CONSTRUCTIONS WITH VARIOUS TOOLS IN TWO GEOMETRY DIDACTICS COURSES IN THE UNITED STATES AND GERMANY

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In this article, I report on a study on experiences and perceptions from two culturally different cohorts of preservice teachers about teaching geometric constructions using different construction tools (compass and ruler, protractor, patty paper, Mira, dynamic geometry software). The analysis of written reports indicated that the teaching experiment challenged participants' existing perceptions about teaching constructions and enabled them to reexamine their future practices. Both groups reflected on the experience both through the lenses of themselves as learners and as future practitioners arguing for and against benefits of using different tools when teaching constructions, and its effect on the development of geometric thinking.

Keywords: geometric constructions, geometric reasoning, construction tools, preservice teachers, reflection

INTRODUCTION

The topic of geometric constructions played an important role in the antiquity; likewise, nowadays, it has an important role in the mathematics curriculum. Euclid's *"Elements"* has been the most enduring and widely used mathematical work in the history of mathematics. In addition, it contains many geometric construction problems that had to be carried out using very specific tools: a collapsible compass and a straightedge, which are nowadays somewhat synonymous for constructions in school. Even though many attribute the teaching of geometric constructions to its long tradition, this topic is tightly connected to the central goals of teaching geometry, such as discovery learning, proving, concept learning and problem solving (Weigand & Ludwig, 2009) and, for these reasons, is of immense importance.

Though compass and straightedge have a long history as construction tools in geometry and in teaching geometry, many other tools are suitable for a constructive access to geometry. These include, but are not limited to, dynamic geometry software, Mira [1], patty paper [2], a protractor, TESE (a two- edged straightedge) and inch cards (Pandiscio, 2002; Serra, 2003; Weigand & Ludwig, 2009). According to NCTM (2000), construction tasks can encourage students to "draw and construct representations of two- and three-dimensional geometric objects using a variety of tools" (p. 308) and to recognize and connect mathematical ideas as a way to "develop robust understandings of problems" (p. 354). Use of these construction alternatives provide both a fresh view on the classical geometry topic and, together with classical tools, fosters different mathematical ideas promoting in-depth understanding of geometric concepts and highlights the connections among different ideas (Pandiscio, 2002; Serra, 2003). However, the pitfall of teaching this topic is that teachers often deliver mere construction steps to their students and emphasize the importance of precision (Schoenfeld, 1988). Such instruction, however, inhibits the students' possibility to develop problem-solving competencies, understanding of geometry, and development of mathematical power (Schoenfeld, 1988).

THEORETICAL CONSIDERATIONS AND AIMS OF THE STUDY

Perception is a central issue in epistemology. They are views or opinions held by an individual resulting from experience and external factors acting on the individual. Individual's perception is a broader look at the construct of beliefs encompassing beliefs, meaning, preferences, concepts, and mental images (Philipp, 2007). Our perceptions are influenced by our past experiences, beliefs and expectations. Researchers often apply conceptual lenses of teachers to interpret teachers' knowledge, beliefs and practices. Beliefs are often robust, permeable mental structures that evolve with experience (Thompson, 1992) and they influence the ways in which teachers perceive and deploy tasks. Perceptions and beliefs are formed very early in life whereas our thoughts and behavior patterns are governed by these. Teachers' individual perceptions, beliefs about the sense and nature of mathematics education and preparation are becoming increasingly recognized as fundamental contributors influencing the way they teach and in which innovative teaching concepts are implemented into day-to-day mathematical lessons (Philipp, 2007; Thompson, 1992). Consequently one's perception of reality is different and beliefs need to be challenged in order for a change to occur (Wilson & Cooney, 2002). Researchers (Thompson, 1992; Wilson & Cooney, 2002) place reflection as the agent of change where as a consequence teachers can consolidate their knowledge, shape the forms of knowledge produced, question their existing perceptions and values about teaching, influence their future classroom practices and so on.

The integration of tools and technologies is an important and actual theme today in teaching mathematics. New and emerging tools and technologies are introduced into the mathematics classroom that continually transform the mathematics classroom and redefine ways mathematics can be taught (Barzel, Drijvers, Maschietto, & Trouche, 2005). Several different perspectives have been developed, in particular instrumental approaches which help explain the use of tool from a cognitive point of view (Rabardel, 2001). The main tenet in the instrumental approach is the difference drawn between an artifact and an instrument; the artifact is the object that is used as a tool, whereas tool is a material artifact that has a purpose to perform a task. However, the instrument involves techniques and schemes developed by the user during its use (instrumentalisation) that then guide the way the tool is being used and the user's thinking (instrumentation). When integrating different tools in the mathematics classroom different dimensions have to be taken into account: the relation between the use of tool and learning, the role of the teacher in technology-rich mathematics education, and the characteristics of technological tools (Barzel et al., 2005).

In light of these considerations, this was designed to seek an understanding about how learners from two different classrooms and learning cultures appreciate and make sense out of construction endeavors using different construction tools. The question arises: Do such opportunities allow development of a deeper understanding why and how can geometry meaningfully be taught? Hence, I explore how different conscious perceptions about different construction tools may influence the intention to implement innovations into their future classroom as suggested by a course initiative. The following research questions guided the study: Which elements of reflection can be identified in preservice teacher communication of experiences on using different construction tools when solving standard construction tasks? How do preservice teachers perceive the importance of using different construction tools with respect to teaching the topic of constructions?

METHODOLOGY

This was an exploratory study with participants from two cohorts of preservice middle and junior high school teachers from the United States and Germany. The research design differed for both countries because of the different classroom and learning culture. However, the similarity of items within the two designs allowed for a comparison study. The following is a discussion of two research designs.

Teaching experiment in the United States

This research was conducted during a mathematics education course "Teaching Geometry and Measurement in the Middle School" in Spring 2011, which is a part of the middle school teacher education program (grades 5-8) at a large southern university. The course focused on teaching methods, curriculum materials, and topics and psychological factors for developing geometric and measurement structures in grades 5-8. The class met twice per week for 75 minutes, where the professor and the teaching assistant (author of the paper) led the discussion and organized the lesson plans, and supported by two additional teaching assistants. In this study, a cohort of 36 prospective mathematics middle school teachers students participated. Data collection took place in the 3rd week of the course when topic of construction began. The class was divided into three groups in which a different manipulative was used: a compass and a ruler, the Mira, and patty paper. The students were instructed to solve standard Euclidean construction problems (duplicating a segment, duplicating an angle, constructing a perpendicular bisector, construction of an angle bisector, constructing a line parallel to a given line and a point not lying on the given line) with the guidance of the instructor. After 20 minutes they rotated to a different room where they engaged in the same activities but using a different manipulative. During the next class meeting time was spent on learning how to work in a dynamic geometry environment, namely in the Geometer's Sketchpad (GSP), performing afterwards the same construction tasks as in the previous class meeting.

Data collection methods for this part of the study consisted of the following: mathematical autobiography, written reports, and field notes. The purpose of the mathematical autobiography was for the students to reflect on their experiences as a mathematics learner in geometry thus far. Written reports were 2 to 3 page reflection papers that students submitted every 2 to 3 weeks designed for students to relate what they were learning in class to their own practice or experience. They needed to choose one aspect of class that was of interest to them, and discuss it in depth both through the lenses of a student and future practitioner. I analyzed 20 of those papers that reflected on the events in the above described class-meeting. All participants reflected on the four construction tools. Additional data were my own field notes I took while observing students performing the above described tasks.

Teaching experiment in Germany

This research was conducted during a mathematics education course "Didactics of geometry for grades 7-10" in the spring 2012 semester which is a part of the teacher education program for grades 5-10 at a large university in Germany. The course focused on developing and consolidating content and didactical competencies relevant to grades 7-10. The class met twice peer week for 90 minutes in a form of a lecture and discussion, respectively. In this study, a group of 20 prospective teachers participated who attended one of the discussion sessions. Data collection took place in the 4th week of the course where the students learned about different tools that can be used when teaching constructions (a compass and ruler, a protractor, GeoGebra) as suggested in the literature (Weigand & Ludwig, 2009). In the discussion the students had the opportunity to have a practical experience with constructions using compass and ruler and patty paper. Similar to the previously discussed case, the students were given a sheet of paper with list of standard constructions they were to perform using first the patty paper and afterwards using a compass and a ruler. The students had 20 minutes to engage in these activities with a particular manipulative. As a part of their homework they performed the same activities using GeoGebra and protractor.

Data collection methods for this part of the study consisted of the following: questionnaire with open-ended questions, written reports, and field notes. The questionnaire was divided into two sections. The purpose of the first sections was to learn more about the general background of the participants and their school experiences with respect to the topic of constructions. The second section asked the participants to circle if a given tool (compass and ruler, protractor, patty paper, technology) should be used in teaching constructions where Likert scale (1–5) was used followed by an open question to elaborate on their decision. The study included a survey yielding quantitative data, however, those data were not intended for statistical analysis but as a way to help the participants reveal their thinking about aspects of their experiences, teaching, and perceptions. In addition, two open questions were posed so that the students could reflect on the benefits of these events on their current professional learning and future teaching which allowed study comparison. Additional data were my own field notes.

Data analysis

Once data collection was completed, the textual analysis (Bernard & Ryan, 2010) of all written data reports, where conceptions and reflections about different construction tools were articulated, was used. For the purpose of this study, two stages of analysis, the within-case analysis and the cross-case analysis (Bernard & Ryan, 2010) were carried out. Within-case analysis focused on an analysis of different written reports. During this phase of the analysis, I read all documents and analyzed participant responses according to topics such as background and perceptions that allowed me to situate the participants according to their experiences as well as to ascertain the soundness of the development of the geometric thinking when using different construction manipulative. Once I identified these themes from the reports, I reexamined the data, taking note of any new evidence that came to light in support of these major ideas. The cross-case analysis was used to create a theory offering general explanations of perspectives on the experience of using manipulative for both groups.

RESULTS

Elements of reflection on the experiences of using different construction tools when solving standard construction tasks

The mathematical autobiography written by the American students revealed previous experience with the topic and the different tools; almost half of the students perceived having experienced a teacher-centered middle- and high-school geometry classroom rather than a student-centered one. Other students reported having ample opportunities to engage in discovery learning and individual or group problem solving using different manipulative. With respect to different construction tools, all participants had instrumented a compass and a ruler when solving construction tasks "I have used a compass and straight edge in the past", but three people reported their teachers also used different technology software "but she [teacher] used it only to demonstrate the construction steps. We didn't get the chance to use it". They perceived geometry as "an exciting and interesting part of mathematics". On the other hand, a questionnaire was administered to German students for the same reasons. The analysis of the questionnaire revealed that, though the participants had a positive experience with geometry in their previous educational experiences, the participants' beliefs about teaching constructions varied. While 11 participants were comfortable with the topic, 8 were rather anxious about the topic. The construction problems they had experienced were easy for most of the participants, while only 3 participants experienced difficulties. All of the participants instrumented the compass and ruler together with a protractor in school, while one participant reported having instrumented the former two and technology. The topic was mainly taught in a teacher-centered classroom where the teacher explained specific construction steps and then they would practice the steps which is similar to previous results (Schoenfeld, 1988). For the rest of this section, I discuss students' reflections on the experiences using different construction tools.

Mira – The Mira was only used with the American students, as this tool was not available for use in the didactics course in Germany. For most students this was their first encounter with the Mira, that is, the tool has not been instrumented thus far by them. Though a teaching assistant shortly introduced them to the tool, the participants struggled with its use, for instance, when duplicating a segment:

> But I have never used the Mira…I had the most difficulty in trying to replicate the dots on the other side of the Mira so that I could connect them with a straight edge. It was after exploring the tool I figured out I need to place Mira adjacent to the segment, look through the Mira to identify the endpoints on the other side of it. The Mira was the hardest tool for me to understand and use correctly.

Hence, after they instrumented the Mira taking into account affordances of the tool, they were able to use it as a construction tool. This experience prompted them to stress the importance of letting the students have an opportunity to "play with the tool before using it". Moreover, all of the participants instrumentalized Mira as a reflection tool in addition that can then be used when introducing the concepts of reflection, congruence and symmetry "As I played with the Mira, I noticed more about symmetry and reflections by moving the Mira…I would definitely use the Mira to teach sections regarding reflection." However, three students did not believe that the Mira should be used as a construction tool because "it may not be very useful in applying and relating constructions and properties to students' day-to-day experiences as easily as other tool."

Patty paper – For both group of students, this was their first encounter with the patty paper and similarly to the experience with the Mira the tool needed to have been instrumented by the participants. This process by German students was very interesting. First they tried to adapt their previous compass and ruler strategies. They used the compass and ruler on the patty paper similar as when constructing on a DIN-A4 paper before getting instructed by the teaching assistant (the author of the paper) that they are only allowed to use the patty paper and a ruler. Only after having considered the affordances of the tool their thinking guided how the tool got used. The use of the patty paper as a construction tool was very well received from the American students. The participants perceived that the tool is "very easy to manipulate", "its translucent appearance helps with copying lines on multiple patty paper sheets as well as visualizing angles and intersecting lines", "inexpensive, and it has so many possibilities" and promotes geometric reasoning

> By using the patty paper I got clarity to many geometric relationships. As a teacher, I will use patty paper because there are many ways that I can use the paper to help my students learn mathematics…not only to visualize, but also to experiment and discover the geometric principles for themselves.

Five students after having instrumented all of the tools in addition offered a trajectory with regards to using different construction tools in the classroom

> I really feel these three activities go hand-in-hand, and feel it might be very beneficial to go through them in the order discussed [patty paper, Mira, compass and ruler]. I would use the patty paper as a opening activity with younger students who might not be necessarily able to correctly use Mira or compass and ruler. After discovering concepts and basic relationship the progression to Mira and then to compass and ruler would come naturally.

The other participant added that technology would be used at the end to “solidify their earlier findings or help with questions that may arise from the other teaching methods”. Hence, such experience allowed the participants to reflect on their current student role and experiences allowing to reexamine their beliefs about the teaching of the construction and shift in their future teaching practices. Perception of German students with respect to patty paper was divided; 7 participants were against its use, 9 were neutral and 4 were for its use when teaching constructions. The participants who were against its use included the following reasons: “it is imprecise”, “does not support understanding of the topic and geometric relationships”, “praxis-irrelevant” and “not usable in later education”. Some participants were extremely vocal and believed that “That is not mathematics!” Opposite to those statements, the rest indicated that it supports “a student-centered classroom where student can learn new concepts and discover new relationships” while also helping students “illustrate and visualize figures”.

Compass and ruler – All of the participants had previously instrumented compass and ruler. Only one student from the United States shared his belief that this tool should be used because it is “a classic approach to constructing and examining shapes, angles, and segments” and part of “old school”, while others mentioned reasons, such as focus on mathematical reasoning, making connections between the geometric relationships, and its ease of use.

> I was forced to recall properties of whatever I was attempting to construct or translate in order to make it work. The mathematical reasoning behind this activity is very deliberate in forcing students to make connections. Further, the tools used are ones that they can more easily use in their daily lives and relate to.

Hence, having reflected as learners and future practitioners on their own activity and reasoning they developed their perception towards the tool and consequently recognized how the tool might promote their students’ reasoning, respectively. On the other hand, 19 German participants (strongly) believed that compass and ruler should be used in teaching construction for the following reasons: “compass and ruler are the foundation of constructions”, usability in praxis, ease of use, effective and precise construction tool, promotes discovering geometric relationships, and teaches precision and finger-abilities. Students in both cohorts recognized the historical value of the use of compass by “viewing” the tool as a part of “mathematics culture”. Hence, beliefs towards the use of the tool in the future was influenced by the cultural, historical value assigned to the tool. The question that remains open is if the shift in belief did not occur because it was not challenged sufficiently or because the value is extremely instilled and therefore was not able to have been changed.

Protractor – The protractor was only used with the German students, as this tool is not a part of the American mathematical culture. The participants have had instrumented this tool throughout their education yielding homogeneous beliefs about its use when teaching construction; seventeen of them argued for its use because it was instrumentalized both as a construction and a measurement tool. They also believed that it is “more precise than compass and ruler”, “integral part of geometry”, “time efficient”, “self-explanatory” and “supports development of finger ability”.

Technology – Participants’ opinions to using technology as a construction tool was similar for both groups. For the first group, some argued against its use because it does not require much thought to make connections and experiment compares to other tools “I really loved the hands-on engagement that this activity requires of the student, rather than simple dumb luck.” The majority of the students were thrilled by the vast opportunities GSP offers such as visualization, illustration, drawing locus, and as an aid tool for lower achieving students

> I also think we would be doing students a disservice if we did not allow them to explore programs like GSP because it makes the relationship much easier to see, understand and to solidify their earlier findings. Technology makes all of this easier.

On the other hand, with respect to using technology when teaching construction, the answers from German students were not as homogenous: in total 14 students were for its use, 3 were neutral and 3 were against its use. Some reasons against its use, but not limited to, were: “not easy to use”, “by a click of the mouse a figure can be constructed failing to promote understanding of the geometric relationships”, “not usable in every day situations”, “lack of availability to students”, and “hinders the need for a proof”. Hence, the participants had a limited view of the affordances of technology a perceived technology as a “product tool”. Reinforcement of proof and visualization of geometric relationships between figures that technological tools allow was not recognized because they did not reflect as future practitioners. The rest argued for its use because of the affordances technology brings in general: visualization, speed, and accuracy.

Preservice teachers’ perception on the importance of using different construction tools with respect to teaching the topic of constructions.

At the end of their report, the participants reflected about the influence of the experience of using different construction tools on their future teaching. All American students and eight German students experienced a shift in their beliefs towards the topic of construction “using different tools gives the students a tangible experience where use of the tool promotes problem solving and proving through the use of reasoning”. However, some did not fully grasp the value of the experience writing that they learnt that “figures can be constructed with different tools”, “construction with software is precise”, and “different tools make teaching fun without connecting it to students’ learning”. In addition, fourteen German students said they will use the tools they used when they were in school. This apprenticeship of observation (Lortie, 1975) provides preservice teachers a limited understanding of teaching, which should not be underestimated. Such preconceptions that preservice teachers hold about teaching should be challenged to allow for a new geometry classroom. The rest remarked they would like to use them in their future career to differentiate instruction and support students’ independent learning. On the other hand all of the American participants expressed that they plan on implementing these activities in their classroom, as these tools will deepen students' understanding of geometry explaining, "those [hands-on activities] are what students get the most out of in class." One of the participants summed up her thoughts by using her high-school teacher's saying: "Tell me and I will forget, show me and I may not remember, involve me and I will understand." Hence, by focusing on hands-on work students can expand their outlook on different tools helping them to broaden and deepen their understanding of mathematics (Pandiscio, 2002).

In summary, both groups of students reflected through the lenses of current learners and future practitioners. The American students reported on elements portraying broadening of their mathematical perspective, such as consolidating their knowledge about constructions and geometry and the mathematics behind each tool. The latter was communicated likewise by the German students. The participants' perceptions to the importance of different tools were centered around these qualities: development and promoting of geometric thinking, problem solving activities and processes, hands-on experience, and visualization. The analysis yielded rather remarkable results with respect to their future profession and with no doubt broadened their didactical perspective. Both group of students recognized that by using different construction tools, they could arrange a student-centered classroom focusing on the students' independent learning and strengthening their conceptual understanding of different topics. On the other hand, stressing the historical and cultural value of a tool, namely the compass and the ruler, was given by the German students.

FINAL THOUGHTS

Construction tasks are of immense didactical meaning in teaching geometry for many reasons, such as development of deductive reasoning, problem solving, argumentation and proving abilities, connections within geometric topics, and creativity (Weigand & Ludwig, 2009) but this topic does not get the necessary attention in mathematics (Pandiscio, 2002; Weigand & Ludwig, 2009). The results of this study show that a short lesson with carefully designed activities with well-chosen tools may enable prospective teachers' to learn about new instructional strategies, shift in their future teaching practices towards more student-centered pedagogies and broaden their content and pedagogical perspective targeted towards promoting geometrical reasoning. Hence, it is clear that beliefs and practices are linked; teachers' perceptions are critical factors in better understanding the development of quality mathematics teaching that we as educators need to confront during their education (Wilson & Cooney, 2002). Preservice teachers should have experience in solving construction tasks using different tools as well as opportunities to discuss curricular, pedagogical, and learning issues in variety of contexts before becoming in-service teachers and taking those responsibilities on themselves. Such experience with new tools has a profound effect on preservice teachers' knowledge of geometry and without a doubt is a powerful medium for the transformation of teaching and learning geometry.

NOTES

1. The Mira is a device constructed of red or green semitransparent plastic with a beveled lower edge that generates reflected images of geometric objects.

2. Patty papers are the waxed square papers often used for baking. They are translucent but also show creases when folded.

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