# DIFFERENT PRAXEOLOGIES FOR RATIONAL NUMBERS IN DECIMAL SYSTEM – THE $0,\overline{9}$ CASE

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*We report the results of an experiment in which students were asked to make some computations involving the (ultimately periodic) decimal expansion of rational numbers and simple algorithms derived from the algorithms in use for decimal numbers. We show in which way these algorithms could be useful to increase the understanding of such a counterintuitive equality as 0.999…=1.*

## INTRODUCTION

It is well known that the understanding of the double representation of decimal numbers in base ten system is an important issue for students. The emblem is the equality $0,\overline{9}=1$ that summarizes the situation and on which teachers and searchers generally focus.

Many searchers already investigated this inequality, among them it is worth mentioning Sierpinska (1985), Tall (1980), Tall and Scharzenberger (1978) and Dubinsky et al. (2005). These studies mainly focus on the infinite character of such a representations and, more specifically, on real numbers and limits. Here, we expose an alternative viewpoint on the equality $0,\overline{9}=1$ which is essentially algebraic and emphasizes on rational numbers. The present work derives from a recent research topic in mathematics, the notion of circular words (Rittaud & Vivier, to appear).

In this paper, we consider the comparison and the sum of repeating decimals. The register of representation (Duval, 1995, 2006) allows us to take into account the difference between semiotic representation (repeating decimals and fractions) and mathematical objects (rational numbers) [1]. We exhibit two different Mathematical Organizations (Chevallard, 1999) related to (1) repeating decimals and (2) rational numbers. The main difference between them is a technology related to the identification of the double representation of decimal numbers. These two praxeologies could explain some difficulties of students when we want them to understand why $0,\overline{9}=1$. It is our opinion that an important issue of the problem is to pass from a repeating decimals praxeology to a rational numbers praxeology. In this perspective, using an algorithm for the sum (Rittaud & Vivier, submitted work [2]) of two repeating decimals, we present a sequence of two activities that we proposed to students at the beginning of university.  This informs on previous teaching at secondary school and at the very beginning of university.

## THE EQUALITY $0,\overline{9}=1$: REGISTERS AND TECHNOLOGY

In this section, we present the problem of the equality of $0,\overline{9}$ and 1 with registers of representation (Duval, 1995, 2006) and praxeologies (Chevallard, 1999).

## Theoretical frameworks

In the Anthropological Theory of Didactics (ATD) the mathematical activity is elaborated around types of tasks, appointed by Chevallard (1999) as mathematical organization. Generally, to perform a type of tasks T, we have at least one technique $\tau$. Type of tasks and techniques are organized in a $[T,\tau]$ appointed block of know-how or praxis. To produce and/or justify a technique $\tau$, it is necessary to have a theoretical look at the problem posed by T. Chevallard defines a new block $[\theta,\Theta]$, called block of knowledge or logos, made up of technology and theory. Type of tasks, techniques, technology and theory form a praxeology or mathematical organization $[T,\tau,\theta,\Theta]$.

Duval (1995, 2006) starts from signs used in the mathematical work grouped into registers of semiotic representations. The essential distinction made by Duval consists in the dichotomy between treatment and conversion. Treatment is a semiotic transformation which remains within the same register of representation. Conversion is a semiotic transformation whose result is expressed in another register. Duval stresses the essential cognitive difference between treatment and conversion. Conversion is much more complex and problematic than treatments, especially for a non-congruent conversion.

## Two numerical registers of representation for rational numbers: $R_f$ and $R_d$

When speaking about rational number, one often thinks about fractions. Fractions constitute the main way of constructing rational numbers, especially in secondary teaching because of proportionality. This is the first register of representation of rational numbers, denoted here by $R_f$.

Duval stressed that one needs two, or more, registers in order not to mix the object with its representation, that is here rational numbers with fractions. The interpretation of fractions *a/b* by *a* divided by *b* allows, by long division, to obtain a new register written $R_d$ related to base ten system. The result of a long division is a repeating decimal which is, traditionally, a non finite expansion for a non decimal number and a finite expansion for a decimal number [3]. As Duval pointed out, conversions between $R_f$ and $R_d$ are non congruent as one can see with 1/4=0,25 or $2/7=0,\overline{285714}$.

Non congruency appears also in treatments. Obviously, the comparison of the two rational numbers above is very different in $R_f$ and in $R_d$. But it is also the case of the four basic operations. These calculations are well known and taught at lower secondary school in France in $R_f$ and we do not describe them. A description of algorithms for the four basic operation in $R_d$ is one of the aim of our submitted work. We only expose in this paper the sum (see the section devoted to the sum below).

## A hidden technology

Comparing two repeating decimals, type of tasks denoted by $T_<$, is quite simple since one has just to compare cipher by cipher from left to right, a technique denoted by $\tau_<$. It is a natural generalization of the comparison of two decimal numbers (and natural numbers too). Students have no difficulty with this technique $\tau_<$ for the type of tasks

$T_<$ as we will see it. But are we comparing rational numbers? It is not so obvious because $\tau_<$ deals with semiotic representations that must be interpreted in order to have mathematical objects.

Indeed, comparing $0,\overline{9}$ and 1 leads to $0,\overline{9}<1$ by $\tau_<$ technique, when computing on repeating decimals, but must lead to $0,\overline{9}=1$ for rational numbers. We identify a new technique $\tau'_<$ close to $\tau_<$ except for decimal numbers. This shows that from the type of tasks $T_<$ arises two praxis related to two different mathematical objects whose writings are exactly the same: $[T_<,\tau_<]_{rd}$ for *pure* repeating decimals and $[T_<,\tau'_<]_{\mathbf{Q}}$ for rational numbers – we do not speak here of fraction representation.

This distinction could be confusing because we see the same writings, the same ciphers, the same signs. The difference stands in the interpretation of what is represented. Furthermore, the technology that justify technique $\tau_<$ and $\tau'_<$ are mainly identical: it is a technology that grounds the base ten system, denoted by $\theta_{bts}$. Obviously, there is a lack for $\tau'_<$ in order to justify the equality of the two representations of decimal numbers. We denoted this *hidden* technology by $\theta_=$. One has to notice that, until now, there is no indication of $\theta_=$, it cannot appears only with the comparison and there is no indication of its nature, of its origin.

### A technology of real numbers theory

Actually, $\theta_=$ is an important technology of the real numbers theory, denoted by $\Theta_{\mathbf{R}}$, that is related to the topology of the set **R** of real numbers [4]. Explications within the APOS theory is quite clear as Dubinsky et al. (2005, pages 261-262) wrote:

> An individual who is limited to a process conception of .999… may see correctly that 1 is not directly produced by the process, but without having encapsulated the process, a conception of the "value" of the infinite decimal is meaningless. However, if an individual can see the process as a totality, and then perform an action of evaluation on the sequence .9, .99, .999, …, then it is possible to grasp the fact that the encapsulation of the process is the transcendent object. It is equal to 1 because, once .999… is considered as an object, it is a matter of comparing two static objects, 1 and the object that comes from the encapsulation. It is then reasonable to think of the latter as a number so one can note that the two fixed numbers differ in absolute value by an amount less than any positive number, so this difference can only be zero.

At the end of this quotation, one can note a technology, that encompasses $\theta_=$, closely related to the topology of **R**. But it is not so obvious, and especially for a student. And one has to notice that this technology does not stand in non standard analysis, nor in the monoïd of repeating decimals we will define later.

Furthermore, in this quotation, one notices that it is question of the “value” of an infinite expansion and of “numbers”. Hence, before talking about topology of **R**, we think that it is reasonable to first justify that we are dealing with numbers. According to (Chevallard, 1989), numbers are objects that could be, almost, compared, added and subtracted with usual properties.

## DIDACTIC UNDERSTANDING OF THE PROBLEM BY TAD THEORY

In this section, we continue the description by TAD in considering the sum type of tasks, $T_+$. The aim of this section is to try to justify $\theta_=$ by the sum according to: (1) the comparison is not sufficient to make $\theta_=$ arise and (2) the topology technology is a to higher level of knowledge. Before investigating the sum type of tasks $T_+$, we discuss the classical way for proving that $0,\overline{9}=1$ using calculations and an equation.

### A classical way to produce and justify $\theta_=$

A classical way to produce and justify $\theta_=$ comes from calculations with repeating decimals, as if they were usual numbers. It is well known by all mathematics teachers that every repeating decimal could be converted into $R_f$ using an equation [5]. There are others calculations that lead to the target equality, see for example (Tall & Schwarzenberger, 1978). All these calculations are justified by $\theta_{bts}$ and the operations are supposed to be well defined – this assumption on operations, especially on subtraction, is not so obvious and is, in fact, directly linked to the problem.

The point of view is quite natural from a mathematical perspective: after generalizing objects one wants to preserve some properties. Here, one has some new *numbers* and the properties are those of the usual operations, even if we do not know if it is possible to define these operations. Obviously, this raises the problem of the consistency of the mathematics produced. It is the same perspective for the multiplication of integers within the set **Z**, and especially the sign of the product (Glaeser, 1981). But the consequence are not identical. Indeed, the result in **Z** are totally new and it is not problematic since there is no opposition with an ancient knowledge – think for example at $(-2)\times(-3)=(+6)$. But in our case it is not so simple. Of course calculations *say* that $0,\overline{9}=1$, but $\tau_<$ *say* that $0,\overline{9}<1$. Hence, there is an obvious contradiction and what a student may believe? It is natural to think that the ancient knowledge is stronger, even if a student answer the equality because of didactical contract. Hence, on one hand with these type of calculations it appears the need of $\theta_=$ but it brings a contradiction too and no explanation could emerge.

### Four processes for the sum in $R_d$

We here describe four processes to compute the sum of two repeating decimals. Each time, we discuss of the possibility to justify $\theta_=$.

The first process is guided by the coding: performing calculations by approximations and inferring the period of the sum. This process requires two technologies: *the sum of two repeating decimals is a repeating decimal* and *the sum is continuous* (according to the usual topology of **R**). For example, to compute $0,\overline{5}+0,\overline{7}$, one successively writes 0,5+0,7=1,2; 0,55+0,77=1,32; 0,555+0,777=1,332; and so on. It is not an algorithm because there is no criteria to stop the approximations (when do we get the period?). We do not retain it, first because it relies on the high-level technology we pointed out previously, second because this process causes some wrong writings to students (see the experimental section below).

The second process is an algorithm that requires a conversion: doing the sum after a conversion into $R_f$. For example $0,\overline{5}+0,\overline{7}$=5/9+7/9=12/9=1+3/9=$1,\overline{3}$. The point is that the conversion of repeating decimals such as $0,\overline{9}$ requires calculations involving repeating decimals, sum and subtraction. It is close to the *doubtful* equation process (see previous section). Moreover, it is quite difficult to understand how repeating decimals could become numbers with this process since calculations are made with fractions. Hence, we also reject this process.

The third process makes an explicit use of $\theta_=$, as in $0,\overline{5}+0,\overline{7}=0,\overline{9}+0,\overline{3}=1+0,\overline{3}=1,\overline{3}$. It is possible to build an algorithm but we do not retain it since it relies strongly on $\theta_=$ itself.

The fourth process is an algorithm we proposed in (Rittaud & Vivier, submitted work) is quite close to the algorithm of summation of two decimal numbers in base ten system: decimal points have to be in the same row, so do the periods. In the simplest case, with no carry overlapping the periodic and aperiodic parts, we get something like in the first example of figure 1. When a carry appears at the leftmost digit of the periodic part, we have to consider it twice: the first one, as usual, at the rightmost digit of the aperiodic part (this corresponds to the exceeding part), the second one, more unusual, at the rightmost digit of the periodic part (such a unusual carry is written between parenthesis). An example is given in figure 1.

$$\begin{array}{rrrrrr}
 & 1 & & & 1 & \\
 & & 4, & 2 & \overline{4} & \overline{2} \\
+ & 1 & 7, & 0 & \overline{4} & \overline{9} \\
\hline
 & 2 & 1, & 2 & \overline{9} & \overline{1}
\end{array}
\qquad\qquad
\begin{array}{rrrr}
 & 1 & & (1) \\
 & 0, & \overline{8} & \overline{2} \\
+ & 0, & \overline{4} & \overline{1} \\
\hline
 & 1, & \overline{2} & \overline{\cancel{3}4}
\end{array}$$

$4,2\overline{42}+17,0\overline{49}=21,2\overline{91}$ $\qquad$ $0,\overline{82}+0,\overline{41}=1,\overline{24}$

**Figure 1: two examples of the algorithm**

Of course, we also have to deal with sums in which the position of the decimal point or the length of the periods, are not the same. For example, to perform the sum $2,45\overline{38}+13,3\overline{192}$, we first rewrite it as $2,45\overline{383838}+13,31\overline{921921}$.

We argue that this last process can help for allowing $\theta_=$ to arise and, meanwhile, to make repeating decimals become numbers. We explain why at next section.

## Monoïd praxeology *versus* number praxeology

The equality $0,\overline{9}$=1 is not relevant when one deals only with the comparison praxis but it is needed for calculations. Hence, we focus here on praxeologies which arise with two types of tasks: comparison, $T_<$, and sum, $T_+$.

We saw several techniques to solve $T_+$ but, except the last one given in the previous section with our algorithm, all rely upon a technology of $\Theta_{\mathbf{R}}$ – $\Theta_{\mathbf{Q}}$ for the equation –

that has to be accepted. Hence, we focus on our algorithm interpreted as a technique denoted by $\tau_+$. First, $\tau_+$ comes uniquely from $\theta_{bts}$ which seems to be very interesting in order to introduce $\theta_=$ since students may justify by themselves the technique $\tau_+$.

We begin to build a praxeology $[T_<,T_+,\tau_<,\tau_+,\theta_{bts}]$ related to the $R_d$ register of the repeating decimals. It is important to notice here that one works with repeating decimals, and quite easily, whether they are interpreted as rational numbers or not.

We easily get that, for any periodic expansion $a$ with a non-zero period, we have $a+0,\overline{9} = a+1$. Hence, in the set of periodic expansions, we do not have the usual simplification property that allows to deduce from $a+c = b+c$ the equality $a = b$. To recover it, it is necessary to identify $0,\overline{9}$ and 1. Here, this is the time for the choice of $\theta_=$: do we accept it or not? This question is directly related to the choice between two mathematical organizations linked to the comparison, $T_<$, and sum, $T_+$, types of tasks:

- $MO_{\mathbf{Q}}=[T_<,T_+,\tau_<,\tau'_+,\theta_{bts},\theta_=,\Theta_{\mathbf{Q}}]$ that could be extend to others operations both in $R_d$ and $R_f$, that is the theory $\Theta_{\mathbf{Q}}$ of rational numbers.
- $MO_m=[T_<,T_+,\tau_<,\tau_+,\theta_{bts},\Theta_m]$ that could not be extend to fraction nor to others operations. Here, we only have a non regular monoïd (it is not a semi-group).

The aim of our investigations is to use this opportunity to show to the students where is the problem, in order to make them understand that only one of these two possible choices leads to the convenient notion of rational numbers.

## AT THE BEGINNING OF UNIVERSITY

A test was given to 29 students in mathematics at the first university year. It was presented in two steps: first an individual test (see annex 1), then, two days later, a team test (see annex 2) with three students by group. The first step was diagnostic: understanding of the coding (questions E1 and E2), comparison (questions C1 to C4), sum (questions S1 to S6) and difference (questions D1 to D3). The team test showed the algorithm, asked for an explanation and then proposed an activity with the intent of presenting the $\theta_=$ alternative.

Students knew the two representations of rational numbers since grade 10 because of the ancient mathematics secondary syllabus (before 2009), and heard of it again at the first semester of university. The case of $0,\overline{9}=1$ was taught as well.

This investigation follows a previous test at grade 10 and at the first university year (Vivier, 2011): at grade 10 a lot of the 113 students of this previous study were able to use the algorithm but they were not able to explain it probably because the base ten system ($\theta_{bts}$) is not enough understood. Hence, we think that *good level* for this kind of experiment in France is at the transition between secondary and university levels.

### Individual and diagnostic test

The coding (see convention in annex 1) caused some problems to 6 students (E1 and E2) and, obviously, they did not succeed the test (they were subsequently dispatched over 6 different teams).

Comparison of the three non problematic cases was successful for all students. They wrote the numbers in extension and used $\tau_<$ for comparing them. As we expected it, $\tau_<$ is not problematic at all since it is the same technique for $MO_{\mathbf{Q}}$ and $MO_m$.

But, for the case with $0{,}\overline{9}$ and 1, only 8 students stated the equality and 21 the inequality [6]. Among the first ones, two students wrote that this case was seen before, one of them qualifying this case of "strange". One other student stated both the equality and the inequality, arguing for the equality that it is "because it is not a real number", and for the inequality by comparing the unit cipher of $0{,}\overline{9}$ and 1 (she used $\tau_<$). One can see here the gap between $MO_{\mathbf{Q}}$ and $MO_m$.

Unsurprisingly, the sums and differences were computed by approximation by 25 students. Some students used sometimes a conversion into $R_f$ and also the equality $0{,}\overline{9}=1$ (see the third process above). As expected, 14 students gave some *infinitesimal answers* (Margolinas, 1988) such as $0{,}\overline{5}+0{,}\overline{7}=1{,}\overline{3}2$.

Finally, 7 students concluded that $2-1{,}\overline{9}=0$. All of them having previously declared that $0{,}\overline{9}=1$. 11 students gave $0{,}\overline{0}1$ as a result (infinitesimal answer again), 6 gave $0{,}\overline{1}$, 2 gave 0,0001 (with a finite number of 0) and one gave $0{,}\overline{10}$. More generally, no student who affirmed $0{,}\overline{9}=1$ wrote an infinitesimal answer.

Equality $0{,}\overline{9}=1$ seems to be an indicator of a more general knowledge on repeating decimals related to $MO_{\mathbf{Q}}$. Even if repeating decimals are written in extension for treatments, interpretation and control of results show a difference between the two mathematical organizations $MO_{\mathbf{Q}}$ and $MO_m$ beyond the $\theta_=$ understanding.

Despite the frequent appearance, in secondary teaching as well as in the university, of rational numbers written in base ten, most of the students do not agree with the equality $0{,}\overline{9}=1$ and about half of them write some infinitesimal answers for a sum. Hence, it seems fair to say that previous teaching gave to them neither sufficient control nor understanding on rational number in decimal writings, even if our experiment involved only mathematics students. $MO_{\mathbf{Q}}$ seems to provide more control and a better understanding for the sum than $MO_m$, even for tasks not involving $\theta_=$.

**The team test**

The justifications of the algorithm were quite good for 4 teams: anticipation of the carry coming from the right, existence of two carries ("local" and "inherited"), periodicity of the periods and some mentions of the stability of the periods lengths. Two other groups proposed some partial justifications, three did not write any justification and one team did not understand the algorithm.

Apart from team I in which the algorithm was not understood, teams gave the answer 1 in the third line of the table (see annex 2). Hence, the activity is adequate to set the problem that $x=0{,}\overline{9}+a$ even if $x-a=1$. Only the four groups in which there was at least one student who wrote $0{,}\overline{9}=1$ in individual test made the remark $0{,}\overline{9}=1$. The groups A, B and E expressed the apparent contradiction between an algebraic calculation (such as $0,\overline{9}+a-a$ is equal to 1) but concluded, as group J, that there are some approximations. Group F wrote: "if one uses the algorithm then $0,\overline{9}=1$" leading us thinking that the validity of the algorithm is quite suspicious for that group.

Hence, in spite of the opposition that emerged, our objective is not fully attained since $\theta_=$ not arose but only some contradictions related to $\theta_=$. A first explanation is that the short time at our disposal was probably not sufficient. Second, the fact that the test was given after recalling to students that $0,\overline{9}=1$ was obviously a bias that might have inhibited remarks and reflexions. It would therefore be interesting to make a comparison with secondary students or non mathematics students or at the very beginning of university.

## CONCLUSION

It appears clearly, and it is not very surprising, that both secondary and first university teaching are not suited to understand the problem of the double representations of decimal numbers in base ten system. Of course, some students say that $0,\overline{9}=1$ and some of them reproduce the equation process that had been taught. But do they really understand the reason? The answer is positive probably only for a few of them. In France, the new syllabus at grades 9 and 10 do not propose synthesis on numbers anymore, the situation will probably worsen in the coming years.

We pointed out the following alternative: do we take $\theta_=$ or not? We think that the use of comparison and sum could help in exhibiting and understanding this necessary choice. Our position is close to the investigation of Weller et al (2009): they shown that working on comparison, sum and difference of rational numbers written in decimal system – they used a software which makes calculations with fractions, hidden for the students – is important in order to understand the equality $0,\overline{9}=1$.

We intend to pursue our investigations for non mathematics students at the university (for instance students who want to become primary teachers) and for secondary students. For this last, grades 11 and 12, with a scientific option, are certainly better than grade 10 as we previously said in (Vivier, 2011). Another point have to be taken into consideration: conversion into a fraction by long division is interesting for validation and control of the sum algorithm.

## NOTES

1. In this paper we only consider numerical registers.

2. A simplified French version is available at <http://hal.archives-ouvertes.fr/hal-00593413>.

3. When dividing $a$ by $b$, one has just to replace the usual condition on remainders, $0\leq r<b$, by the new one $0<r\leq b$ in order to obtain the alternative representation of decimal numbers, with an infinite sequence of 9.

4. Without identifying the double representation of decimal numbers, the obtained set is not $\mathbf{R}$ but a Cantor set.

5. This kind of exercises appears in some French textbooks at grade 10. The meaning of the equation is quite unusual since the unknown is not the number but one of its representations: for $a=0,\overline{9}$, one has $10a–a=9$ and therefore $a=1$.

6. In (Vivier, 2011), 7 students over 14 answered the equality and, in (Tall, 1980), that was 14 students over 36.

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**ANNEX 1: THE INDIVIDUAL TEST**

CONVENTION : In the decimal writing of a rational number, we write the period by a bar above it. Hence, the number 0,127272727… with period 27 could be writen as $0,1\overline{27}$.

E1) Circle the number which is different from the other ones:

$5,0\overline{0100}$ $\quad$ $5,001\overline{0001}$ $\quad$ $5,0\overline{01000100}$ $\quad$ $5,0\overline{0101}$ $\quad$ $5,00\overline{1000}$

E2) Write in four different ways the number $14,\overline{121}$.

Circle the right answer and then justify your choice:

| | | | |
|---|---|---|---|
| C1) | $8,13 < 8,\overline{13}$ | $8,13 = 8,\overline{13}$ | $8,13 > 8,\overline{13}$ |
| C2) | $3,\overline{4} < 3,\overline{40}$ | $3,\overline{4} = 3,\overline{40}$ | $3,\overline{4} > 3,\overline{40}$ |
| C3) | $0,\overline{9} < 1$ | $0,\overline{9} = 1$ | $0,\overline{9} > 1$ |
| C4) | $45,\overline{101} < 45,1\overline{01}$ | $45,\overline{101} = 45,1\overline{01}$ | $45,\overline{101} > 45,1\overline{01}$ |

Compute the following sums:

| | | |
|---|---|---|
| S1) $0,\overline{24} + 0,\overline{57}$ | S2) $6,7\overline{1} + 1,8\overline{5}$ | S3) $0,\overline{5} + 0,\overline{7}$ |
| S4) $0,0\overline{8} + 0,\overline{2}$ | S5) $0,\overline{9} + 0,\overline{4}$ | S6) $0,\overline{5} + 0,\overline{72}$ |

Compute the following differences:

| | | |
|---|---|---|
| D1) $2,1\overline{7} - 0,\overline{7}$ | D2) $2 - 1,\overline{9}$ | D3) $1,\overline{28} - 0,\overline{72}$ |

(*Two different individual tests, of equal difficulty, was given to avoid cribbing.*)

**ANNEX 2: GROUP TEST**

Q1) We propose to discover a new algorithm to compute the sum of two rational numbers in decimal writing. In your opinion, is this algorithm give the good result? Justify your answer. (*The two examples of figure 1 was given.*)

Q2) In trying to find all the cases, give at least five other sums involving two rational numbers in decimal writing and compute these sums with the proposed algorithm. Note that, whether the case you consider, some adaptations of the algorithm are required.

Q3) Chose, each of you, a number $a$ with a period. One defines, for each number $a$ chosen, the number $x = 0,\overline{9} + a$. Fill the following table:

| | Student 1 | Student 2 | Student 3 |
|---|---|---|---|
| $a$ | | | |
| $x = 0,\overline{9} + a$ | | | |
| $x - a$ | | | |

Which remark(s) this table may suggest?