CONCEPTUALIZING AND ASSESSING SECONDARY MATHEMATICS TEACHERS' PROFESSIONAL COMPETENCIES FOR EFFECTIVE TEACHING OF VARIABILITY-RELATED IDEAS

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The importance of statistics education in secondary school has been emphasized in numerous mathematics curriculum reforms carried out recently in many countries, it being noticeable that variability may arise within all the statistical objects studied in such curricula. Despite this, there have been few attempts to conceptualize or assess empirically teachers' professional competencies (sensu Döhrmann, Kaiser & Blömeke, 2012) for teaching variability-related ideas. This article introduces a conceptual framework for examining teachers' statistical knowledge for teaching alongside teachers' beliefs and conceptions of variability, as well as a survey instrument developed based on it. Preliminary results of an ongoing exploratory study are reported, and implications for teaching and teacher training are discussed.

Keywords: Teachers' professional competencies, statistical knowledge for teaching, teachers' beliefs, teachers' conceptions of variability.

INTRODUCTION

In recent years, curricular reforms in many countries have brought into the secondary school mathematics curriculum topics related to statistics (e.g., NCTM, 2000), aiming towards statistical literacy. It is noticeable that variability—a property of an statistical object which accounts for its propensity to vary or change, which is considered by several researchers as a fundamental concept in statistics (e.g., Shaughnessy, 2007)—may arise in many different ways in such topics. Therefore, nowadays secondary mathematics teachers must instruct several variability-related ideas—such as the one of distribution, since through the lens of this idea statisticians examine data variability (cf. Pfannkuch & Ben-Zvi, 2011, p.326)—, and such work demands from them specific professional competencies, without which the aims of the mathematics curriculum regarding statistics education cannot be achieved.

Döhrmann, Kaiser and Blömeke (2012) point out that "successful teaching depends on professional knowledge and teacher beliefs" (Ibid., p. 327), and, with this in mind, they framed mathematics teachers' professional competencies in terms of cognitive and affective-motivational facets (cf. Figure 1). In such framework—which is the theoretical basis of the international study Teacher Education and Development Study in Mathematics (TEDS-M)—, Döhrmann and her colleagues highlighted subject matter knowledge (SMK) and pedagogical content

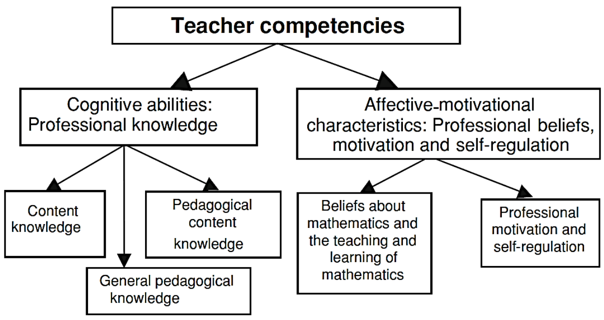


Figure 1: Conceptual model of teachers' professional competencies, according to Döhrmann et al. (2012)

knowledge (PCK) in the cognitive facet, as well as teachers' professional beliefs in the affective-motivational facet, as fundamental criteria for effective teacher education.

In the case of statistics education, scarce studies can be found in the literature focused on the SMK and PCK entailed by teaching variability-related contents to help students achieve the aims of statistics education (cf. Shaughnessy, 2007), as well as on the beliefs held by in-service teachers on statistics teaching and learning of such contents. Hence, it is by no means surprising the urgent call for increasing research on these areas made by a number of concerned researchers, particularly for studies on teachers' professional knowledge and practices while teaching variability (e.g., Sánchez, da Silva & Coutinho, 2011, p.219), as well as for teachers' beliefs on statistics itself and on what aspects of statistics should be taught in schools and how (e.g., Pierce & Chick, 2011, p.160). Accordingly, the purpose of this paper is to respond to such calls by proposing a conceptual framework for secondary teachers' professional competencies to teach variability-related contents, which integrates statistical knowledge for teaching—henceforth SKT, the knowledge, skills, and habits of mind needed to carry out effectively the work of teaching statistics—, conceptions of variability, and statistics-related beliefs, aiming to identify indicators that could serve to examine such competencies—since examination of mathematics teachers' competencies is one of the most important parameters of school quality (cf. Blömeke & Delaney, 2012, p.224), and thus it may help to get a clearer picture about what the level of competence of secondary mathematics teachers to teach variability-related contents is, and about the existence of any deficiencies that may need to be improved.

THE MKT MODEL

Ball, Thames and Phelps (2008) developed the notion of mathematical knowledge for teaching—henceforth MKT—focusing on both what teachers do as they teach mathematics, and what knowledge and skills teachers need in order to be able to teach mathematics effectively. This model describes MKT as being made up of two domains—SMK and PCK—, each of them structured in a tripartite form (cf. Figure 2). Moreover, this model clarified the distinction between SMK and PCK, and refined their previous conceptualizations in the literature.

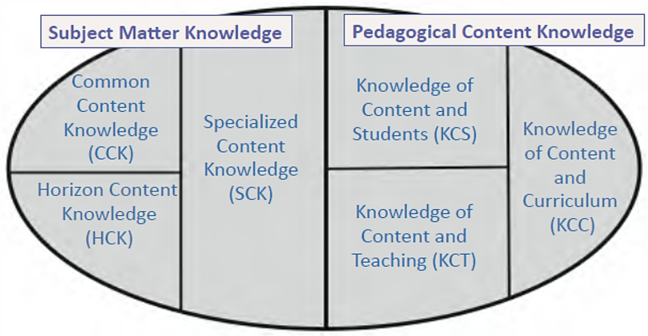


Figure 2: Domains of MKT (Ball et al., 2008)

According to Ball et al. (2008), SMK can be divided into common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). Furthermore, Ball et al. presented a refined division of PCK, comprised by knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) (the interested reader should refer to the original article for a detailed discussion of these constructs).

Through this model, Ball and her colleagues made significant progress in identifying the relationship between teacher knowledge and student achievement in mathematics. However, as highlighted by some researchers (e.g., Petrou & Goulding, 2011, p.16), Ball et al.'s (2008) model does not acknowledge the role of neither beliefs nor conceptions about the subject matter in teachers' practices, which could be a drawback, since it is well documented in the literature that both beliefs and conceptions are important factors affecting the work of teaching (cf. Philipp, 2007).

CONCEPTUALIZING TEACHERS' PROFESSIONAL COMPETENCIES FOR EFFECTIVE TEACHING OF VARIABILITY-RELATED TOPICS

While several models have been developed in the literature to conceptualize mathematical knowledge for teaching, few have been done on SKT. Within those few conceptualizations of SKT proposed to date—which almost all of them having assimilated some of the categories present in the aforementioned model for MKT (cf. Groth, 2007; Burgess, 2011; Noll, 2011)—, none of these cognitive-oriented models takes into account neither all the six components identified by Ball et al. (2008) and the role of beliefs in teachers' professional practice, nor the conceptions of variability held by the teachers, which could result in an inadequate picture of teachers' preparedness to teach statistical contents.

Aiming to remedy such gaps in the literature, a conceptual model for secondary mathematics teachers' professional competencies to teach variability-related contents is proposed. This model is a two-faceted one: it includes a cognitive as well as an affective facet. The cognitive facet is a sixfold conceptualization of SKT, comprised by all the professional knowledge subdomains identified by Ball et al. (2008) in their model of MKT, with the construct CCK—defined as the mathematical knowledge and skills expected of any well-educated adult—being adapted to meet the case of teaching statistics. In this regard, statistical literacy will be seen as CCK, since its acquisition is expected from any individual after completing school education, and the obtainment of its related skills—e.g., identifying examples or instances of a statistical concept; describing graphs, distributions, and relationships; rephrasing or translating statistical findings, acknowledging the omnipresence of variability in any statistical context, or interpreting the results of a statistical procedure—are also regarded as one of the main goals of both statistics education and mathematics curricula at all educational levels (cf. Gal, 2004; Pfannkuch & Ben-Zvi, 2011).

The affective facet of the model proposed in this article is comprised by two components: teachers' beliefs about statistics, its teaching and learning; and teachers' conceptions of variability, since both beliefs—defined by Philipp (2007, p. 259) as "psychologically held understandings, premises, or prepositions about the world that are thought to be true"—and conceptions—the set of internal representations and the corresponding associations that a concept evokes in the individual, often explained in the literature as "conscious beliefs"—, have been regarded in by a number of studies as factors influencing every aspect of teaching, including the instructional method and the course content (cf. Philipp, 2007). A detailed discussion of the conjectures that informed the development of this conceptualization can be found in González (2012).

ASSESSING TEACHERS' PROFESSIONAL COMPETENCIES FOR EFFECTIVE TEACHING OF VARIABILITY-RELATED TOPICS

The Survey Instrument.

Based on the conceptual model previously outlined, a pen-and-paper instrument, designed to be completed in one hour and comprised by tasks addressing variability-related concepts present in the secondary school mathematics curriculum, was developed, in order to elicit and assess each one of the eight components of teachers' professional competencies to teach variability-related contents identified by this study. Each item in the instrument was developed based on questions used in previous studies with similar aims reported in the literature (e.g., Ball et al., 2008; Isoda & González, 2012).

In order to provide a comprehensive framework for conceptualizing the cognitive aspects of teachers' competencies in the context of teaching variability-related ideas, twelve indicators were identified and selected for assessing SKT from teachers' answers to each of the designed items (see Table 1).

A: Indicators associated to Statistical Literacy (CCK):

1. Is the teacher able to give an appropriate and correct answer to the given task?
2. Does the teacher consistently identify and acknowledge variability and correctly interpret its meaning in the context of the given task?

B: Indicators associated to SCK:

1. Does the teacher show evidence of ability to determine the accuracy of common and non-standard arguments, methods and solutions that could be provided on a single question/task by students (especially while recognizing whether a student's answer is right or not)?
2. Does the teacher show evidence of ability to analyze right and wrong solutions that could be given by students, by providing explanations about what reasoning and/or mathematical/statistical steps likely produced such responses, and why, in a clear, accurate and appropriate way?

C: Indicators associated to HCK:

1. Does the teacher show evidence of having ability to identify whether a student comment or response is mathematically/statistically interesting or significant?
2. Is the teacher able to identify the mathematically/statistically significant notions that underlie and overlie the statistical ideas involved in the given task?

D: Indicators associated to KCS:

1. Is the teacher able to anticipate students' common responses, difficulties and misconceptions on the given task?
2. Does the teacher show evidence of knowing the most likely reasons for students' responses, misconceptions and difficulties in relation to the statistical ideas involved in the given task?

E: Indicators associated to KCT:

1. In design of teaching, does the teacher show evidence of knowing what tasks, activities and strategies could be used to set up a productive whole-class discussion aimed at developing students' understanding of the key statistical ideas involved in the given task, instead of focusing just in computation methods or general calculation techniques?
2. Does the teacher show evidence of knowing how to sequence such tasks, activities and strategies, in order to develop students' understanding of the key statistical ideas involved in the given task?

F: Indicators associated to KCC:

1. Does the teacher show evidence of knowing at what grade levels and content areas students are typically taught about the statistical ideas involved in the given task?
2. Does the designed lesson (or series of lessons) show evidence of teacher's understanding and support of the educational goals and the intentions of the official curriculum documents in relation to the teaching of the statistical contents present in the given problem, as well as statistics in general?

Table 1: Set of indicators proposed to assess SKT through the answers to the survey items

In regard to the affective facet of the conceptual model proposed here, the conceptions of variability that might be distinguished in teachers' answers will be classified using the eight types of conceptions of variability identified by Shaughnessy (2007, pp. 984-985). In the case of teachers' beliefs, these could be identified through examining the features of the lesson plans that teachers produce—such as the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson, being both related to the construct KCT—, as the limited research on teachers' beliefs about statistics teaching and learning suggests (e.g., Pierce & Chick, 2011, p.159). These beliefs will be categorized as beliefs about the nature of statistics, and beliefs about learning statistics (cf. Tatto et al., 2012, pp.154–156).

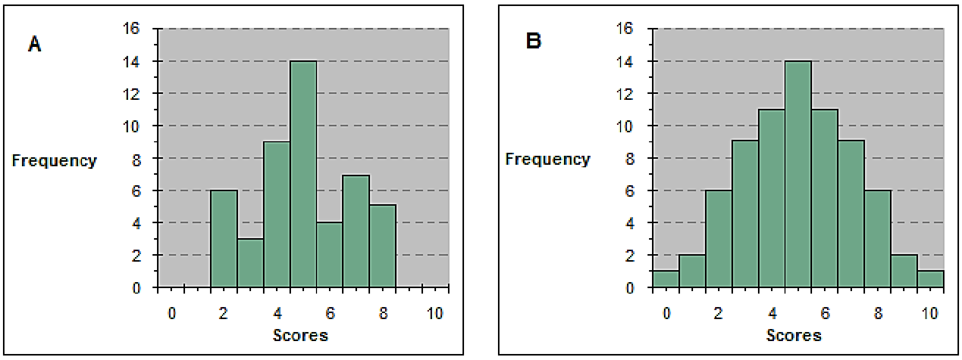
Profile of Item 1.

In a first stage of this study, a survey instrument comprised of one item—Item 1, which deals with several ideas of descriptive statistics, and is depicted in Figure 3—was designed, and was sent by postal mail to three secondary schools in Hiroshima Prefecture, Japan. The fact that the majority of the statistical contents present in the Japanese mathematics curriculum are ideas related to descriptive statistics was crucial in the selection of the task in Item 1. Two more stages of this study are planned in the future, each of them using a one-itemed questionnaire dealing with the ideas of probability and sampling, respectively.

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(a) Answer this task in as many different ways as you can. Please, be sure to show every step of your solution process.

(b) What are the important ideas and concepts that students might use to answer this task?

(c) Suppose that, after posing this task to your students, three of them come up with the following answers:

Student 1: "Distribution A has more variability because it's not symmetrical."

Student 2: "Distribution A ranges from 3 to 14, while Distribution B ranges from 1 to 14. Then, Distribution B has more variability."

Student 3: "The bars in Distribution A are clumped closer to the central bar than they are in Distribution B. Then, Distribution B has more variability."

Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer.

(d) Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), misconceptions and difficulties you would expect from them? Briefly explain why you think so. (Regarding to the most likely answers that you might get from the students, please do not include those mentioned in part (c).)

(e) Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects. (Please, focus your response on whether there is a significant mathematical/statistical insight in the student's answer, and whether there are forthcoming contents in future classroom subjects overlying or related to the notions/concepts being said or implied in such answer.)

(f) Briefly describe how the important ideas and concepts involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling.

(g) Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the context of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order.

Figure 3: Item 1 – "Choosing the distribution with more variability" task

The original version of the task in Item 1—developed by Garfield, delMas and Chance (1999), and reported in the literature as an effective means of investigating teachers' conceptions of variability in the context of histograms (e.g., Isoda & González, 2012)—was modified to facilitate the calculations that could be made while the respondent gives answer to the task, and was also enriched with questions aiming to elicit all the facets of teachers' professional competencies to teach variability-related contents identified by this framework. A mapping between the components of SKT that would be elicited by each question in Item 1, as well as the indicators associated to each cognitive aspect considered by this framework, can be appreciated in Table 2.

The context of the task posed in Item 1—comparing distributions—requires from teachers the mastery of several variability-related concepts—e.g., distribution, measures of variation, frequency distribution table, and histogram. Therefore, the task in Item 1 was selected in order to see, among others, (a) whether by looking at the histograms of two distributions of scores, teachers could figure out which one has more variability, and then use data-based arguments to defend their answer; and (b) how the respondents conceptualize variability in the context of the given task.

Elicited Knowledge Component of SKT	Associated Indicator of SKT	Question
Statistical Literacy (as **CCK**)	A1	(a)
	A2	(a)
Specialized Content Knowledge (**SCK**)	B1	(c)
	B2	(c)
Horizon Content Knowledge (**HCK**)	C1	(e)
	C2	(b)
Knowledge of Content and Students (**KCS**)	D1	(d)
	D2	(d)
Knowledge of Content and Teaching (**KCT**)	E1	(g)
	E2	(g)
Knowledge of Content and Curriculum (**KCC**)	F1	(f)
	F2	(g)

Table 2: Knowledge components of SKT elicited by each of the questions posed in Item 1

About the Data.

At the time of writing this article (September 2012), the survey process—which began in July 2012 in Hiroshima Prefecture, Japan—was ongoing, and expected to be completed by the end of September 2012. In this paper, a preliminary analysis of the data gathered from one of the schools participating in this study, comprised of the written responses to Questions (a) and (g) on Item 1 given by four senior high school teachers working in such school, will be reported. The respondents are between 28 and 56 years old; they have between one and thirty-four years of teaching experience—with three of them with at least 13—, and were the first group of teachers that voluntarily and anonymously responded and mailed back the survey booklets.

Results and Findings regarding Question (a).

Three out of four teachers answered this question. From those who answered, two teachers—Teachers 1 and 2—used three different approaches: Teacher 1 answered the task by comparing the range, variance and interquartile range of both distributions; while Teacher 2 answered the task by comparing the range, the shape, and the mean absolute difference from the mean of both distributions. Teacher 4 answered using only one approach: by comparing the largest data span from the mean in both distributions.

It is quite surprising that all these teachers made computation errors in every approach that involved calculations. Among all the calculation errors done by them, one is recurrent: although both Teacher 1 and Teacher 2 identified correctly Distribution B as the one with more variability via comparing the ranges, when computing them they used as minimum and maximum values 2 and 8 in Distribution A, and 0 and 10 in Distribution B, respectively; that is, they used the class marks instead of the lower and upper class limits, which are 1.5 and 8.5 for Distribution A, and are –0.5 and 10.5 for Distribution B.

In a similar way that Teacher 1 and Teacher 2 calculated the distribution ranges, Teacher 4 calculated the largest data span from the mean in both distributions. This teacher mistakenly argued that the largest data span from the mean is 3 and 5 units for Distribution A and B, respectively. Once again, it is noticeable that this teacher used for the calculations the class marks instead of the lower and upper class limits.

Regarding the conceptions of variability held by the respondents, the answers given by Teacher 1 and Teacher 2 indicate they hold the eight conception of variability identified by Shaughnessy (2007)—"Variation as distribution"—, since teachers were able to use theoretical properties of the histograms to calculate numerically—although mistakenly—the measures of variation associated to each distribution in order to make their decision. In the case of Teacher 4, his answer indicates he holds the fifth conception of variability identified by Shaughnessy (2007)—"Variability as distance or difference from some fixed point"—, since this teacher was able to visually identify the bar which represents the mean class in each of the histograms, and from there consider the variability of the endpoint values from the mean. Teachers holding this conception do not exhibit an aggregate view of data and distribution, since they are predominantly concerned with the variability of one data point at a time from a measure of central tendency, rather than with the variability of an entire data distribution from a center (cf. Shaughnessy, 2007, p.985).

Results and Findings regarding Question (g).

The purpose of this question is to elicit evidence of the indicators associated to KCT outlined in Table 1—namely E1 and E2. In order to determine the presence of these indicators in teachers' answers, a criterion-referenced assessment rubric was designed, based on the characteristics of effective classroom activities to promote students' understanding of variability compiled by Garfield and Ben-Zvi (2008).

All the four teachers answered this question. In relation to Indicator E1, the answers given by Teacher 1 and Teacher 4 (cf. Figure 4) are the ones that seem to exhibit a higher level of knowledge on the key characteristics of effective activities that promote students' understanding of variability identified by Garfield and Ben-Zvi (2008), such as the implementation of tasks involving comparisons of data sets, aiming towards describing and representing variability with numerical measures when looking at the given data, and promoting a whole-class discussion on how measures of central tendency and variation are revealed in data sets or graphical representations of data (Ibid., pp. 207-209).

Regarding Indicator E2, the answers given by Teacher 1 and Teacher 4 are the ones evidencing more knowledge on how to sequence activities and strategies intended to promote students' understanding of variability. For example, both answers explicitly state that the lesson must start by presenting students with some simple data, in order to then represent and interpret it (Garfield & Ben-Zvi, 2008, pp.135-137). However, Teacher 4's answer is at an even higher level compared to the others, since explicitly states that variability should be described and compared informally at first—e.g., by describing verbally how the data is spread out—, and then formally, through measures of variation (cf. Ibid., p.208).

Teacher 1	Teacher 2
Task: 「Among 2 distributions, which one do you think has more variability?」 **Activities:** ① Check different ways (range, variance, standard deviation, interquartile range) for examining variability. ② Place students in groups, asking to each group to use only one of the methods in ① to discuss about what things could be told about the variability of the given distributions. ③ Each group will share with the rest of the classroom what they considered in ②. ④ Depending on the method used, and while checking different considerations, think about how to look at variability. Students will experience personally the need of using several methods and finding out the appropriate one in order to consider data trends.	1. To make students think about which of 2 given histograms, A and B, has more variability. 2. To make students think about whether they can make their decision based only on the sample size. 3. To judge the variability using the variance. 4. Practice problems.
Teacher 3	**Teacher 4**
In mathematics there are a large number of approaches in many directions concerning "variability": Introduction of the formulas related to variability. Studying variability through the use of computer technology. Based on the aforementioned approaches, bring up for discussion various topics in society and the corporate world, such as product development, among others, as well as their connections with practical applications.	− Give 2 histograms, A and B. − To make students think about in which histogram the variability is larger, and to make them expose about what they think. …At this stage, a detailed explanation about "variability" has not yet been provided. − After their presentations, explain about "variability", and make students think again about which histogram has more variability. − Explain, among other things, different terms besides "variability", provide different histograms, and practice.

Figure 4: Translation of answers to Question (g) given by the four surveyed teachers

In relation to the beliefs about the nature of statistics held by the surveyed teachers, the answers given by three of them—Teachers 1, 2 and 4—provide evidence that they see statistics as a process of inquiry; that is, as a means of answering questions and solving problems. For example, Teacher 1 explicitly states that examination of data variability can be correctly performed in many ways, which can be tried out by the students themselves. Regarding beliefs about learning statistics held by the surveyed teachers, Teachers 1 and 4 planned lessons in which they encourage students to find their own solutions to statistical problems, while fostering the development of statistical discourse and argumentation in the classroom (cf. Pfannkuch & Ben-Zvi, 2011, p.329), which provide evidence that they see statistics learning as being active learning. The answers given by Teachers 2 and 3 give evidence that they see statistics learning as a teacher-centered individual work.

CONCLUSIONS

Based on teachers' performance in Question (a) of Item 1, some answer tendencies shown by the group of surveyed teachers can be identified; for example, mistakenly using the class marks instead of the lower and upper class limits of the first and last classes, respectively, to calculate particular measures of variation. Only one teacher mistakenly used the shape of the histograms to answer, interpreting the variability in the given histograms as the differences in the heights of the bars, which is a common misconception in this kind of problems (cf. Meletiou & Lee, 2003; Isoda & González, 2012). Despite of this, evidence of two teachers in this group exhibiting an aggregate view of data and distribution—i.e., holding the conception of variability known as "Variation as distribution" (cf. Shaughnessy, 2007, p.985)—is noteworthy.

Regarding teachers' performance in Question (g) of Item 1, the one of Teachers 1 and 4 stands out from the others. Some of the characteristics identified in their answers are consistent with those of effective classroom activities to promote students' understanding of variability made by the specialists (cf. Garfield & Ben-Zvi, 2008). Nevertheless, besides the answer given by Teacher 3, the lessons planned by the respondents lack consideration of an explicit daily-life context, which is vital to internalize in the students that statistics helps solve everyday problems and tasks (cf. Gattuso & Ottaviani, 2011, pp.122-123, 129).

From the answers to Question (g), it is evident that two of the teachers surveyed believe that statistics is a process of inquiry, and its learning should be achieved through active involvement of the students, instead of a teacher-centered way. The other two seems to see statistics learning as a teacher-centered individual work.

The fact that teachers' answers showed, among others, a lack of knowledge about how to relate the given task to different data representations—such as boxplots and frequency distribution tables—, and due to the importance of making an appropriate interpretation of variability for statistics, courses where Japanese senior high school in-service teachers could learn more about developing intuitive ideas of variability, as well as variability-related ideas and the interrelationship among them; describing and representing variability; using variability to make comparisons; being able to map the characteristics of a given histogram to alternate representations; and so on, could be required.

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