AN INVESTIGATION ON STUDENTS' DEGREE OF ACQUISITION RELATED TO VAN HIELE LEVEL OF GEOMETRIC REASONING: A CASE OF 6-8TH GRADERS IN TURKEY

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The aim of this study was to examine 6-8th grade students' performances when they were asked to identify, name, and draw geometrical objects. In order to investigate students' geometrical structures, van Hiele theory was employed with the help of a model developed by Gutierrez et al. (1991, 1998). The data was collected from 809 6th to 8th grade middle school students in Ankara, Turkey. The analysis of the data revealed that most of the students had difficulty to think 3 dimensionally, and so to reach 1st van Hiele level with complete acquisition. The findings are discussed and implications for educational field are presented.

Key Words: Geometric reasoning, van Hiele theory, degree of acquisition

Geometrical Understanding

Learning geometry improves students' mathematical understanding and helps students develop their reasoning abilities (Baykul, 2000; National Council of Teachers of Mathematics [NCTM), 2000). As essential elements of school mathematics, while geometry is strongly emphasized in the Turkish elementary mathematics curriculum (Ministry of National Education [MoNE], 2006), Turkish students show low performance in both national and international exams (EARGED, 2003; 2005).

The literature points out that understanding students' thinking may improve the quality of instruction (Anderson, 2000; Carpenter, Franke & Levi, 1998). Studies investigating students' geometrical understanding (Battista & Clements, 1996; Ben-Chaim, Lappan, & Houand, 1985; Ng, 1998) emphasize that to design and implement effective instruction for meaningful learning, students' thinking structures should be understood. Since developing three-dimensional understanding is important, many researches have focused on the ways of improving students' abilities on visualizing, drawing, naming, and constructing geometric solids (Ben-Chaim, Lappan, & Houang, 1988; Meng & Idris, 2012; Pittalis & Christou, 2010). While most of those researchers investigated the effect of using manipulatives (Meng

& Idris, 2012; Moyer-Packenham, & Bolyard, 2002), 3-D simulations, games, and virtual environments (Dalgarno & Lee, 2010); some others focused on drawing solids (Ben-Chaim, Lappan, & Houang, 1985, 1988; Mohler, 2007). Those studies revealed important findings such that Ben-Chaim and colleagues (1985) found that 5-8th grade students had difficulties in relating isometric type drawings to the rectangular solids which those drawings represent. Another study explored students' thinking in 3D geometry, particularly, the representation of 3D objects and conceptualization of mathematical properties of 3D objects, and argued that there is a close relation between the representation reasoning and the mathematical properties reasoning, and so between the figural and the conceptual aspects in 3D geometry (Pittalis & Christou, 2010). The authors also suggested to curriculum developers to enrich the activities by addressing different types of reasoning. Moreover, Duval (1999) made a remarkable contribution to the literature by distinguishing four types of apprehensions for a "geometrical figure"; namely, perceptual, sequential, discursive and operative. Deliyianni and colleagues (2009) used Duval's framework to understand primary and secondary students' geometrical understanding and found that perceptual and recognition abilities appeared as first order effect on developing better geometric understanding. The authors suggested giving sufficient emphasis on geometrical figure apprehension in both primary and secondary school levels (Deliyianni, Elia, Gagatsis, Monoyiou, & Panaoura, 2009).

All those studies in the literature highlighted the importance of investigating how students construct geometrical understanding and how understanding students' geometric thinking informs teachers to improve their instruction (Panaoura, & Gagatsis, 2009). The present study will contribute to the literature by investigating Turkish 6-8th grade students' level of naming, identifying and drawing 3D geometric solids. Specifically, this study intended to understand at which van Hiele level Turkish 6-8th grade students identify a geometric solid and to what degree they acquired this level of reasoning. In order to understand how 6th-8th grade students approach geometry problems, their responses to a geometry task where they were asked to identify and draw geometrical objects were investigated via the van Hiele theory.

Van Hiele Theory

The van Hiele theory was developed by Dina van Hiele Geldof ve Pierre van Hiele in 50s in order to examine children' geometrical thinking structures, and it has been used since 80s to understand the difficulties of teaching geometry (Pegg et al., 1998). In the van Hiele theory, children' geometrical thinking structures is classified hierarchically, the levels are sequential, and they are not experience or age dependent (van Hiele, 1986). In this model, children' geometrical thinking structures are classified into 5 different levels (visual level-level 0, descriptive level-level 1, theoretical level-level 2, formal logic level-level 3, rigor level-level 4 (van Hiele, 1984, 1986).

While the van Hiele theory is a strong model to analyze students' geometrical thinking structures, it has also some limitations. Since van Hiele geometry test is a multiple choice test, it does not provide any space for students to explain their answers or reflect their ideas. More importantly, students' answers to van Hiele test sometimes may conflict to the hierarchical structure, so another test to analyze students' thinking is needed. For these reasons, another test developed by Gutierrez, Jaime, and Fortuny (1991) was adapted in this study.

Gutierrez, Jaime, and Fortuny (1991) suggested a way to assess the level of reasoning of students via creating an instrument with open-ended questions and providing explanation on evaluation of student responses through utilizing van Hiele model of reasoning. In the present study, one of the task adapted from Gutierrez, Jaime, and Fortuny (1991) was employed to determine both van Hiele level and reasoning/acquisition level of the students. One point to underline here is that in degree of acquisition analysis, the 5^{th} van Hiele level was not considered since this level was not believed to exist in 6^{th}-8^{th} grade students (Van de Walle, 2007).

The purpose of this study was to investigate students' geometrical structures, and so to examine 6-8th grade students' performances when they were asked to identify, name, and draw geometrical objects on a dot paper. The research questions that we wanted to answer were: 1) At which van Hiele levels 6-8^{th} grade students responded to the questions in Gutierrez Test? and 2) What was the students' degree of acquisition related to van Hiele levels in Gutierrez Test??

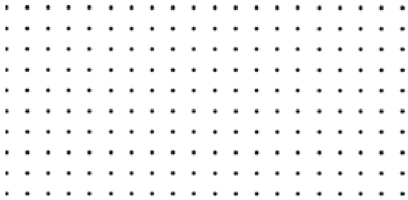
METHOD

Data Collection Tool and Procedure

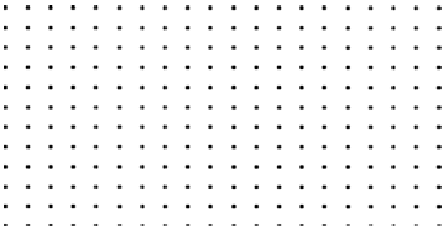
To collect data, an item adapted from Gutierrez et al. (1991) was employed to 6^{th}-8^{th} grade Turkish students. The original item provided six solids which are more complex than the ones in the adapted item below, and a set of conditions. Then students were asked to draw a different solid which satisfies five conditions given. Following that, students were requested to identify the minimum conditions which will establish the geometric shape drawn (Gutierrez et al., 1991). With a group of mathematics teachers and mathematics educators, the item was decided ask reversely. In other words, the smallest set of criteria was given and students were asked to draw the geometric shape satisfying those criteria and name it. Considering 6-8^{th} grade mathematics curriculum, following three geometric objects are selected to ask. The properties describing the objects are written by a group of researchers based on literature. The item was piloted with 75 sixth to eight grade students and pilot study suggested to provide a dot paper because 6-8^{th} graders had difficulties in drawing, and a box to name the object because most of the students forgot to name the object. The administered item is provided below.

Please draw the objects on the dot paper provided below according to the given properties, and write the names of the objects into the boxes.

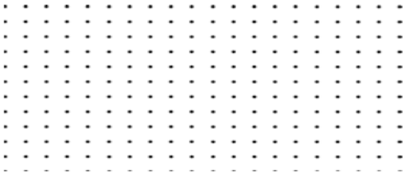
1) A vertical geometric object with rectangular lateral faces, and parallel and congruent opposing faces



2) A vertical geometric object with parallel triangle bases, and rectangular lateral faces



3) An object with a polygonal base and lateral faces which meet in one point



As seen above, the item was composed of three sub-items. Since each sub-item has potential to display whether students considered only one or more than one property to draw and/or to name the geometric object, it serves well to understand students' performances on identifying, naming, and drawing geometrical objects on a dot paper. Moreover, it measures students' performance in two ways: (1) determining van Hiele level of students, and (2) determining reasoning processes that students went through (Gutierrez & Jaime, 1998). The reasoning processes and brief descriptions are presented below (Table 1).

Table 1: Reasoning processes and descriptions

Reasoning Process	Descriptions
Recognition	Identification of types, attributes and properties of geometric shapes
Definition	Defining geometric concepts. This level of reasoning includes two different aspect: (1) **Formulation** of definitions of the concepts learned, and (2) **Utilizations** of definitions which are either read in the book or learned from teacher and peers
Classification	Classifying geometric shapes and concepts into different groups
Proof	Proofing properties and statements

Gutierrez and Jaime (1998) also determined which van Hiele levels were associated with the item and which reasoning process were included. Table 2 briefly explains that the item might be in different levels since students may produce answers in different levels with different reasoning processes (Gutierrez & Jaime, 1998).

Table 2: Item's levels of van Hiele and reasoning process

Levels of Van Hiele				Reasoning Processes				
					Definition			
0	1	2	3	Identification	Utilization	Formulation	Classification	Proof
Sub-item 1	Sub-item 1				Sub-item 1	Sub-item 1		
Sub-item 2	Sub-item 2				Sub-item 2	Sub-item 2		
Sub-item 3	Sub-item 3				Sub-item 3	Sub-item 3		

As seen in Table 2, students may produce answers to this item in the level 0 and level 1 of van Hiele theory by using *Definition* as a reasoning process.

Participants

The item was piloted with 75 sixth to eight grade students in Ankara, Turkey. After the pilot study, the item was revised and administered to 809 6-8th grade students (283 6th, 259 7th, 267 8th grade students) during two class hours in 2009-2010 academic year. Considering the grade level differences, students are varied in terms of the previous knowledge about 3-dimensional geometry. Moreover, no training was provided to students before administration of the test because this study intended to understand students' performances in detail and how those performances vary in terms of grade level.

Data Analysis

Students' responses to each sub-item were examined and analyzed by the researchers. The quality of students' responses indicated which of the van Hiele levels were attained by the students. Researchers conducted a series of meetings to form a coding process following the van Hiele Model and test the reliability of scoring. Then, students' responses to each sub-item were carefully read and classified according to van Hiele levels. Additionally, students' degree of acquisition of the levels 0 to 3 was analyzed by using the "percentage intervals" developed by Gutierrez and Jaime (1998) (Table 3). In fact, Gutierrez and Jaime (1998) asserted to determine degree of acquisition for each level instead of assigning students into a particular van Hiele level because they viewed the development of students through van Hiele levels continuous.

Table 3: Acquisition levels and their intervals

Percentage Value of The Interval	Acquisition Level
0%-15%	No Acquisition
15%-40%	Low Acquisition
40%-60%	Intermediate Acquisition
60%-85%	High Acquisition
85%-100%	Complete Acquisition

Each sub-item was analyzed separately since van Hiele level of students' responses varied in each sub-item. Students who could not answer the item or produced unrelated answers were removed from the data set.

FINDINGS

Below, selected findings are presented for the first sub-item. Because of the place limitations, the results of second and third sub-items are only summarized briefly.

It should be noted that in the case that students did not answer the item or answered it incorrectly, their van Hiele level could not be determined. If students drew or named the geometric object by considering only one property, then these students were assigned to the level 0. Students who correctly drew and named the geometric object by considering all of the properties given were grouped into the van Hiele level 1.

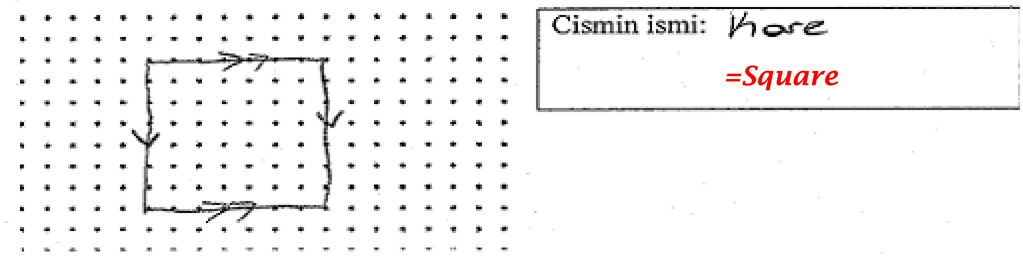
Sub-item-1

The first sub-item described a geometric object which is vertical and has rectangular lateral faces, and parallel and equal opposing faces; and asked students to write the name of the geometric object and draw it to the given dot paper.

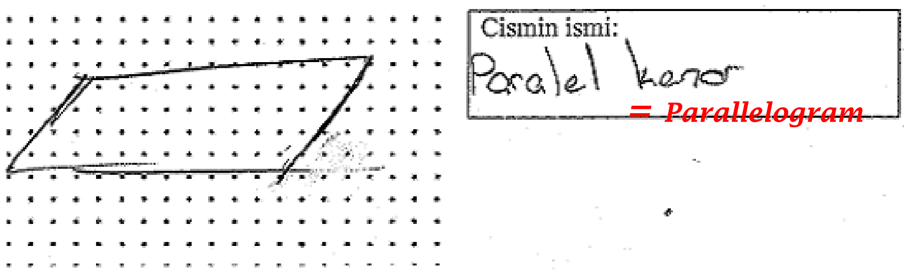
Sub-item-1: Van Hiele level not determined

Considering the evaluation described above, van Hiele level of 171 (20.5%) students could not be determined. The student responses resulted with no interpretation are illustrated below.

Sample student response #1:



Sample student response #2:

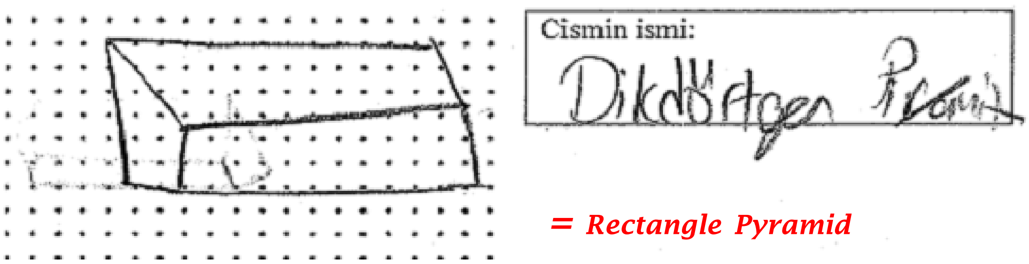


As seen from the sample student answers to the sub-item-1, some students could not visualize in three dimensions, and so could not answer the question correctly. Thus, their van Hiele level could not be determined.

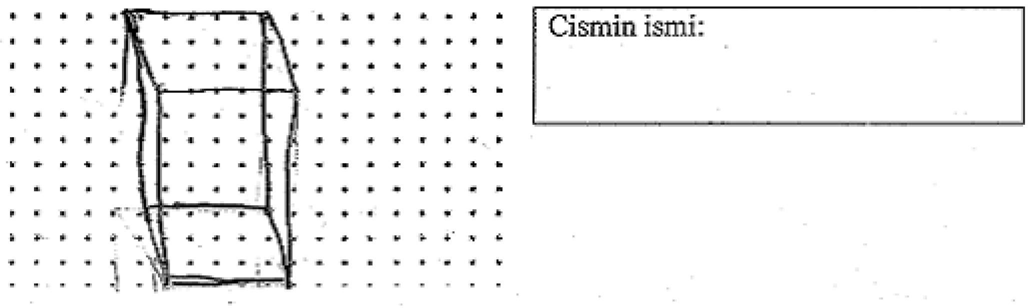
Sub-item-1: van Hiele Level 0

316 (38.1%) students answered this sub-item in the level 0 of van Hiele theory (112 sixth, 115 seventh, 89 eight graders). Sample student responses reflecting level 0 are given below.

Sample student response #1:



Sample student response #2:



As seen in student responses above, students either correctly wrote the name of the geometric object but could not draw it properly, or drew the object correctly but could not name it or named it wrongly.

When the results were analyzed according to degree of acquisition, it was found that majority of 8th graders (88.8%) had *no acquisition* of the level. Half of the 6th and 7th graders had *no acquisition* of the level while the other half of them possessed the level with *low acquisition*. In this sub-item, there were no 6th graders, but a few 7th and 8th graders in *intermediate level of acquisition*. Again none of the 6th and 8th graders acquired the level completely. Only one 7th grade student reached *complete acquisition* of the level.

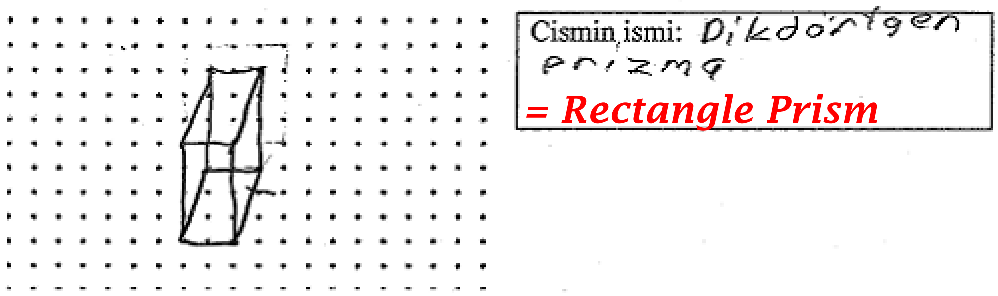
Table 4: Distribution of students' responses on sub-item-1 at van Hiele Level 0 across grade levels

Level of Acquisition	6th grade		7th grade		8th grade	
	N	**%**	**N**	**%**	**N**	**%**
No Acquisition	56	50,0	59	51,3	79	88,8
Low Acquisition	56	50,0	53	46,1	9	10,1
Intermediate Acquisition	0	0,0	2	1,7	1	1,1
Complete Acquisition	0	0,0	1	0,9	0	0
TOTAL	112	100	115	100	89	100

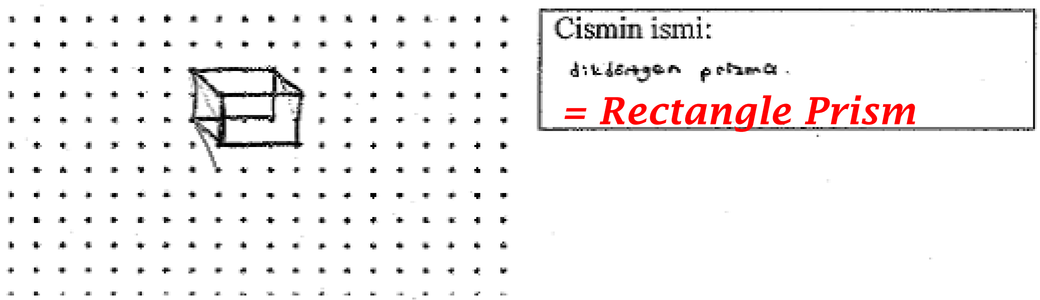
Sub-item-1: van Hiele level 1

The analysis showed that 346 (41.4%) students produced responses in the first van Hiele level (111 sixth, 115 seventh, 120 eight graders).

Sample student response #1:



Sample student response #2:



The sample student answers reveal that some of the students were able to identify the object, draw it, and name it correctly.

With respect to the students' degree of acquisition, majority of the students in all three grade levels had *complete acquisition* of the level. In other words, 68.4% of 6th graders, 73.9 % of 7th graders and 67.5% of 8th graders attained the second van Hiele level completely. As seen in Table 5, approximately a quarter of students in each grade level had *intermediate acquisition* of the level for sub-item-1. Even though there were few students having the level with *low acquisition* and *no acquisition*, the students' responses reflected 1st van Hiele level with a high degree.

Table 5: Distribution of students' responses on sub-item-1 at van Hiele level 1 across grade levels

Level of Acquisition	**6th grade**		**7th grade**		**8th grade**	
	N	**%**	**N**	**%**	**N**	**%**
No Acquisition	3	2,7	1	0,9	4	3,3
Low Acquisition	1	0,9	3	2,6	5	4,2
Intermediate Acquisition	31	28,0	26	22,6	30	25,0
Complete Acquisition	76	68,4	85	73,9	81	67,5
TOTAL	111	100	115	100	120	100

Sub-item-2: Van Hiele level not determined, and van Hiele levels 0 and 1

In this sub-item, students were given properties such that "*A perpendicular object with parallel triangle bases, and rectangular lateral faces*", and asked to name and draw this geometric object. Similar to Sub-item-1, students produced answers in van Hiele levels 0 and 1. While 296 (35.7%; 116 sixth, 95 seventh, 85 eight graders) students answered the sub-item in the level 0 mostly with *low* and *no acquisition level*, 283 students (33.9%; 81 sixth, 108 seventh, 97 eight graders) were in van Hiele level 1 with *complete* and *intermediate level of acquisition*. Also, there were 254 (30.4%) students the answers of whom could not be determined.

Sub-item-3: Van Hiele level not determined, and van Hiele levels 0 and 1

This sub-item asked students to name and draw a 3D geometric object which had a polygonal base and lateral faces meeting in one point. The results were similar to that of sub-item 1 and 2. In other words, there were 288 (34.5%) students of whom van Hiele level could not be identified; there were 291 (35.1%; 100 sixth, 91 seventh, 100 eight grade) students in van Hiele level 0; 254 (30.4%; 85 sixth, 96 seventh, 73 eight grade) students in van Hiele level 1 with *complete* and *intermediate level of acquisition*.

DISCUSSION AND CONCLUSION

In order to examine our students' acquisition degrees, we employed an assessment technique developed by Gutierrez et al. (1991) and Gutierrez and Jaime (1998) through utilizing van Hiele model of reasoning.

The findings of the study revealed that approximately quarter of 6-8th grade students could not think in 3D, and their van Hiele level for 3D geometric thinking could not be identified. Approximately, 36% of the students classified in van Hiele level 0. Moreover, students had this level mostly with *no acquisition*.

Findings also revealed that 36% of the students in average drew 3D object properly and named it correctly, and reached to van Hiele level 1. Most of the students attained the 1st van Hiele level 1 with *complete acquisition*. The number of 6-8th grade students slightly differed from each other. This finding contradicts to Wu and Ma (2006)'s findings where they found that the higher the grades the higher the van Hiele levels. In their study with 5581 randomly selected 1-6th graders in Taiwan, Wu and Ma (2006) concluded that the higher the grades the higher the van Hiele levels. At this point, the fact that the methodologies, cultural aspects, and students' experiences with the topic were different in these two studies should be taken into account.

In short, more than 25% of the 6-8th graders' van Hiele level could not be determined; 36% of them had van Hiele level 0 mostly with *no and low acquisition;*

and other 36% was in van Hiele level 1 mostly with *complete and intermediate level of acquisition.* This finding reveals that 64% of the students could not reach the 1st van Hiele level. Thus, this finding supports that students' geometrical understanding and reasoning is low (Duval, 1998; Gutierrez et al., 1991). This result might be attributed to the fact that the teaching strategies might be based on memorization (Battista, 2001).

Van Hiele (1986) stated that if students can not reach even to the descriptive level of geometry, it might be because they had no chance to experience geometric problems before. Instruction is believed to be the way for development through the van Hiele levels (Koehler & Grouws, 1992), and quality instruction is one of the most effective ways to improve students' understanding of geometry (Usiskin, 1982).

In this study, considering a quarter of 6-8th grade students' van Hiele levels were not being determined, we suggest curriculum developers and teachers to provide more opportunities for students to deal with three dimensional geometry. Moreover, even though 36% of students were found in the level 0 of van Hiele theory, they had either low acquisition or no acquisition at all. This might be another evidence of their need of geometrically rich experiences like the item utilized in this study or the original item developed by Gutierrez et al. (1991) in order to develop three dimensional thinking in geometry. From this idea, further studies are suggested to be conducted to understand how the classrooms could be transformed to support students' three dimensional reasoning. It is also recommended to analyze reasons of low or no acquisition in a particular van Hiele level so as to inform teachers and curriculum developers about the needs of students.

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