easily accept elaborating, reading and defending the reports due to the difference compared to other study devices used in other subjects. Despite all the resistance put up by students to the changes introduced during the implementation of SRC – working in groups, scheduling the study on their own, formulating questions, selecting mathematical contents, using a computer and bibliographical resources, writing and defending temporary answers, etc. – all these responsibilities (traditionally assumed by teachers) were progressively accepted by them. This increasing autonomy taken on by the students during the SRC seems a necessary condition to carry out the activity of mathematical modelling.

3.6. The dialectics of 'media' and 'milieu'

The implementation of a SRC can only be carried out if the students have some pre-established responses $R_i^\lozenge$ accessible through the different means of communication and diffusion (that is, the *media*), to elaborate the successive provisional answers R_i. These *media* are any source of information such as, for instance, textbooks, treatises, research articles, class notes, etc. However, the answers provided are constructions that have usually been elaborated to provide answers to questions that are different to the ones that may be put forward throughout the mathematical modelling process. Thus they have to be 'deconstructed' and 'reconstructed' according to the new needs. Other types of *milieus* will therefore be necessary to put to the test and 'check' the validity of these answers.

In our experimentations with SRC, this dialectics was crucial for mathematical modelling. In the process of construction of a model from a certain system, it was essential for the student to have access to answers that are not reduced to the 'official' answer of the teacher (or textbook), as well as to the means to validate them. Students were systematically asked to look for information into *media* about the types of models they provided. In particular, they had to look whether these already existed and whether they were important enough so as to be assigned a specific name, and so on. The validity of the models constructed or provided was carried out from data – which in our case the teacher had provided – and through numerical simulation with Excel or the symbolic calculator Wiris (www.wiris.com). We find good examples of this in the 3rd branch of SRC based on Leslie' matrix models (see figure 2 and 3) where, for example, to be able to forecast the short- and long-terms distribution of users into groups or to simulate the *n*-power of a Leslie matrix, the numerical simulation provided by Excel or Wiris could work, on the one hand, as *media* to be able to formulate some conjectures about their pattern but, on the other hand, they can work as *milieu* to check or refuse the conjecture they could have formulated.

4. DISCUSSION AND FURTHER RESEARCH

Using the 'ecological' metaphor, we can say that for mathematical modelling to be able to normally 'live' in a teaching institution (in concrete, at university level), an in-depth study the *conditions* that facilitate and the *constraints* that hinder the type of mathematical activities has to be carried out.

In our research, we have used the notion SRC as a reference model of didactic organisations, which are proposed to allow mathematical modelling to 'live' in educational systems. In this paper, we have introduced some of what we consider as their main characteristics that can be used as principles for their design, testing and analyses. Far from being closely-characterised, what it is important to underline is that most of their traits appear to face some constraints that generally come from the dominant 'epistemological' and 'pedagogical' models and that make the life of mathematical modelling difficult (Barquero et al. 2010 and 2012). We have mentioned several of them along the paper: coming from 'monumentalistic' school epistemology, from the classical organization of mathematics following the 'logic of mathematic contents', from the 'traditional' didactic and pedagogic contract, from the rigidity of the classical structure 'lectures – problem sessions – exams', from students' passiveness, etc.

Given the fact that the origin of most of these constraints is located at the generic levels of 'school' and of 'society', it seems obvious that they are not to be directly modified through only changes introduced by the teacher in the classroom. We thus propose a different way, which, in a sense, is the opposite approach. We suggest to begin by proposing and changing the *gestures of the study*, which requires the introduction of *new didactic devices* which make the carrying out of gestures possible. After more than six years of implementation of SRC at university level, we can say that SRC have became more and more consolidated as a normal didactic device. Although their initial difficulties, our present research move forward a progressive and generalized introduction of certain '*study gestures'* and the appropriate *'didactic devices'* that could make it possible to transform the type of scientific activity carried out in the universities' classroom effectively.

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