HOW EXPERTS GRAPH FORMULAS

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Introduction

In secondary school algebra students learn to manipulate algebraic expressions, solve equations and to graph formulas: to make graphical representations of formulas, which is an important approach in problem solving. To be able to graph formulas easily, one has to read the formula 'grasping the structure of expressions' (Sfard & Linchevski, 1994), one of the aspects of symbol sense (Arcavi, 1994). Symbol sense is a topic rarely addressed in schoolbooks. As a consequence, teachers have to develop their own teaching methods in this area, which requires pedagogical content knowledge including subject matter knowledge and teaching strategies (Hill et al., 2008). In this research we investigate Dutch teachers' knowledge in these respects.

In the first stage of the project we investigate the following question:

> For which algebraic formulas do experts immediately recognize a global graph? And if instant recognition is lacking, which strategies do they use to identify the graph?

We want to investigate how experts perform on these tasks. Expert analysis is important to identify knowledge and heuristics needed for problem solving in this domain.

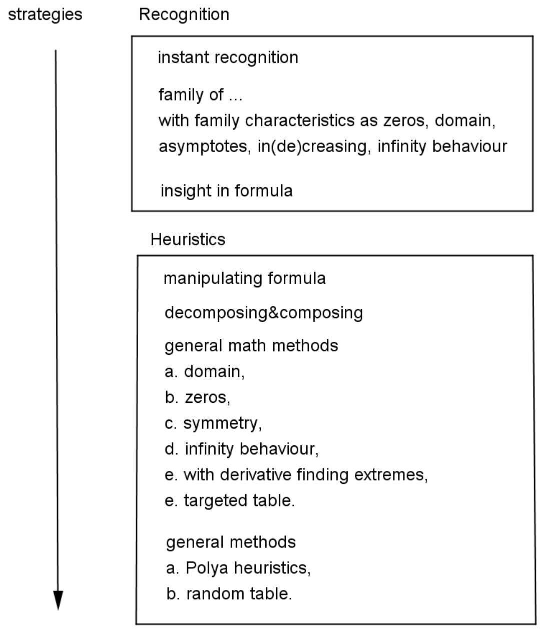
The results will allow for a comparison with the results of teachers and students on the same tasks, as to get a clear view on differences between experts and novices. In this way, we will establish whether experienced and novice teachers and students use the same heuristics when working on these tasks.

Theory

Chi et al. (1979) and De Groot (1965) both claim that experts, compared to others, recognize more, see deeper patterns and are more sensitive to critical features. They use larger units of knowledge (chunks), hierarchically organized. In problem solving they perform qualitative analyses, consider potential actions and categorize problems. Executing such tasks they apply self-monitoring, evaluating product and process and looking for alternative solutions. More specifically for mathematics, Polya (1945) formulates heuristics as strategies for problem solving. In line with this, Van Streun (1989) uses heuristics based on Polya's, but he identifies recognition as an explicit start of the problem solving process. On basis of these general perspectives we formulate a framework for specific strategies for graphing a formula (figure 1), in which we distinguish two steps, recognition and heuristics.

Method

For this stage of the project we developed three tasks in order to elicit experts' algebraic knowledge and strategies. Task 1 is selecting a global graph that can represent a given formula and vice versa. In task 2 we want to the experts to move beyond their recognition-zone through working with more complex formulas and graphs. In both tasks we ask experts to think aloud. Task 3 is a card sorting task. Given 60 formulas, the experts are asked to categorize them on the (global) graph, and to describe characteristics and a prototype of each of the categories. We use our framework to analyse the data.



Two of the experts we have chosen work as mathematicians at university and teach first year students; another works as a teacher and develops mathematical schoolbooks, the fourth works at the National Institute for National Examens (Cito) and the last one works as a teacher educator at the university of Utrecht. All of them are masters or PhD in mathematics, have more than 10 years experience in their work, in which they often graph formulas.

Results

In the protocols of the experts' interviews we identified the strategies mentioned in our framework. As formulas become more complex, experts recognized less and use more general methods. However, we also found differences among experts; in order to take these into account our framework must be adapted. In our poster we will give examples of the tasks and results of one of the experts.

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