SPECIALIZED CONTENT KNOWLEDGE OF MATHEMATICS TEACHERS IN UAE CONTEXT

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This study aimed to characterize the gap between the mathematics teachers' current knowledge of mathematics and the ideal mathematical understanding for teaching in the UAE context. Specifically, it aims to investigate this gap by focusing on the specialized content knowledge of mathematics teachers in UAE. A fairly representative sample was chosen from UAE public schools and a questionnaire including open-ended and multiple-choice questions was applied to 142 mathematics teachers from grade 1-12. The results suggest that there is quite a gap between the current and ideal levels in terms of understanding core mathematical ideas, interpreting student work and knowledge, understanding purpose of assessment and conceptualizing the development of mathematics curriculum.

INTRODUCTION

The purpose of this research project[1] was to characterize the mathematics teachers' knowledge of mathematics in UAE public schools in order to portray the gap between what mathematics teachers should know and what they already know. For this purpose, I sampled a number of teachers from different public schools of United Arab Emirates (UAE) and applied a comprehensive research-based mathematics questionnaire assessing their knowledge of and about mathematics.

RELEVANT LITERATURE AND RESEARCH QUESTIONS

Even though there is not much solid research-based evidence, it is the perception shared in the relevant research literature that teacher knowledge has an impact on students' understanding, especially in the field of mathematics education (Ball & Forzani, 2009). Such shared belief in this area puts extra responsibilities on the shoulders of researchers' to investigate the relationship between teacher knowledge and student success. However, before doing so, researchers need to learn more about the nature of teacher knowledge so that they can then use it to pursue the aforesaid endeavor. Therefore, in this study, I aimed to investigate the following research questions in a geographic region whose voice has not been heard well enough in the relevant literature, specifically UAE. The research questions for this study were:

1. How do in-service mathematics teachers understand and think about fundamental mathematical ideas in major strands (geometry, algebra, measurement, probability and statistics, and numbers)"?
2. What is the nature of the gap between 'what teachers currently know' and 'what they should know' to teach mathematics effectively in UAE schools?

In light of these research questions the current study aimed to contribute to the field's understanding of mathematics teacher knowledge and its nature. To pursue these research questions I used the following framework and methodology. The data analysis is still in progress but some major results will be shared in this article.

THEORETICAL FRAMEWORK

Searching through the relevant research literature gave me the chance to identify the types of knowledge (in mathematics) and the ideal qualities of mathematics teachers with respect to these knowledge types. Following on Shulman's (1986) previously proposed knowledge types, Ball and her colleagues in a number of studies (e.g., Ball, Lubienski, & Mewborn, 2001; Hill, Schilling, & Ball, 2004; Hill, Ball, & Schilling, 2008; Ball, Thames, & Phelps, 2008) identified the types of knowledge teachers should have that is required in teaching mathematics, called mathematical knowledge for teaching (MKT). In this framework, MKT consists of two major components, namely subject matter knowledge (SMK) and pedagogical content knowledge (PCK). PCK consists of knowledge of content and students, knowledge of content and teaching and knowledge of curriculum. SMK consists of common content knowledge, mathematical knowledge at the horizon and specialized content knowledge (SCK). The subcomponent SCK is the focus of attention for this study since it is directly related to teachers' mathematics subject matter knowledge and it is the most essential one. SCK refers to the "mathematical knowledge that allows teachers to engage in particular *teaching* tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill et al., 2008, p.378). This account of SCK was initially described in Ball et al. (2005). In explaining SCK, Ball and her colleagues working on teacher knowledge suggested that teachers should have some SCK that is required to manage everyday tasks of teaching mathematics. Even though these tasks of teaching mathematics vary a lot and these scholars did not provide a complete list in a single publication, it was possible to compile a list of these everyday tasks of teaching from their writings that have previously appeared in different publications (e.g., Ball, Lubienski, & Mewborn, 2001; Hill, Schilling, & Ball, 2004; Ball, Hill, & Bass, 2005; Hill, Ball, & Schilling, 2008; Ball, Thames, & Phelps, 2008). Below is a list of tasks mathematics teachers need to deal with regularly, which require special kind of knowledge (especially SCK) on the part of the teachers. According to Ball and her colleagues mathematics teachers who have SCK have the knowledge of or ability to do the following:

SCK#1: Unpack" mathematical knowledge in order to provide meaning for learners; **SCK#2:** Have knowledge of interpretations and contexts; knowledge of common errors; diagnosing errors in student work; **SCK#3:** Design mathematically accurate explanations that are comprehensible and useful for students; **SCK#4:** Know mathematical explanations for common rules, procedures or algorithms; represent mathematical ideas (and operations) carefully; **SCK#5:** Provide explanations and justifications for mathematical ideas and procedures; evaluate mathematical explanations; present mathematical ideas; **SCK#6:**

Generate examples; find an example to make a specific mathematical point; **SCK#7:** Analyze mathematical treatments in textbooks; Appraise and adapt the mathematical content of textbooks; deploy mathematical definitions or proofs in accurate yet also grade-level-appropriate ways; **SCK#8:** Build connections among mathematical ideas; know underlying mathematical structures; **SCK#9:** Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems and insights (both predictable and unusual); respond to students' "why" questions; **SCK#10:** Assess aspects of understandings students show; listen to and interpret students' responses; analyze student work; pose questions; (+recognize what is involved in using a particular representation; select representations for particular purposes); **SCK#11:** Analyze a superficial understanding of an idea; be able to respond productively to students' mathematical questions and curiosities; evaluate the plausibility of students' claims (often quickly); attend to ambiguity of specific words; make mathematical practices explicit; **SCK#12:** Choose and develop useable definitions; use mathematically appropriate and comprehensible definitions; **SCK#13:** Connect a topic being taught to topics from prior or future years; sequence ideas; **SCK#14:** Think about multiple representations; map between a physical or graphical model, the symbolic notation and the operation or process, and make connections among the representations; link representations to underlying ideas and to other representations; construct and/or link non-symbolic representations of mathematical subject matter; use mathematical notation and language and critique its use; **SCK#15:** Inspect equivalencies; **SCK#16:** Know alternative solution methods, and claims; evaluate mathematical methods, claims and (alternative) solutions

In the current study, this list of competencies is considered to be "what mathematics teachers should know." To learn about teachers' current competencies in each of the above issues (what they currently know), I designed a questionnaire consisting of questions with several subquestions per competency, the details of which are given in the following section. Such design helped me to identify the gap in between what mathematics teachers already know and what they should know.

METHOD

A questionnaire testing the aforesaid competencies was generated based on the relevant research literature and applied to a sample of teachers. The sample was chosen from among all public schools throughout UAE. I used proportional stratified sampling to sample 100 schools out of 499 by considering cycles, gender, and region. Even though I sampled 100 schools out of all UAE public schools I could only work with volunteered teachers from 55 of those 100 schools because of bureaucratic limitations. The distribution of participant teachers is illustrated in Table 1.

City name / Cycle	Cycle 1 (Gr.1-5)	Cycle 2 (Gr.6-8)	Cycle 3 (Gr.9-12)	Common Cycle (Gr.1-12)	Total
Al Ain	12	25	11	17	65
Western Region	0	1	4	22	27
Abu Dhabi	12	23	15	0	50
Total	24	49	30	39	142

Table 1: Participant mathematics teacher distribution based on cycle and region

This sample is about the 10% of the whole school population and mathematics teacher population. The sample of teachers is fairly representative of the public school mathematics teacher population. The participants were 85 male (60%) and 57 (40%) female mathematics teachers.

SOME OF THE QUESTIONS ASKED IN THE QUESTIONNAIRE

Once those competencies (as laid out in the Theoretical Framework section) were identified, I developed a questionnaire targeting each competency (group). Then these questions were tested on mathematics teacher groups online and revised and finalized. Some of the questions from this questionnaire and their corresponding competencies are given within the results.

DATA ANALYSIS PROCEDURE

The gathered data included two main sections; one is about demographics and the other is about mathematical knowledge. Demographic information is analyzed to highlight the background characteristics of the participant teachers, which is partly shared in the Method section. The data about mathematical knowledge was mainly qualitative but to save time in overall data analysis process, I transformed this data into quantitative form and make the necessary analyses using SPSS. For example, in analyzing the participant answers about the division of fractions problems (targeting SCK#4, as explained in page 8 of this paper), the responses were coded as follows: "1 = "Completely wrong answer and/or rationale", 2 = "Says "it is right" but no rationale", 3 = "Turn it into invert-multiply and find solution", 4 = "Size or division matches rationale", 5 = "Size and division match rationale", 9 = "No answer". Once such coding is completed for this question, I then checked the frequencies of different responses and then move into the qualitative analysis of these responses to learn more about the nature of participants' SCK. A detailed qualitative and quantitative analyses are still ongoing. Because of space limitations, the nature of this data analysis is briefly included in the paper. Some of the results are highlighted and briefly discussed in the following section.

BRIEF DISCUSSION OF SOME OF THE MAJOR RESULTS

The current study revealed that in-service mathematics teachers in UAE have significant problems regarding mathematics content, analyzing student work, curricular issues and assessment. This also suggests that there is quite a gap between where they are and the ideal SCK that they should have. Therefore, this paper only highlights the weaknesses of the participant teachers to reveal the nature of the aforesaid gap. The results about these issues are briefly explained below.

ISSUES RELATED TO CONTENT

Participant in-service mathematics teachers can mostly carry out the basic mathematical procedures (e.g., finding solution of a basic division problem), which is a strength on their part, but most of them have serious problems with interpreting the conceptual meanings embedded in those procedures. For example, in analyzing whether a given 3-dimensional graphical representation for $g(x,y) = xy$ as in Figure 1 represents a function (targeting SCK#13), almost 90% of the teachers mentioned that a 3-dimensonal graph cannot represent a function, which is also one of the major misconceptions seen among students. In earlier grades whether a given graph represents function is tested through vertical line test (VLT). However, when the given representation is a three-dimensional (3D) graph, as given in higher grade mathematics classes, the participants had a hard time applying this VLT to the given graph. They even left VLT aside and think that a 3D graph cannot represent a function. This seems to be because it is a challenging task for them to connect a topic being taught in early grades to topics from future years (a requirement for SCK#13).

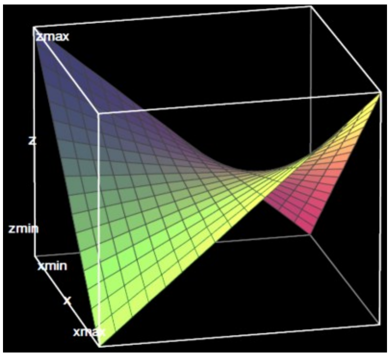

Figure 1: A given 3-dimensional graphical representation for $g(x,y) = xy$.

The participant teachers, for example, analyze equations through a single lens, mostly through algebra, as opposed to referring to several lenses including geometry. In analyzing a given first degree equation, $2x+4 = 3x+8$ (targeting SCK#15), about 95% of the teachers found its solution as $x=-4$, which is a strength on the teachers' part, but could not provide another way to analyze it (e.g., as a point of intersection of two lines). This suggests that their analysis of equivalencies is limited (opposite to the requirement of SC#15) and they did not seem to be able to interpret equivalencies, such as $2x+4 = 3x+8$, by referring to different lenses like analytic geometry.

One final example that illustrates a challenge for participants regarding conceptual meanings of mathematical ideas is the way they approach to the concept of parallelism. In one of the questions, the participants were given a drawing-a-parallelogram scenario in a dynamic geometry software environment (DGS) (with pictures only as illustrated in Figure 2). In this scenario the participants were told that a student, Ahmed, was going through the following steps. In a dynamic geometry environment, a line is drawn (passing through two arbitrary points, A and B), then a point (out of line AB) is put on screen (point C), and the program is asked to draw a parallel line (passing through that outside point) to the initially drawn line. Then the DGS draws a parallel line passing through the initial point (point C) but it automatically puts another point on this newly drawn line (point D).

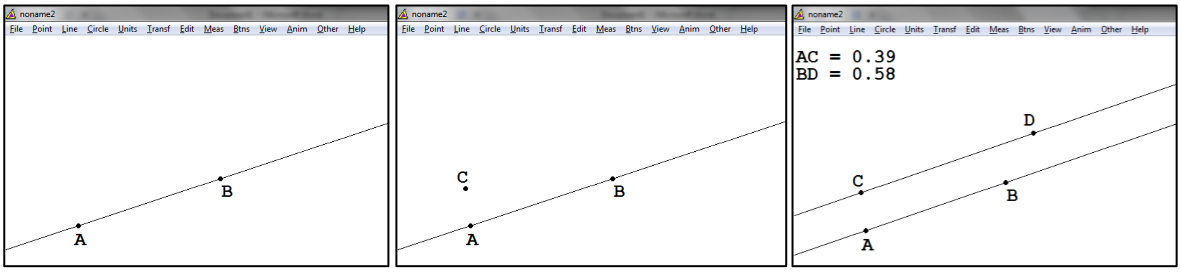

Figure 2: Pictures given in question #9 targeting SCK#9.

They are then asked: "*After going through Step#3 and finding the measures for AC and BD, he got puzzled. He thought to himself, "If Wingeom did not draw the parallel line based on the distances between A and C, and B and D, how did it decide on how to draw the parallel line CD? On what basis did the program created this extra point (point D) automatically?" If Ahmed was in your class, how would you explain the answers to these two questions mathematically to Ahmed?*"

It is obvious from the participants' responses that they could only think about parallelism by focusing on the distance between the two lines or the ratios of distances AB, AC, CD and DB. A sample response from one of the participants is as follows: "I confess utter confusion as to the reason Wingeom would create such a pt. D at exactly the place it did. However, it could have picked a random point on $\overleftrightarrow{AB}$ and gone 0.3 inches away to put a point D and then draw the parallel line. It may have created point D so that the ratio of $\overline{AB}$ to $\overline{CD}$ was equal to AC to DB but without knowing the values of $\overline{AB}$ and $\overline{CD}$. I cannot verify such a claim."

The above response suggests that this teacher focused his attention to the distance between the two lines as well as the ratios of certain distances in the given scenario, whereas he ignored the fact that parallelism also requires same sloped lines. The participant teachers had difficulty in interpreting and making mathematical and pedagogical judgments about hypothetical unusual student questions and in responding to students' "why" questions, a requirement for SCK#9.

As seen in these given examples, interpreting the mathematical meanings of core mathematical ideas is quite a challenge for the participant mathematics teachers.

ISSUES RELATED TO ANALYSIS OF STUDENT WORK

The participants' analysis of student work is not at a desired level when it comes to an alternative solution method. For example, when given a scenario about a fraction division solution, like $\frac{3}{8} \div \frac{1}{3} = \frac{9}{24} \div \frac{8}{24} = \frac{9}{8}$ (called common denominator algorithm), they only turned it into a well-known invert-and-multiply algorithm to check whether it is accurate or not instead of analyzing the accuracy of the following method: This suggests that their ability to provide mathematical explanations for common algorithms, which is a requirement for SCK#4, falls short in reaching the ideal level.

The study also revealed that the participant mathematics teachers are also quite weak in identifying possible student errors for a given problem situation. They were asked to talk about possible student errors for the following problem targeting SCK#2:

Question #2: *A computer game store is having a sale. They have advertised 10% off everything in the store. They also have just purchased a new shipment of computer games. These games cost the store 32.11AED each. They want to price the game so that they will make at least a 40% profit, even at the sale price. What is the lowest regular selling price for the game that will allow this profit? (Bair & Rich, 2011)*

a) *If a student brings this question to your mathematics class, to what extend would you feel confident (or comfortable) in analyzing this problem situation before you actually solve it?*
   ☐*5-highly confident* ☐ *4-somewhat confident* ☐ *3-confident*
   ☐*2-little confident* ☐ *1-not confident*
b) *What mathematical knowledge or understandings are required to solve this problem? [Please be specific in your answer. For example, saying that it requires algebra or geometry is not informative! If you need, you can solve the problem here and then talk about the mathematical components of it!]*
c) *What common mathematical errors (or mistakes) would you expect from students in solving such a problem?*

In approaching such a problem, instead of focusing on the student errors, participants mostly solved the problem first, and then talk very superficially about possible student errors such as “students will most like make calculation errors.” This suggests that the participant mathematics teachers need significant support in how to analyze student work in mathematics classes. Such support can be in a form where they are given opportunities to “listen” to the student ideas, interpret those ideas, and think about what it means to think like a student in certain problem situations.

ISSUES RELATED TO ASSESSMENT

The study revealed a very interesting result about mathematics teachers’ understanding of assessment. When participants were asked to choose from among three assessment items so as to test student understanding of infinity (given below), about 95% of the participant teachers chose the easiest to handle representation to ask, Example 3. A typical response given by the participants who chose Example 2 or 3 as the best one to ask students are: “Since sets are given side by side [in Example 1], students can’t really analyze them” and “Because it is well cleared [in Example 3] to the student that both groups are infinite.”

Question #10 (targeting SCK#10): *Which of the following sample questions would be nice to assess student understanding of infinity? In answering the question please also consider the way the questions are represented. (Tsamir & Dreyfus, 2002)*

☐*1-Example#1* ☐ *2-Example#2* ☐ *3-Example#3*

Example #1: Consider the following sets. Which one do you think has more elements than the other?

$A = \{1, 2, 3, 4, 5, 6, ...\}$; $B = \{1, 4, 9, 16, 25, 36, ...\}$

Example #2: Consider the following sets. Which one do you think has more elements than the other?

$$A = \{1, 2, 3, \ 4, \ 5, \ \ 6, \ ...\}$$
$$B = \{1, 4, 9, 16, 25, 36, ...\}$$

Example #3: Consider the following sets. Which one do you think has more elements than the other?

$$A = \{1, \ 2, \ 3, \ 4, \ \ 5, \ 6, \ ...\}$$
$$B = \{1^2, 2^2, 3^2, 4^2, 5^2, 6^2, ...\}$$

Such focus on the teachers' part suggests that the participants think about assessment in mathematics as a way to help students as opposed to as a way to test knowledge of students. One of the purposes of assessment is to figure out how and to what extent students understand a targeted concept (Airasian & Russell, 2008) and a teacher can assess students' knowledge through probing questions that do not include any hints about the solution of the problem. The fact that almost every participant choose the easiest question to ask students to figure out how students think about infinity suggests that they seem to misinterpret the purpose of assessment in mathematics classes.

ISSUES RELATED TO CURRICULUM

From curricular standpoint, almost all of the participant teachers follow the traditional ways to explain curricular decisions made in the books. For example, they think about trapezoid by referring to exclusive definitions (definition #1 as given below) rather than inclusive definitions (definition #2 as given below).

Question #7 (targeting SCK#7): *A well-known mathematics educator in USA, Zal Usiskin, checked through the geometry textbooks used in USA since 1800s and realized that there are mainly two definitions given for "trapezoid" as shown below.*

Definition #1: *Trapezoid is a quadrilateral with exactly one pair of parallel sides.*

Definition #2: *Trapezoid is a quadrilateral with at least one pair of parallel sides.*

A) *If we accept Definition #1 which one of the following figure(s) would be considered as trapezoids? Mark all that apply.*

☐ *1-Parallelogram* ☐ *2-Rectangle* ☐ *3-Rhombus* ☐ *4-Square* ☐ *5-Isosceles Trapezoid*

Why? .....

B) *If we accept Definition #2 which one of the following figures would be considered as trapezoids? Mark all that apply.*

☐ *1-Parallelogram* ☐ *2-Rectangle* ☐ *3-Rhombus* ☐ *4-Square* ☐ *5-Isosceles Trapezoid*

Why? .....

C) *If you were to use one of these definitions to teach students in your math classes, which definition would you use?*
☐ *1) Definition#1*　　☐ *2) Definition#2*
Why? .....

In addition to this, when they need to make curricular decisions about the sequence of mathematical topics (e.g., which topic should come first, triangles or circles?), their decisions do not reflect a solid understanding of those mathematical concepts. For example, most of them do not know that having the knowledge of circle is necessary to make sense of triangles, and therefore, it may be appropriate to teach circles first and then triangles. Therefore, their responses suggest that their analysis of the mathematical treatments in textbooks and deploying mathematical definitions in accurate and grade-level-appropriate ways, as suggested by SCK#7, need to be improved in ways that allow them to critique mathematical definitions and their treatments in textbooks.

IMPLICATIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

Considering the above general results about mathematics teacher knowledge, there seems to be a considerably negative large gap between where teachers are and where they should be – teachers currently teaching mathematics in UAE are quite weak in understanding the core mathematical ideas they teach, in interpreting and analyzing student work, in the assessment of understanding mathematical ideas, and finally in making and criticizing curricular decisions. Their SCK needs to be improved in order to close this gap. Therefore, considering their background information it seems quite reasonable to think that teachers who currently teach mathematics in UAE public schools need considerable professional development support in the aforesaid areas. The nature of this support will depend on identifying the weaknesses of the teachers, which was done through the current research study, and then preparing professional development programs that specifically address those needs and weaknesses throughout long term programs.

These results also suggest that teachers are not prepared well throughout the Education and Science faculties of the universities in the Gulf region. This is not to say that they do not learn anything from those programs. It is rather saying that the experiences teachers gain from their undergraduate education does not seem to support them well for the aforesaid areas. Obviously teaching the subject for many years (some teachers have the experience of 20 years of teaching) or having credentials (e.g., master, PhD) did not help them improve their understanding of the core mathematical ideas either. Neither the academic background nor the teaching experience that they had helped them in answering even very simple questions about core mathematical ideas such as drawing a triangle (e.g., one question was about given three side measures and investigation of whether those three side measures give a triangle) or interpreting a first degree equation.

Finally, the mathematics teacher competencies should be carefully reconsidered and revised, and the teachers who will teach this subject needs to be eliminated through certain examination process. The SCK construct seems to be useful in targeting certain competencies for mathematics teachers regarding their content knowledge and in generating questions to test those. However, to what extend these competencies and testing of them would tell us about student success is not clear. The connection between student success and these competencies is beyond the scope of this paper.

NOTES

1. The current paper describes part of a major research study funded by United Arab Emirates University with the fund number 31D000 for the years 2010-2012 and includes some preliminary results about mathematics teacher knowledge.

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