PROSPECTIVE PRIMARY SCHOOL TEACHERS' ERRORS IN BUILDING STATISTICAL GRAPHS

Pedro Arteaga, Carmen Batanero, Gustavo R. Cañadas y J. Miguel Contreras

University of Granada, Spain

We analyze the graphs produced by 207 prospective primary school teachers when comparing data collected by them in a statistical project. These graphs are classified according to their correctness, and the type of error in case of incorrect graph. The influence of using computers on the errors produced is also analyzed. Results show the prospective teachers' difficulties in building statistical graphs, in spite that they will have to teach these graphs, since many of them shared errors reported in previous researches with children.

Keywords: statistical graphs, errors, graph construction, primary school teachers' education.

INTRODUCTION

Nowadays we are surrounded by statistical information that is often presented through statistical graphs, and building and interpreting statistical graphs is an important part of statistical literacy (Watson, 2006; Ridgway, Nicholson & McCusker, 2008). Taking into account the relevance of the topic, this research was aimed to evaluate the formative needs of Spanish prospective school mathematics teachers in relation to their competence in building elementary statistical graphs. Below we describe the research rationale and background, method, results and implications for teacher education.

STUDY RATIONALE

In Spain, curricular guidelines (MEC, 2006) include statistical graphs since first level of primary school education (6 year-olds children). The success of this curriculum depends on the correct preparation of teachers, that, until now did not include statistics education.

Moreover, few studies have focused on teachers' knowledge about statistical graphs and most of them are related to prospective teachers (González, Espinel, & Ainley, 2011). In this study we continue our previous research (Arteaga & Batanero, 2011) where we analyzed the graphs produced by 207 prospective primary school teachers in an open statistical project where they had to compare three pairs of distributions. In that paper we classified the graphs built by the prospective teachers according to its semiotic complexity and the participants' reading levels in Curcio's (1989) categorization. Results showed that, although most participants produced graphs with sufficient semiotic complexity to solve the task posed, only a minority reached an adequate reading level to get a correct conclusion from the graph. In this new study we re-analyse the same data with a different perspective: The focus now is the correctness of the graphs built by these teachers and the influence of using computers on possible errors.

STUDY BACKGROUND

Errors in building graphs

Several studies evaluated the competence in building statistical graphs by students (see Friel, Curcio, & Bright, 2001; Tiefenbruck, 2007 for a survey). Li and Shen (1992) found the following errors: selecting a graph that is inadequate to the type of variable or representing not related variables on the same graph; using inadequate scales; omitting the scales in at least one axe; not specifying the origin of coordinates and not using enough divisions on the scales. Wu (2004) found the following errors by secondary school students when building and reading statistical graphs: (a) errors related to scales, (b) errors in tittles, labs or specifiers (c) errors in pie charts, (d) difficulties with proportionality in a pictogram, (e) confusion between apparently similar graphs (for example, between histogram and bar chart), (f) confusing frequencies and variable values. Lee and Meletiou (2003) described four wrong reasoning when working with histograms: (a) Interpreting histograms as representation of discrete variables, assuming each rectangle refers to an isolate value instead to an interval of values; (b) Comparing frequencies in histograms using only the vertical axis (instead of areas) (c) Lack of appreciation of randomness in the data represented, and (d) Interpreting histograms as a bivariate graph.

Technology does not facilitate the work with statistical graphs, because the students need to learn the software options in addition to the graphs features. Ben-Zvi and Friedlander (1997) analyzed the graphs produced by some students when working with computers and suggested that some of these students used the software in a non critical way, since they were unable to select the most adequate options of the software.

Teachers or prospective teachers' competence in building graphs

Bruno and Espinel (2005) described the following errors made by prospective primary school teachers when building a histogram: misrepresenting intervals, omitting null-frequency intervals, using non-attached rectangles with continue variables**.** In Burgess's (2002) study some teachers were unable to integrate the knowledge they got from the graphs produced in their reports with the problem context.

Tiefenbruck (2007) investigated fourteen primary school teachers' understandings of graphical representations of categorical data with a questionnaire, where only a few questions asked the participants to build a graph. The teachers had a basic knowledge of graphical representations of categorical data. However, some of them incorrectly suggested that the histogram and stem and leaf plot were adequate for categorical variables. They also were confused when defining the scale and in describing how to create a scale from data. In this study we continue all this previous research in analysing the type of errors the prospective teachers made when building graphs. Contrary to Espinel and her colleagues or Tiefenbruk, instead of using questionnaires we gave the teachers an open-ended project; we also study the influence of computers on the possible correctness of the graph. Arteaga & Batanero (2011) is also a basis for this research.

THE STUDY AND METHOD

Participants in our study were 207 prospective primary school teachers in Spain who were enrolled in a mathematics education course; in total 6 different groups (35-40 participants per group). The participants studied statistics the previous academic year, including statistical graphs, which are an important component in the primary school curriculum (MEC, 2006). The data were collected as part of a practical activity where prospective teachers were encouraged to carry out an experiment to decide whether the group had good intuitions on randomness or not. The experiment consisted of trying to write down apparent random results of flipping a coin 20 times (without really throwing the coin, just inventing the results) in such a way that other people would think the coin was flipped at random (simulated sequence). Participants recorded the simulated sequence on a recording sheet. Afterwards participants were asked to flip a fair coin 20 times and write the results on the same recording sheet (real sequence). At the end of the session, in order to confront these future teachers with their misconceptions, participants were given the data collected in their classroom. These data contained six statistical variables: number of heads, number of runs and length of the longest run for each of real and simulated sequences from each student (part of the data are presented in Table 1; the total data consisted of 35-40 rows similar to those presented in Table 1. The prospective teachers were asked to analyse the data and produce a report with their conclusion (about the similitude or differences in the three pairs of distributions).

	Simulated sequence			Real sequence		
Student	N. of heads	N. of runs	Longest run	N. of heads	N. of runs	Longest run
1	10	14	4	11	9	4
2	12	9	4	11	16	2
3	11	12	4	11	16	2
4	10	9	4	8	9	4
5	11	11	3	7	11	4
…						

Table 1: Data collected by the students

RESULTS AND DISCUSSION

Although we did not asked the prospective teachers to use graphs in their analyses, 181 of them (87.4%) built some graphs to compare the number of heads 146 (70.5%) to compare the number of runs and 129 (62.3%) to compare the longest run in the real and simulated sequences. These high proportions of students who built graphs suggest they carried out a *transnumeration* process (Wild and Pfannkuch 1999) to obtain new information that was hidden in the raw data. These graphs were firstly classified into basically correct, partially correct and incorrect graphs and secondly, according the types of errors presented. Results are described below.

Categories of graphs

1. *Basically correct graphs.* Here we include *correct graphs* (correct title, axes, scales and labs) that are adequate to both, the problem posed and the variables being displayed (1.1). We also include unusual, but correct representations (1.2).

2. *Errors in the graph scale.* Watson (2006) warns about the need to be careful with the graph scales; however we found the following errors (Bruno & Espinel, 2005):

2.1. *Non proportional scales* where distances between different pairs of points that should be equal were different.

2.2. *Wrong representation of natural numbers on the number line, f*or example, omitting null-frequency variable values.

2.3. *Titles or scale values missing or confuse* Although the title, labels on the axes and scales are essential part of graphs (Curcio, 1987) because they provide the contextual information needed to interpret the variables represented and the relationships between the graph different elements, Bruno and Espinel (2005) found many prospective teachers with difficulties to include a correct and meaningful title. In our study many students provided imprecise titles or labels, but only a few built graphs with no tittles or labels.

2.4. *Not centred bars.* The variables in our study were discrete; however, many participants built histograms, which are used to represent continuous variables or when we need to group the variable values. Furthermore, some participants did not centre the rectangles in the class centre, although the variables only took integer values. This error was also reported by Bruno and Espinel (2005) and Espinel (2007)

2.5. *Wrong representation of intervals on the X axis.* Some participants made errors in representing intervals; for example they displayed intervals with a common point as disjoint. This error, also reported by Bruno and Espinel (2005) was more common between the participants who used Excel to produce their graphs.

2.6. *Inappropriate scales.* Li y Shen (1992) found some students who built a scale not wide enough to cover the range of variation of the variable represented. In our study we found this error as well as participants who built too wide scales, error described by Wu (2004).

3. *Incorrect graphs.* We found the following subcategories (See examples in figure 1):

3.1. *Lack of proportionality in the specifiers*. Participants did not take into account the conventions for each particular graph. For example in Figure 1a is difficult to read the frequency associated to each variable because the width of circular sectors is not proportional to frequencies.

3.2. *Confusing variable values and frequencies.* In the distribution, each variable value is associated to its frequency. Some participants confused both and exchanged variable values and their associated frequencies in the graph (Figure 1b).

3.3. *Representing variable values and frequencies together.* Some participants built attached bar graphs, representing each variable value with its frequency in two attached bars, as if they were two different variables; they usually used Excel. Figure 1c shows an example of this category where a prospective teacher shows a limited knowledge about the software options; therefore, he makes an uncritical use of the software (Ben-Zvi & Friedlander, 1997).

3.4. *Representing variable values multiplied by frequencies.* A few participants working with Excel displayed in an attached bar graph both the frequencies and the product of frequencies by the variable value. These participants also showed an uncritical use of the software and misunderstanding of statistical of distribution.

3.5. *Inadequate graph.* Some participants selected graphs that were inadequate to the problem they had to solve. For example some participants did not form the distribution and displayed bar graphs were a too high number of bars to be able to interpret the graph (Figure 1d).

3.6. *Representing non-related variables in the same graph.* Some participants plotted variables non comparable together; for example representing in the same graph the three variables under study (number of heads, number of runs and longest run) or their averages (Figure 1e).

3.7. *Non comparable statistics displayed in the same graph.* In this case some participants displayed averages and measures of spread in the same graphs and therefore they confused the meaning of central tendency and spread (Figure 1f).

3.8. *Several errors.* Some students made several of the errors described before.

In Table 2 we present a summary of results. Basically corrects graphs had the highest percentage in each variable, although the percentage of incorrect graphs or graphs with errors in scales (partially correct graphs) was also very high. About half participants constructed basically correct graphs in comparing the variables at the project: 47% (85 participants) built correct graphs for the number of heads, 43.8% (64 participants) for the number of runs and 45.7% (59 participants) for the longest run.

About 20% of errors were related to scales (non proportional scales, wrong representation of numbers on the number line, etc...) in spite of Watson's (2006) claim that special attention should be paid to scales, because through them people can display misleading graphs. Bruno and Espinel (2005) also indicated that these errors were widespread among prospective primary school teachers in their research.

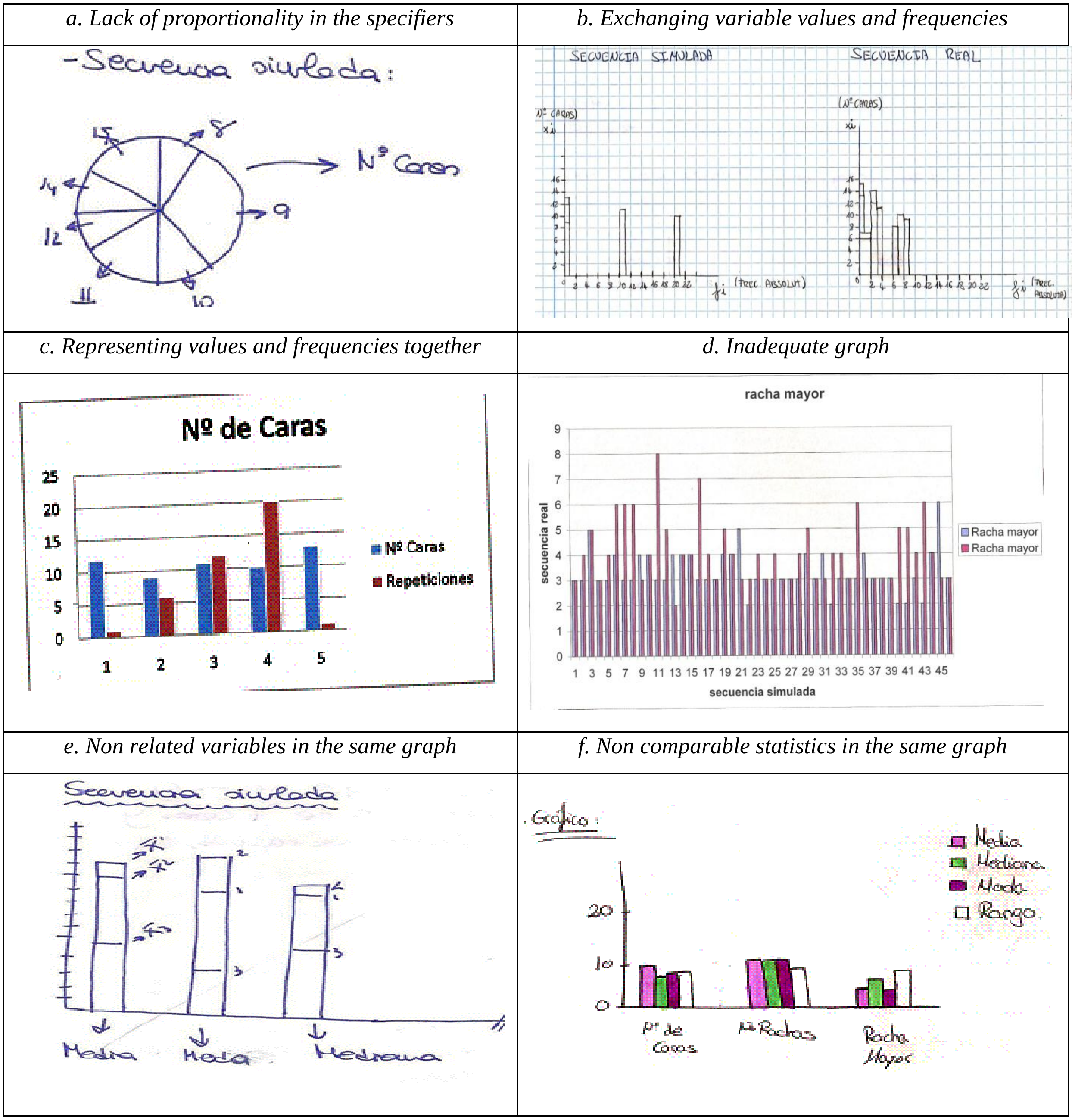

Figure 1. Incorrect graphs

The percentage of incorrect graphs was about 30%; adding the 20% errors in scales, this involves about half of the prospective teachers doing some kind of error. The differences in percentage of correct, mostly correct and incorrect graphs between the three variables posed in the project, were not statistically significant in the chi-square test of homogeneity for the distribution of errors (Chi = 1.03, 6 degrees of freedom, p = 0.9). This suggests that the errors in the graphs did not depend on the variable represented but are due to lack of the necessary competence related to graph construction.

		Number of Heads	Number of runs	Longest run
Basically correct	1.1. Correct	78(43.1)	57 (39.0)	53 (41.1)
	1.2. Correct and unusual	7(3.9)	7(4.8)	6(4.8)
Errors related to scales	2.1. Non proportional scales	0(0)	2(1.4)	3(2.4)
	2.2. Wrong representation of numbers in the number line	4(2.2)	10(6.8)	5(3.8)
	2.3. Confuse titles or scales values	6(3.3)	4(2.7)	4(3.1)
	2.4. Non centered bars	8(4.4)	6(4.1)	7(5.4)
	2.5. Errors in representing intervals	4(2.2)	3(2.1)	3(2.4)
	2.6. Inappropriate scale	13(7.2)	8(5.4)	8(6.3)
Incorrect graph	3.1. Lack of proportionality in the graph specifiers	3(1.7)	4(2.7)	1(0.7)
	3.2. Exchanging variable values and frequencies	2(1.1)	1(0.7)	1(0.7)
	3.3. Representing variable values and frequencies together	3(1.7)	2(1.4)	2(1.5)
	3.4. Representing variable values multiplied by frequencies	2(1.1)	2(1.4)	2(1.5)
	3.5. Inadequate representation	7(3.8)	3(2.1)	1(0.7)
	3.6. Representing non related variables in the same graph	14(7.7)	14(9.6)	13(10.1)
	3.7. Non comparable statistics displayed in the same graph	2(1.1)	2(1.4)	2(1.5)
	3.8. Several errors	28(15.5)	21(14.4)	18(14)
Total		181(100)	146(100)	129(100)

Table 2. Frequency and percentage of participants according to the graph correctness

Influence of computers

Participants in our study were free to use computers or not to solve the task proposed. There were 50 prospective primary school teachers who did their statistical graphs using computers (all of them with Excel); around a forth of the sample and less than a third part of those participants producing graphs.

An added problem when using computers is that, apart the statistical knowledge needed, it is necessary to know and manage the different software options. This provides an added difficulty to represent data using a graph; consequently many students simply accepted the output provided by the software without using the possibilities of changing the scale, graph type, etc.., i.e. they made an uncritical use of the software that was also observed in Ben-Zvi (2002). This happened in our study, where, although a minority of students used computers (Excel), in general, these students had more errors than those who made graphs with only pencil and paper. Table 3 presents the results obtained in our study.

	Number of heads		Number of runs		The longest run		Total	
	Computer	No	Computer	No	Computer	No	Computer	No
Correct	18(36)	67(51.1)	16(40)	48(45.3)	17(42.5)	42(47.2)	51 (39.2)	157 (48.2)
P. Correct	7(14)	28(21.4)	4(10)	29(27.4)	6(15)	24(27)	17 (13.1)	81 (24.8)
Incorrect	25(50)	36(27.5)	20(50)	29(27.4)	17(42.5)	23(25.8)	62 (47.7)	88 (27)
Total	50	131	40	106	40	89	130	326

Table 3. Frequency and percentage of prospective teachers according to the correctness of the graph and the use of computers

50 prospective teachers (27.5% of those who made graphs) used the computer to represent the number of heads in the real and simulated sequences, 40 (27.4% of those who made graphs) the number of runs and 40 (31%) the longest run. We note that the proportion of correct graphs was always higher if the participants did not use the computer. There were fewer errors in scales, possibly because the software automatically builds the scales, but there were much more significant errors when using the computer, such as choosing an inadequate graph or plotting frequencies and variables together.

When performing the chi-square test, to test homogeneity in the distribution of the three categories in the global data (last two columns of Table 3) among participants who performed the graph with or without a computer the test was statistically significant (Chi = 19.72, p = 0.0001 with 2 degrees of freedom). Therefore the use of software increased the participants' errors.

STUDY IMPLICATIONS

The strength of this study is that we provide specific information about the difficulties that prospective primary school teachers have when building statistical graphs. This is relevant, since graph construction is an important part of the graphical competence that a citizen should have (Wu, 2004; Watson, 2006) and because these teachers will have to teach this content in future. Consequently, we prove the need for these prospective teachers to have more education in working with statistical representations. In agreement with Bruno and Espinel (2009) and Monteiro and Ainley (2007) our research suggests that building graphs is a complex activity for prospective school teachers. We agree with these authors in the relevance of improving the prospective teachers' levels of competences in both building and interpreting graphs (Arteaga & Batanero, 2011), in order that they later can transmit these competences to their own students.

A limitation of the study is that we did not analyse the pedagogical content knowledge about statistical graphs in prospective teachers. Since Espinel, Bruno, and Plasencia (2008) observed lack of coherence between the graphs built by participants and their evaluation of tasks carried out by fictitious future students there is also need to evaluate this pedagogical knowledge

Acknowledgments. Project EDU2010-14947 (MCINN); group FQM126 (Junta de Andalucía).

REFERENCES

Arteaga, P. & Batanero, C. (2011). Relating graph semiotic complexity to graph comprehension in statistical graphs produced by prospective teachers. *Proceedings of the seventh congress of the European society for research in mathematics education (in press).* Rzeszow: ERME. *Http://www.cerme7.univ.rzeszow.pl/index.php?id=wg5*

Ben-Zvi, D., y Friedlander, A. (1997). Statistical thinking in a technological environment. En J. Garfield y G. Burrill (Eds.), *Research on the role of technology in teaching and learning statistics* (pp. 54-64). Voorburgo, International Statistical Institute.

Bruno, A. y Espinel, M. C. (2005). Recta numérica, escalas y gráficas estadísticas: un estudio con estudiantes para profesores. *Formación del Profesorado e Investigación en Educación Matemáticas* 7, 57-85.t

Bruno, A., & Espinel, M. C. (2009). Construction and evaluation of histograms in teacher training. *International Journal of Mathematical Education in Science and Technology*, *40*(4), 473-493.

Burgess, T. (2002). Investigating the "data sense" of preservice teachers. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics,* Cape Town, South Africa: International Association for Statistics Education. Retrieved from www.stat.auckland.ac.nz/~iase/publicatons.

Curcio, F.R. (1987). Comprhension of mathematical relationships expressed in graphs. Journal for Research in Mathematics Education 18 (5), 382-393.

Curcio, F. R. (1989). *Developing graph comprehension*. Reston, VA: N.C.T.M.

Espinel, C. (2007). Construcción y razonamiento de gráficos estadísticos en la formación de profesores. *Investigación en Educación Matemática* 11, 99-119.

Espinel, C., Bruno, A., & Plasencia, I. (2008). Statistical graphs in the training of teachers. In C. Batanero, G. Burrill, C. Reading & A. Rossman (2008). *Proceedings of the Joint ICMI /IASE Study Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education.* Monterrey, México: ICMI &IASE. CD ROM.

Friel, S., Curcio, F., & Bright, G. (2001). Making sense of graphs: critical factors influencing comprehension and instructional implications. *Journal for Research in mathematics Education 32*(2), 124-158.

González, T., Espinel, C., & Ainley, J. (2011). Teachers' graphical competence. In C. Batanero, G. Burrill & C. Reading (Eds.). *Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. A Joint ICMI and IASE Study.* New York: Springer.

Lee, C., & Meletiou, M. (2003). Some difficulties of learning histograms in introductory statistics. Paper presented at the *Joint Statistical Meeting. Section on Statistical Education.* Retrieved from http://www.statlit.org/PDF/2003LeeASA.pdf

Li, D. Y.,& Shen, S. M. (1992). Students'weaknesses in statistical projects. *Teaching Statistics 14* (1), 2-8.

MEC(2006). *Real Decreto 1513/2006, de 7 de Diciembre, por el que se establecen las enseñanzas mínimas correspondientes a la Educación Primaria.* (Royal Decrete establishing the minimum teaching contents for Primary Education).

Monteiro, C., & Ainley, J. (2007). Investigating the interpretation of media graphs among student teachers. *International Electronic Journal of Mathematics Education* 2 (3), 188-207. Retrieved from http://www.iejme/.

Ridgway, J., Nicholson, J. y McCusker, S. (2008). Mapping new statistical Literacies and Iliteracies. International Conference on Mathematics Education, Trabajo presentado en el *11th International Congress on Mathematics Education,* Monterrey, Mexico.

Tiefenbruck, B. (2007). Elementary teachers conceptions of graphical representations of categorical data. Unpublished Ph.D. University of Minnesota.

Watson, J. M. (2006). *Statistical literacy at school: growth and goals*. Mahwah, NJ: Lawrence Erlbaum Associates.

Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review 67* (3), 223-265.

Wu, Y. (2004, Julio). Singapore secondary school students´understanding of statistical graphs. Trabajo presentado en el *10th International Congress on Mathematics Education*. Copenhagen, Dinamarca.