GENERALIZING AND JUSTIFYING PROPERTIES OF REAL NUMBERS: A STUDY AT GRADE 9

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ALGEBRAIC REASONING PROCESSES

Developing mathematical reasoning is an essential condition to understand mathematics and also to use mathematics in a proficient way. Also, as reasoning is recognized as inseparable from the representations and language through which it may be expressed (Arzarello, Bazzini, & Chiappini, 2001), algebra, which forms the basis of the symbolic language of mathematics, is one of the privileged topics for its development. To give some structure to the complexity of ideas, notations and activities involved in Algebra and algebraic reasoning, Kaput (2008) advocates a perspective of symbolism. In this perspective, the author emphasizes two central aspects of algebra: (i) Algebra as symbolizing systematically generalizations of regularities and constraints, and (ii) Algebra as syntactically guided reasoning and actions as generalizations expressed in conventional symbolic systems. One of the areas these aspects are embedded to is Algebra as the study of structures and systems of abstract relationships and procedures, including those from Arithmetic (algebra as generalized arithmetic) (Kaput, 2008).

Reasoning ability is crucial to students. Developing students' algebraic reasoning is not just memorizing concepts and routine procedures. Instead, focus on memorization leads the students to develop a vision of mathematics as a disconnected set of rules rather than a logical and coherent science (ME, 2007). However, to develop reasoning is challenging to teachers, particularly in everyday classrooms, in all mathematical topics. In order to understand how to promote the development of students' algebraic reasoning, a fruitful step is to regard students' mathematical reasoning processes as generalizing and justifying. Generalizing plays an essential role in the understanding of mathematics as this process of reasoning is one of the foundations of mathematics as a science. Also, justification is an essential process as it allows students to clarify their reasoning, contributing to the development of a deeper understanding of algebra.

AIMS AND METHODOLOGY

This study aims to analyse grade 9 students' algebraic reasoning processes while working on tasks involving real numbers and to know how those tasks contribute to develop students' mathematical reasoning in an algebraic context. Based on a collaboration with a grade 9 mathematics teacher, the study is developed within a

teaching unit supported by exploratory tasks involving real numbers. Following an interpretative and qualitative methodology, data collection includes videorecording of the lessons and students' written tasks. Data analysis is undertaken according to two main categories related to reasoning processes – generalization and justification. Two other transversal dimensions that go along with reasoning processes – representations and sense making – are also considered in the analysis. This poster focuses on two students working on a particularly significant task on the properties of operations with real numbers, involving a mixture of closed and open questions.

CONCLUSIONS

The results suggest that exploratory tasks with real numbers promote the development of students' conjecturing processes, particularly generalizations, but are not enough to make them use justifying processes. While making generalizations, most students follow an inductive approach, generalizing the relations observed in particular cases to a larger class of objects. Sometimes these generalizations have a deductive nature. In contrast, justifying is not done spontaneously, but, in response to the teacher's questioning, students show to be able to make justifications based on previous knowledge of properties or mathematical concepts and based on counterexamples that refute a statement. Therefore, we conclude that, besides exploratory tasks, teaching practices such as questioning needs to play a key role in promoting the use of reasoning processes. Exploratory tasks allied with teacher questioning practices, emerged as key elements to develop students use of reasoning processes as well as to promote their deeper understanding of real numbers properties.

CONTENT OF THE POSTER

The poster presents the aims of the study, a pictorial theoretical framework on mathematical reasoning, some of the students' written work on tasks with properties of real numbers, conclusions regarding those tasks and some implications for the development of algebraic reasoning.

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