THE EFFECTS OF A TEACHING METHOD SUPPORTING METACOGNITION ON 7TH GRADE STUDENTS' CONCEPTUAL AND PROCEDURAL KNOWLEDGE ON ALGEBRAIC EXPRESSIONS AND EQUATIONS

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The purpose of this study is to investigate the effects of a teaching method supporting metacognitive strategy use on improving seventh grade students' conceptual and procedural knowledge on algebraic expressions and equations. The study was conducted in two seventh grade classes from a public school in the 2010–2011 academic year. Quasi experimental design with pretest- posttest control group was utilized for the study. Conceptual Knowledge Scale and Procedural Knowledge Scale were conducted to control and experimental groups before and after the implementation of the instructions. Data were analyzed by using t-tests. Results showed that there was a significant difference between experimental and control groups in terms of gain scores on conceptual and procedural knowledge in favor of experimental group.

Key Words: *Elementary Mathematics Education, Conceptual Algebra Knowledge, Procedural Algebra Knowledge, Metacognition*

INTRODUCTION AND RELATED LITERATURE

Algebra is a branch of mathematics, which turns relations examined by using symbols and numbers to generalized equations. Not only does it represent letters and quantities, it also allows making calculations using these symbols at the same time (Kieran, 1992). Ideally, algebra lessons lead students to develop a profound understanding of algebraic concepts and the ability to see algebra as a central and connected branch of mathematics and the ability to apply algebra to a wide range of topics. If this happens, then students can be said to have a high algebraic competency (Oldenburg, 2009).

There are two concepts underlying algebraic expressions and equations. These concepts are "variable" and "equality". Variable concept is usually expressed through literal symbols. According to Philipp (1992), literal symbols have such different uses as label, constants, unknowns, generalized numbers, varying quantities, parameters, and abstract symbols. Equal sign is used for various meanings in algebra. Research demonstrated that children perceive the equal sign as a symbol indicating action instead of a relational symbol (Kiearan, 1992; Yaman et al., 2003). Küchemann determined that children have 6 different thoughts regarding perceiving letters. These are; 1) letters have numerical values, 2) letters do not have a meaning in mathematics, 3) letters are abbreviations of concrete objects, 4) letters are unknown numbers and they have only one value, 5) letters are generalized numbers and have only one value, and 6) letters are variables.

Dede et al. (2002) put forth reasons of the hardship students undergo in learning algebra as not knowing about different uses of variables, not knowing about the role of variables in making generalizations, not being able to interpret variables, and failure to perform operations with variables. Baki (1998) listed students' misconceptions as errors in inclusion in parentheses and using operators, carelessness, and turning non-numerical expressions into algebraic expressions. Perso (1992) grouped the misconceptions in algebra under three main headings as the location of the letters, use of variables, and algebraic rules.

For elimination of those misconceptions in learning algebra, the students need to understand concepts like variable, equation, and have preliminary knowledge such as arithmetical operation knowledge. Algebraic comprehension depends not on knowledge of the students of the formulas and understanding the calculations right, but instead understanding of the concepts and operations, and development of mathematical thinking. Therefore, concepts and relations should be attached importance instead of procedural means of solution, and learning should be realized through conceptual learning that involves the knowledge of operations and concepts in a balanced manner (Baki & Kartal, 2004).

To sum, research reviewed above show that students have difficulties in understanding the concepts of variable and equation, forming and solving algebraic equations, using algebraic expressions, and in algebraic problem solving (Baki & Kartal, 2004; Dede & Peker, 2007; Herscovics, 1989; Kieran, 1992; MacGregor & Stacey, 1993).

CONCEPTUAL AND PROCEDURAL KNOWLEDGE

Researchers describe two kinds of knowledge in learning mathematics; conceptual knowledge and procedural knowledge (Van de Walle, 2004). Conceptual knowledge can be defined as any concept, rule, generalization and the relation between them (Hiebert & Lefevre, 1986; Rittle- Johnson & Alibali, 1999). Procedural knowledge is explained with the two parts. First part of procedural knowledge consists of the symbols and language of mathematics. The second part of the procedural knowledge consists of rules, algorithms or procedures used to solve mathematical tasks (Hiebert & Lefevre, 1986).

When algebra knowledge of students is examined in the context of conceptual and procedural knowledge, it can be seen that the nature of algebraic knowledge of students is not based on conceptual learning, where conceptual and procedural knowledge is balanced (Baki & Kartal, 2004).

METACOGNITIVE INSTRUCTION

Constructivist learning approach advocates that knowledge is not independent from the learner, and that the individual constructs knowledge him/herself in his/her mind (Olkun & Toluk, 2003). For the students to construct mathematical concepts and thoughts in their mind in a meaningful manner, they should have skills such as monitoring and regulating their own thought processes and mental activities as well as self-control of learning. These abilities are defined as metacognitive skills. Metacognition means one's awareness of its own thought processes, and his or her ability to control those (Beauford, 1996; Brown, 1978; Flavell, 1979).

Instructional practices such as writing, thinking aloud, using behavior cards (Demircioğlu, 2008; Pugalee, 2004; Özsoy, 2007); promoting learning environments that are conducive to the construction and use of metacognition (Schraw, 1998); supporting interactive problem solving (Kramarski, Mevarech, Liebermann, 2001; Schraw, 1997); asking reflective questions (Mayer, 1998; Schoenfeld, 1985); using control lists (Schraw, 1998) are used to improve students' metacognitive skills.

It was revealed that children who are applied educational processes for development of metacognitive skills had positive and meaningful increases in mathematical success (Naglieri & Johnson, 2000; Özsoy, 2007; Teong, 2002). Even though research has shown positive impacts of metacognitive skills (prediction, planning, monitoring, and evaluation) on mathematical success, little is known about the effect of a metacognitive instruction on students' conceptual and procedural knowledge on algebra, particularly on algebraic expressions and equations.

PURPOSE OF THE STUDY

This research aims at examining the effect of a teaching method supporting the use of metacognitive strategies on conceptual and procedural knowledge of the students about algebraic expressions and equations. Besides, an answer was sought on which one of the conceptual and procedural knowledge the provided education had more effect.

METHODS

Quasi experimental design with pretest-posttest control groups was used in the study. Participants consist of 80 7th grade students attending in a public school in one of the low socio-economic level districts of Istanbul province. Among the four seventh grade classes taught by the first researcher, two of them were arbitrarily selected. One of these classes was randomly assigned as experimental group while the other group was assigned as control group. The experimental and control groups show a balanced distribution in terms of gender. Both groups also showed no meaningful difference in terms of preliminary test scores assessing their conceptual and procedural knowledge on algebra.

The implementation was held in fall semester of 2010-2011 educational year, and lasted for a total of 6 weeks (24 hours) for both groups. The first researcher, who is the teacher of the experimental and control groups, taught the classes of both groups throughout the research. Before starting the process, the experimental group students were informed on metacognition for 2 lesson hours, and the implementations were told to be held throughout the process. For internalization of the metacognition strategies by the students, pre-implementation lessons of the experimental group were taught for two weeks using metacognitive strategies and with writing exercises.

Instruction supporting the use of metacognitive strategies was applied to the experimental group. This process includes teaching through structured applications based on metacognition together with problem-based learning activities used and suggested in many researches (Goldberg & Bush, 2003; Kramarski et al., 2001; Özsoy, 2007; Schraw, 1998; Schoenfeld, 1985; Wilburne, 1997). The lessons involved the exercises of thinking aloud, solving problems with groups of two, class discussions, writing, and keeping learning diaries. Students were expected to use "Problem Solving Metacognitive Behavior List", adapted from Goos et al. (2000) while solving problems and "List of The Explanation of Thinking Process", adapted from Beyer (1988) while explaining their thinking processes.

In thinking aloud exercises, sudents were requested to think (regarding targets, plans, strategies, etc.) and make decisions aloud. In solving problem with groups of two exercises, one of the students talked about his/her problem solution (what he/she understands from the problem, solution plan, etc.) and his/her friend asked questions that would clarify his/her thinking process. They summarized and reviewed the process from time to time and checked their comprehension. When the thinking processes of the group are shared through class discussion, alternative solution methods were shared and the solution method, which is the most suitable for the problem was discussed. Writing studies involved predictions regarding easiness or difficulty of the problem and the time needed to solve it, planning regarding solution of the problem, operations during the solution, views regarding the decisions taken and evaluation of the solution process. The teacher's task is to make sure that the students use metacognitive steps and explain their thinking processes. The teacher asked several questions in order to improve students' prediction, planning, monitoring and evaluation skills such as, "Can you solve this problem? How long does it take you to solve the problem? How are you going to solve the problem? What are you doing now? Can you summarize what you have done so far? What you should do after this point? How are you doing it? Is this method going to work? Do you think you have done everything correctly? Do you think another method can be tried? What is the best solution method?" An example of writing study related to acquisition of "Explains linear equations" are shown below;

"Please solve the problem below, writing down your thinking processes.

Construction equipment, which has 250 litres of diesel in its tank, consumes 10 litres of diesel per hour;

a) Please write down the equation of the relation between amount of diesel and working time.
b) How many liters of diesel is left in the tank of this construction equipment?

c) For how many hours has this construction equipment worked when there is 140 liters of diesel in this construction equipment's tank?
d) How many hours does this construction equipment need to work to finish all the diesel in its fuel tank?
e) Since this construction equipment can work 5 hours per day at the most, how many days are needed to finish all the diesel in its fuel tank?"

In the control group, the student-focused, ordinary instruction was implemented in line with the educational program. All the problems solved throughout the activities in the experimental group were also solved in this group. Active participation of the students was ensured, methods like question-answer, discussion were used, nevertheless, metacognitive activities surfacing the thought processes of the students such as writing, thinking aloud, keeping learning diary and solving problems with groups of two were not used in this group. Teachers and students solved the questions without explicitly expressing their own thinking processes.

Two hour classes in the experimental and control group were observed by the other maths teacher of the school and notes were taken regarding the functioning of the classes. These notes were used for the purpose of collecting information regarding whether the teaching processes in the experimental and control groups were conducted as planned.

Conceptual Knowledge Scale (CKS) formed using the body of literature was applied in the study for the purpose of measuring the conceptual knowledge of the students on algebraic expressions and equations (Akkuş, 2004; Hart, et al., 1985). The test involves different uses of the variable. Correct answers count as 1 point and wrong answers count as 0 point for every item. The KR-20 reliability coefficient of the scale consisting of 64 items with sub-problems was found 0.86. Some examples of the test items are shown below;

Item 3) Which one of the "2n" and "n+2" statements is greater in which case? (n is a natural number.)

Item 14) When is the expression k+m+n=k+n+p correct?

a) Always b) Never c) Only when m=n d) Only when m=p

Procedural Knowledge Scale (PKS) formed using the body of literature was applied for the purpose of measuring the procedural knowledge of the students on algebra (Akkuş, 2004). The test consists of 17 open-ended questions, which involves routine algebra problems requiring symbolic manipulation and calculations.A five-point rubric (0-4) was used to score the students' responses. Cronbach alpha coefficient of the scale was found 0.94. t-test was administered in analysis of the data, and α value was taken as 0.05. Some examples of the test items are shown below;

Item 4) $\frac{3x}{2}+7=x+1$ please solve the equation of the first degree with one unknown.

Item 10) Please find the value of y for x=0 on 3x+2=y line.

RESULTS

A comparison of the experimental and control groups in terms of their mean scores from CKS shows that their pre test mean scores are low with low variability ($\bar{x}_{Experimental} = 10.73$, $\bar{x}_{Control} = 11.13$) but there is an evident difference in favor of the experimental group between their post test mean scores ($\bar{x}_{Experimental} = 28.63$, $\bar{x}_{Control} = 16.93$). However, considering the fact that the highest score achievable from CKS is 64, it was observed that the means of both groups are low.

A comparison with t test of the means of the gain scores ($\bar{x}_{Experimental} = 17.90$, $\bar{x}_{control} = 5.80$) of the two groups revealed a meaningful statistical difference in favor of the experimental group (t (78) = 5.701; $p<0.05$) (Table 1). Cohen d value 1.28 shows high effect size (Cohen, 1988, as cited in Gravetter & Wallnau, 2004).

Group	n	$\bar{X}$	s	sd	t	p
Experimental Group	40	17.90	7.90	78	5.701	.000
Control Group	40	5.80	10.85			

Table 1: t Test Results for CKS Gain Scores of the Experimental and Control Groups

A comparison of the experimental and control groups in terms of their mean scores from PKS shows, in a way similar to the previous finding, that their pre test mean scores are low with low variability ($\bar{x}_{Experimental} = 2.80$; $\bar{x}_{Control} = 3.80$), while there is an evident difference in favor of the experimental group between their post test mean scores ($\bar{x}_{Experimental} = 28.05$; $\bar{x}_{Control} = 14.25$). However, considering the fact that the highest score achievable from PKS is 68, it was observed that the means of the groups are low. Besides, it is observed that the standard deviation values are quite high in distribution of the post test scores ($s_{Experimental} = 17.51$; $s_{Control} = 15.32$). This shows that the implementations might have had different effects on the procedural knowledge of the students with different personal characteristics.

A comparison of the means of the gain scores ($\bar{x}_{Experimental} = 25.25$, $\bar{x}_{Control} = 10.45$) of the groups revealed a meaningful statistical difference in favor of the experimental group (t (78) = 4.633; $p<0.05$) (Table 2). Cohen d value 1.04 shows high effect size.

Groups	n	$\bar{X}$	s	sd	t	p
Experimental Group	40	25.25	15.55	78	4.633	.000
Control Group	40	10.45	12.90			

Table 2: t Test analysis Results for PKS Gain Scores of the Test and Control Groups

To examine whether or not the use of metacognitive strategies is more effective on either of the conceptual knowledge or procedural knowledge gains, the scores from the two scales were transformed to a 100-score scale, and the gain scores of the experimental group was recalculated (Table 3). Accordingly, an evident difference is observed between the mean scores of the conceptual knowledge gain and procedural knowledge gain of the experimental group ($\bar{x}_{CKS} = 27.97$ $\bar{x}_{PKS} = 37.13$).

Group	Test	n	$\bar{X}$	s
Experimental Group	CKS	40	27.97	12.34
	PKS	40	37.13	22.87

Table 3: Descriptive Statistics on the Gain Scores of the Experimental Group from CKS and PKS

An analysis with t test of the conceptual and procedural knowledge scale gain scores of the experimental group revealed a meaningful statistical difference in favor of PKS (t (78) = 2.230; p<0.05) (Table 4). Cohen d value 0.50 shows moderate effect size.

Test	n	$\bar{X}$	s	sd	t	p
CKS	40	27.97	12.34	78	2.230	.030
PKS	40	37.13	22.87			

Table 4: t Test analysis Results for CKS and PKS Gain Scores of the Experimental Group

The mean of procedural gain score of the group instructed with the method supporting the use of metacognitive strategies has been statistically higher at a significant level compared to the mean of conceptual gain score.

CONCLUSION

The first two findings of this research showed that both of the conceptual and procedural knowledge gains of the experimental group has been significantly higher than the conceptual and procedural knowledge gains of the control group. Considering these findings, it can be said that instruction supported with metacognition has positive effects on conceptual learning balancing conceptual and procedural knowledge. These findings are parallel to the prior research findings supporting metacognitive instruction (Mevarech & Fridkin, 2006; Pilten, 2008). However, a review of the post-test scores shows that success in the experimental group is low as in the control group. This may be related to the general low success of the students participated in the study. These findings indicate that instruction supporting use of metacognitive skills may not be adequate alone in order to increase success of students with low mathematics achievements.

The third finding of the research is that the procedural knowledge gain of the experimental group is significantly higher than its conceptual knowledge gain. This can be due to formation of the conceptual foundations of the procedural knowledge, and the interest and familiarity of the students to the problems requiring procedural skills through traditional exams. Most of the investigations examining the effect of gaining either conceptual or procedural knowledge on gaining the other showed that development of conceptual knowledge substantially entailed development of procedural knowledge (Baki & Kartal 2004; Hiebert & Waerne, 1996; Perry, 1991; Rittle- Johnson & Alibali, 1999). Supporting the procedural knowledge with conceptual knowledge can ensure the use of procedures in a right way, and results to an increase in procedural success. Besides, it was specified that the preparation of the students for exams might have had effects on this finding. Students taking the Placement Test at the end of the year, would more likely to encounter problems requiring procedural skills. Hence they are more eager and familiar with these skillsthrough the private teaching institutions they attend and through their test books. Thus, they might have higher opportunities to improve their procedural knowledge and skills. Furthermore, researches comparing the current conceptual and procedural knowledge and skills of the students showed that procedural success is higher compared to conceptual success. It was specified that that can be explained by the students' encountering mostly with questions requiring their procedural knowledge (Baki & Kartal, 2004; Bekdemir & Işık, 2007; Bekdemir, Okur & Gelen, 2010; Star, 2000).

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