EXAMINATION OF PRE-SERVICE MATHEMATICS TEACHERS' INSTRUCTIONS IN TERMS OF DEVELOPING STUDENTS' MATHEMATICAL THINKING

Berna TATAROĞLU TAŞDAN **Ayten ERDURAN** **Adem ÇELİK**

Dokuz Eylül University

The aim of this study was to examine the instructions of pre-service teachers in terms of developing students' mathematical thinking. In the study, Advancing Children's Thinking framework developed by Fraivillig, Murphy and Fuson (1999) was adopted as theoretical framework. Case study was used and participants were determined as two pre-service mathematics teachers. The focus of the study was the instructions of pre-service teachers. Four lessons for each participant were observed via video camera and the data were analyzed by using descriptive analysis technique within framework components. It was found that pre-service teachers performed the component of eliciting in the classroom more than other components.

Key words: mathematical thinking, pre-service mathematics teachers, instruction

INTRODUCTION

Mathematical Thinking (MT) is one of the basic skills emphasized by standards and programs developed for mathematics learning and teaching. National Council of Teachers of Mathematics (NCTM) (2000) indicates the increase in MT and problem solving level beside an increase in the mathematical level required for individuals in business life (from health to graphical design). This change has also taken place in objectives of mathematics education in mathematics curriculum renewed in our country, and MT has been incorporated into the skills targeted to be developed by the curriculum (Ministry of National Education [MNE], 2011).

According to NCTM (2000), effective teaching includes monitoring the students, carefully listening to their thoughts and explanations, having mathematical objectives, and using the knowledge when taking instructional decisions. Teachers using these applications motivate the students to engage them in MT and reasoning and provide compelling learning opportunities for the students at every understanding level. In this context, the teacher must know the mathematical thoughts of the students and develop her/his instruction within the frame of these thoughts (Olkun and Toluk, 2004). Cooper (2009) indicates that the teacher can arrange a more individualized education and thus increase the learning of the students by focusing on their MT. Crespo (2000) also suggests that analyzing the students' MT will help the teachers in taking more appropriate decisions and developing their practice in their classrooms. Researchers put forth that the interest of pre-service teachers in the students' MT contributes them in terms of the development of their teaching. For example, the interest in the students' MT allows the pre-service teachers to question their mathematical knowledge and learning (McLeman & Cavell 2009; Philipp, 2008).

Theoretical Framework

The aim of this study was to examine the instructions of pre-service teachers in terms of developing students' MT. In the study, Advancing Children's Thinking (ACT) framework developed by Fraivillig, Murphy, and Fuson (1999) was adopted as theoretical framework. Fraivillig, Murphy & Fuson (1999) presented a pedagogical model supporting the development of conceptual mathematical understanding of the students in their study. The model consists of three components: eliciting the solutions of the students, supporting their conceptual understanding, and extending their mathematical thinking. The instructional strategies which must be used by the teacher for developing students' MT in a questioning classroom environment revealing the thoughts and solutions of the students are presented in Table 1.

Instructional Components of ACT Framework		
Eliciting	**Supporting**	**Extending**
Facilitates students' responding Elicits many solution methods for one problem from the entire class Wait for and listen to students' descriptions of solution methods Encourages elaboration of students' responses Conveys accepting attitude toward students' errors and problem solving efforts Promotes collaborative problem solving Orchestrates classroom discussions Uses students' explanation for lesson's content Monitors students' levels of engagement Decides which students need opportunities to speak publicly or which methods should be discussed	Supports describers' thinking Reminds students of conceptually similar problem situations Provides background knowledge Directs group help for an individual student Assists individual students in clarifying their own solution methods. Supports listeners' thinking Provides teacher-led instant replays. Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method Supports describers' and listeners' thinking Records symbolic representation of each solution method on the chalkboard Asks a different student to explain a peer's method Supports individuals in private help sessions Encourages the students to request assistance (Only when needed)	Maintains high standards and expectations for all students Asks all students to attempt to solve difficult problems and to try various solution methods Encourages mathematical reflection Encourages students to analyze, compare, and generalize mathematical concepts Encourages students to consider and discuss interrelationships among concepts Lists all solution methods on the chalkboard to promote reflection Goes beyond initial solution methods Pushes individual students to try alternative solution methods for one problem situation Promotes use of more efficient solution methods for all students Uses students' responses, questions, and problems as core lesson Cultivates love of challenge

Table 1: Instructional Components of ACT Framework (Fravillig, Murphy & Fuson; 1999)

METHOD

Among the qualitative research methods, case study was used in the study. Participants of the study were determined as two pre-service mathematics teachers consisting of one female and one male receiving education at the senior class of a faculty of education. Purposeful sampling was used in the determination of the participants and attention was paid for having a heterogeneous distribution in terms of academic grade point average (Aslı: 3,41 and Ege: 2,97 out of 4) and the gender. When we looked the grade point average of the participants in lessons regarding content knowledge it was seen that Ege had 2,48 and Aslı had 3,23; in lessons regarding pedagogical content knowledge Ege had 3,38 and Aslı had 3,71. Participants were told that their real names would be undisclosed and nicknames (Aslı and Ege) were used.

The study was carried out within the frame of instructional practice in the scope of Teaching Practice of pre-service teachers. At the beginning of the study, individually semi-structured interviews were performed with pre-service teachers about MT and mathematics teaching. Subsequently, two participants and two researchers came together and discussed the answers given by pre-service teachers to the interview questions. Then two videos of math classes were watched and participants were asked to evaluate these lessons in the context of MT. The purpose of this group meeting was to provide participants to have a consensus about MT and its importance for math education. Later on, a section of the studies about MT was presented to the pre-service teachers. In this scope, ACT Framework was also presented to them. Following that, pre-service teachers were asked to prepare a lesson (4 hourly lessons) plan that develops students' MT. Each pre-service teacher examined his/her plan by coming together with a researcher when he/she prepared the lesson plan, and made some changes in line with the feedbacks. Then, pre-service teachers performed the lessons they prepared. These four lessons were observed by using a video camera. After they had completed their teaching, a researcher watched the videos and re-interviewed the pre-service teachers and asked them to evaluate their performance. The focus of this study was the instructions of pre-service teachers. Data were analyzed by using descriptive analysis technique in the frame of ACT Framework developed by Fraivillig, Murphy and Fuson (1999). Two of the researchers watched the lessons individually, took notes in accordance with the framework, then came together to discuss about coding and reached a consensus.

FINDINGS

In this section, summaries about instructions of participant pre-service teachers were included at first, and then findings were presented in the scope of framework components. First case was Ege. Ege carried out the instruction of Conics in 11th class at an Anatolian High School. First he looked at "What is a conic?", and then "What are the different types of conic?" He wrote and explained basic elements of ellipse, hyperbola, and parabola among types of cones and allowed the students time to write things down. Then, he included the examples related to the subject. During his instruction, he emphasized the determination of the type of cone according to the eccentricity (e). At the post-interviews he expressed that at the beginning of the lesson he could not let students watch a video related to the cones due to a technological failure. Generally, Ege played a central part of the lesson although he did try to ensure active participation of the students. Second case was Aslı. Aslı performed the topic of Inverse Trigonometric Functions in 10th class at a vocational high school. She benefited from technology during her four hour instruction. She carried out four activities with the students and also applied one worksheet. Aslı was attentive in examining the preliminary knowledge of the students and allowing the students to reach the knowledge themselves. Aslı used the question-answer technique and frequently gave positive reinforces to students who gave the correct answer. She also provided a very comfortable classroom environment for the students in relation to tackling points they did not understand.

Eliciting

First component of ACT Framework was eliciting. When the instructions of Ege and Aslı were analyzed the following findings were obtained.

Generally, reaching only one solution of one question was sufficient for Ege. So he did not reveal different solutions during his instruction. Namely, he didn't elicit many solution methods for one problem from the entire class. When Ege asked the students questions or took a student to the blackboard, he gave the students sufficient amount of time for explaining his/her thought or solution and listened to them. He waited for and listened to students descriptions. However, he directly explained what to do when the student could not give any explanation on a particular point. Furthermore, he sometimes helped students who had difficulty in determining the type of the cone by indicating the graphic of the cone with his hand. Also, he did not support the students in explaining their thoughts in detail and did not attempt to elicit further explanations from them. He did not question the answers by asking why or how, and focused only on the correct answer. A part of the instruction of Ege for this situation is as follows:

> Ege: I drew two cones; they are symmetrical according to this point. Right cone, its base is circle. If we intersect it with a plane like that (by showing with hand), what can we obtain? What kind of shapes?
> Student 1: Triangle
> Ege: Triangle?
> Student 1: Can't we obtain a triangle?
> Student 2: Ellipse is obtained.
> Student 1: Ellipse, sorry, ellipse is obtained.
> Student 3: Circle
> Ege: Circle, we obtain a circle of the simplest form. Now I'll intersect this with a plane, if I intersect it with a plane parallel with the circle on that base (drawing), of course our drawings did not appear good but we were relying on the projection, we'll handle it like that anyway, I'll obtain a circle, if I intersect it in a manner parallel to the base, again, likewise, I'll obtain a circle since I have a circle at the base. Only its radius will be smaller. What else? This is a parallel intersection. What if I intersect it with a little slope?
> Student 2: Semi thing... ellipse
> Student 4: Trapezoid.
> Ege: Let me take it like that guys (draws an inclined planed intersecting the conic). This time, it becomes an ellipse guys, even if we do not see it visually. Something like that will occur (drawing an ellipse). The first one, the previous one was like that (drawing a circle) this also seemed like an ellipse anyway but I tried to draw a circle below. The first

one is a circle and the second one is an ellipse. It is somewhat elliptical, only some more oblate than the circle.

In this dialogue, it is seen that Ege did not question student answers that were wrong such as triangle and trapezoid and continued his lesson by considering correct answers like circle and ellipse. It can be said Ege didn't encourage elaboration of students' responses. Ege gave responses to student questions with alternative explanations during his lessons. He continued his explanations until clearing the confusion in the minds of the students. However, he did not support the students in reaching the correct answer on their own. This situation can be considered as an indicator of accepting attitude toward students' errors and problem solving efforts in eliciting component of the ACT. Furthermore, Ege exhibited an approach supporting the collaborative problem solving in his teaching. However, he conducted only one group work session during his four lessons. He allowed the students to work in groups consisting of three and four persons by distributing work sheets containing the questions and some graphics providing convenience for the solution. He started exercising this group work to allow the class to question whether the cones have common characteristics. However, any relation could not be established with this purpose in the examination phase of the questions. Ege used student explanations for lesson's content and continued the lessons by focusing on the comments of those who gave a correct answer. Ege did not determine the participation levels of the students. When a question was asked or a wrong answer was given to the question, he did not orientate the students towards thinking about the question or the thought. He preferred giving the correct answer himself. This also prevented the entire classroom from engaging in the lesson. Therefore he didn't monitor students' levels of engagement. He also tried to bring different students to the blackboard; however, since the students did not volunteer, he conducted his lessons with actively and voluntarily participating students. Ege was not successful in deciding which students need opportunities to speak publicly or which methods should be discussed.

In her lessons Aslı did not give the solution herself when studying on a question or a problem and wanted students to share their solutions. She tried to elicit different solutions for one problem from the entire class by means of questions such as "Who solved it in a different way?", "Did anyone do it differently?" She asked if there were different solutions to the solution of the student she brought to the blackboard, and if any, she wanted the students to share them. For example, after having examined if the function f: $\mathcal{R} \rightarrow [-1, 1]$, f(x) = sinx whose graphic is given in her first activity is a bijection, she passed to the question "*Is there any interval where this function is bijective? If any, please show it*". The student she brought to the blackboard wrote: $[0, \pi] \rightarrow [-1,1]$.

Student 1: Is it right, teacher?

Aslı: If you thought something different, come and write that, too.

Student 2: Teacher, my friend has also done it as $[\frac{\pi}{2}, \pi]$.

Student 1: $\frac{\pi}{2}$ is also there, teacher.

Aslı: Okay. You come and write it too, let's have a look and see if it’s correct.
Student 1: No teacher, no need if it's correct.
Student 2: Please tell me teacher, is it correct?
Aslı: Guys, if you're making another interval, let's talk about that, too. For example, did you say $[\frac{\pi}{2}, \pi]$?

Aslı wanted the students to explain the solution, waited for and listened to them. She always questioned the answers given by the students and expected from them detailed explanations. So she could encourage the students to elaborate their responses. She asked questions such as "Why yes?" or "Why no?" to the students giving yes/no answers. She did not directly say correct or wrong in response to students’ answers and appreciated all of the opinions. Thus she was able to determine what the students thought and to take measures against possible mistakes. She listened to the explanations of the students giving wrong or irrelevant answers and made summarizing or reminding explanations to eliminate the existing difficulty. She provided a comfortable classroom environment for the students so that students asked about points they did not understand without hesitation with questions such as "Has anyone had any difficulties so far?", "Is there a point you haven’t understood?" Aslı’s approach also showed that she has an understanding attitude towards student mistakes. She conveyed accepting attitude toward students’ errors and problem solving efforts. During her lessons, Aslı motivated the students in a collaborative working environment with four activities and one worksheet. During this process, she continuously walked between the desks. She took care of almost all of the groups, answered the questions, and guided the groups for solutions. Questions included in the activities focused not only on the operational skills of the students but also their conceptual knowledge. She shaped her lessons according to the approaches of the students and used students’ explanations for lesson’s content. Aslı tried to engage the students in the lesson by using expressions such as "look at the blackboard, did you do it like that?", "are you thinking as your friend thinks?" So she could monitor students’ level of engagement. She was careful to bring different students to the blackboard to show the solutions or explain her/his opinions when she decided which students need opportunities to speak.

Supporting

The second component of the framework was supporting.

Ege was contented with showing only one solution to the questions he solved or he wanted students to solve during his instruction. He did not make any comments about whether different solutions exist. That is to say, he did not cause students to perceive that there may be different solutions. He did not ask whether anyone had a different solution either among the students. Ege made instant replays at points needed by students during his lessons. The information he highlighted most frequently was the determination of the type of the cone according to the value of the eccentricity. An exemplar video part for this situation is below:

Ege dictates a question to the students. The student cleaning the blackboard notes the data given in the question on the blackboard: *Please determine the type of the cone with focus F (-3, 2), directrix 3x-2y-6=0, and passing through point P (0,6).* The student draws a coordinate axis on the blackboard. Ege again summarizes the data given in the question and directs the question to the classroom.

Ege: Just remember, how do we determine the type of conicity?

Student: Now it has a focus, it has a directrix, so this is an ellipse.

Ege: You can't know. It may be hyperbola, parabola. As you see, during the previous lesson, it is the most important one of the section we've seen until now.

Another student: You see, we were telling it by looking at that "e".

Ege: We were looking at the eccentricity. What was the eccentricity? It was the proportion of the distances from the focus and from the line of the point.

This part is an example of the evidence of the fact that Ege highlights previous knowledge with instant replays as well as the approach of non-consideration of student's wrong answer and not helping the student in explaining his/her individual thought indicated by a eliciting component. It cannot be said that he encouraged the students much to ask for help when they needed it. He inspected the classroom only with questions such as "Do you understand?" and also gave answers to individually asked questions. A comfortable environment could not be created for the students in terms of asking whether they understood it or not.

In her lessons, Aslı lead students to establish interrelations in the definition of inverse functions of sine, cosine, tangent, and cotangent functions and reminded them of conceptually similar aspects. For example, she expected from the students that learned arcsin function to write x = arc cos y if y = cos x for arccos function. She called a student to the blackboard. The student wrote y^{-1} under y = cos x expression, and then arccosine and then arccos y after a warning from Aslı. When the student got stuck on this section, the class shouted out $f^{-1}(x)$ to help them. Then the student wrote arccos y = -x. Meanwhile Aslı made the following explanation by noticing that the student experiences difficulty: "*What's going on guys? She/he changed its place. What were we doing while writing the definition and the range sets? What did we do while writing its inverse? We've changed its place. We're also changing the place while writing these.”* Aslı reminded the students of previous (background) knowledge when necessary. For example, at the beginning of the lesson she started a classroom discussion about what conditions must be satisfied so that inverse of function can exist. And then, she found it necessary to remind them what the function was. However, in the 4th lesson, she created a discussion environment about how the factorization while transitioning to sum and difference formulas can be used in trigonometry. Also, since the students could not continue to the study because they could not remember the Sine Theorem in the 4th activity, she reminded them of the Sine Theorem by calling a student to the blackboard and guiding the student. She helped each student in the explanation of individual solutions in the discussion of the activities by the classroom. She also made instant replays in line with the explanations or questions of the students. Aslı did not adhere to only one solution and expressed that she's open to different solutions during her instruction. She showed her own solution where students experienced difficulty. She ensured that all of the students see the different answers she got verbally from the students by noting them on the blackboard. She frequently asked whether there is any point that is not understood and encouraged the students to ask for help when they needed.

Extending

Extending was the third component of ACT Framework.

When Ege's instruction was analyzed within the frame of ACT, positive findings could not be obtained for instructional components at extending level; because Ege did not ask students different and challenging problems and did not encourage them to think from different aspects during his lessons. He did not give students the opportunity to analyze, compare or generalize mathematical concepts. He played an active role in reaching general equations of cones, making comparisons between conic types, but did not ensure the participation of the students. However, he asked questions such as "What is a circle?", "What is a geometric locus?" in the first lesson to establish relations between the concepts, but when he could not get any answer, he made the definition of geometric locus, circle, and line without changing tack. Subsequently, he went on to talk about cones and explained that the circle and the line are also a cone. He tried to correlate the concepts of ellipse and circle. However, here again he explained the relation without compelling students to think.

It was observed that Aslı confronted her students with questions that might be different for them and of a type they are not accustomed to in the activities. In this sense, the questions were challenging for her students. Aslı asked each student to solve these questions. She supported the students in trying ways that might compel them individually. She took the answers and the solutions of the students in the center of the lessons and guided her lessons in this direction. She encouraged the students to analyze the concepts, to make comparisons, and to generalize during her instruction. At the same time, she supported the students in establishing relations between the concepts. For example, she tried to enable the students to reach the sum formula for the sine function in her 4th activity.

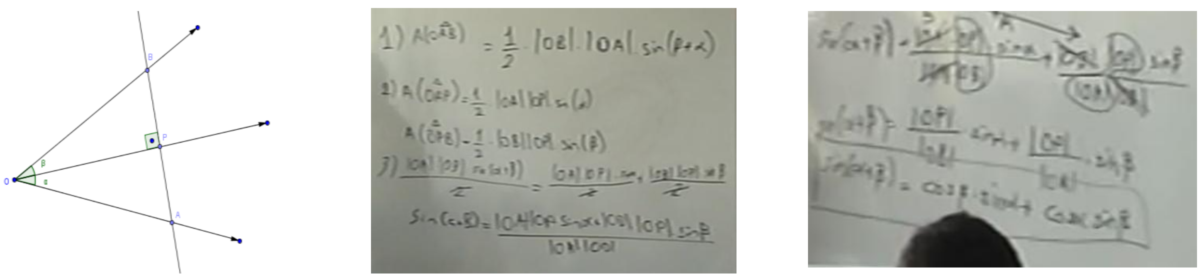

Figure 1: A figure and screen extractions from the activity performed by Aslı

Here she asked the students to find the area of OAB, OAP, and OBP triangles with the help of the Sine Theorem and to show the relation between these areas. Thus, a formula for sin(α+β) was obtained together with the students.

CONCLUSIONS

In this study, the instructions of two pre-service teachers were analyzed by considering the instructional components of ACT Framework developed by Fraivillig, Murphy and Fuson (1999). In facilitating students' responding of eliciting component, Ege only conveyed accepting attitude toward students' errors and problem solving. This may stem from being a pre-service teacher. But unfortunately it can be said that he was not very successful in doing other instructional components of this sub-component. In orchestrating classroom discussions, Ege focused the students who said the correct answers and used these students' explanations for lesson's content. But he couldn't engage whole class. On the other hand Aslı tried to perform all sub-components of eliciting during her instruction in the expected way. However, in the supporting component of the ACT, while Ege only made instant replays, Aslı performed almost all of the sub-components. However she didn't direct the group to help for an individual student and not ask a different student to explain a peer's method. The grade point average of the students was not very good and the students were not accustomed to do these activities in a lesson so these may have been the reason for her difficulties. She couldn't assist individuals in private help sessions. The reason of this may be being a pre-service teacher and not their regular teacher. While Ege did not perform any activity towards the extending component of the ACT, it is determined that Aslı encouraged the students to analyze, compare, and generalize the mathematical concepts and to think about the relations between the concepts. Also Aslı asked all students to think and solve the problems that were difficult for them. As a result it can be said that performance of Aslı was more successful for developing students' MT in the frame of ACT. Ege's non-performing in most of the instructional components may stem from his lack of pedagogical content knowledge. Also in post interviews it was determined that Ege was inclined to do his instruction in a traditional way. This view of Ege is supporting the result we reached. On the other hand in post interviews, Aslı stated that she was decisive in doing student centered instruction. Determination of Aslı might be seen the rationale of her success in performing ACT components. Another result of this research was that participant pre-service teachers of the research often elicited students' mathematical thinking but less often supported and extended. Differently Fraivillig, Murphy and Fuson (1999) found that their participants often had supported students' mathematical thinking. Source of this difference may be researched in further studies.

Pre-service teachers expressed in the pre-interviews that although they have theoretical knowledge about the MT, they would reflect this knowledge in the practice for the first time. In this sense, it can be said that the pre-service teachers made efforts in the development of the MT and tried to realize an instruction suitable for ACT framework in general. This situation was also seen in their lesson plans. Similarly to the results of this study, Hughes (2006) also determined that 10 mathematics pre-service teachers learned to deal with MT of the students in lesson planning before and after a lesson they take. It is reported that they showed a meaningful development in terms of their skills for dealing with the MT of the students from the beginning until the end of a lesson they take at the university. Similarly to the study of Hughes (2006), this study can also be performed by monitoring the instructions of pre-service teachers before and after the study process and comparing the results. Furthermore, handling the instructions of the pre-service teachers in the same concept may create different results. In line with the results obtained, it is thought that it will be useful if pre-service teachers are informed about the ways of developing the thinking in detail, gain experience about reflecting the knowledge they have theoretically, and this is included in the curriculum. Furthermore, the preparation and application of the lesson plans related to how the lessons that will contribute to the components of supporting and extending MT must also be included in the process by teacher education institutions.

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