INTEGRATING TECHNOLOGY INTO TEACHING: NEW CHALLENGES FOR THE CLASSROOM MATHEMATICAL MEANING CONSTRUCTION

Angeliki Mali*, Irene Biza*, Michalis Kaskadamis**, Despina Potari** and Charalambos Sakonidis***

*Loughborough University (UK), **University of Athens (Greece), *** Democritus University of Thrace (Greece)

The paper examines the ways in which technological tools shape teachers' and students' activity and hence the meaning construction related to linear function in a Year 10 classroom. The Extended Mediational Triangle of Cole and Engeström *(1993) is used to analyze the aforementioned activity and to interpret conflicts and contradictions. Results show that students face difficulties to follow their teacher's effort for conceptual understanding via connections of different representations and mathematical context. The teacher's knowledge and flexibility helps him to exploit contradictions between his and students' objectives mostly productively. Technological tools are generally supportive to this direction but also giving rise to complications in teacher-students' communications.*

INTRODUCTION

The learning and teaching challenges of using technology in the mathematics classroom have been repeatedly addressed in Mathematics Education. In exploiting computational environments, teachers' main responsibility is to act as orchestra conductors, aiming at students' interaction with the provided artifacts in ways that allow mathematical meaning to collectively emerge. To this direction, they are expected to tailor scaffolding conditions, exploiting technology in ways that promote students' transpositions towards mathematical meanings. However, this is not a straightforward process, as complications may arise in its course, due to conflicting interests and understandings emerging in the context of the resultant classroom activity. This paper presents an attempt to explore such a situation, aiming to contribute to the wider discussion on students' conceptual understanding via the exploration with technological tools.

THEORETICAL BACKGROUND AND LITERATURE REVIEW

Mathematics teaching is characterized by complexity as it is framed by the classroom interactions, the tasks assigned to the students and the overall social context. Skott (2010) talks about teachers' patterns of participation in different practices that frame their teaching. In technology-based mathematics lessons the situation becomes even more complex, as the nature of tools and management issues complicate student – teacher interaction in moving from the technological to the mathematical objects (e.g., Marracci & Marriotti, in press).

Activity Theory embodies the individual and the society in a unity in a way that the individual acts on his/her society at the same time as he/she becomes socialized to it (Mellin Olsen, 1987). This interplay between the individual and the society could capture the systemic nature of mathematics teaching by addressing its complexity. Recently, Jaworski and Potari (2009) used the activity theory and in particular the Extended Mediational Triangle (EMT) (Figure 1) of Cole and Engeström (1993) to consider the role of the broader social frame in which classroom teaching is situated. In this study, the activity of a teacher and of his students is contrasted and certain contradictions are identified.

The topmost of the subtriangles represents the visible actions of the subject, in our case of the teacher and the students, who use a number of tools to reach a goal (the object). Engeström (1998) refers to this as the "tip of the iceberg" and argues that "the 'hidden curriculum' is largely located in the bottom parts of the diagram: in the nature of the rules, the community and the division of labour of the activity" (p. 79).

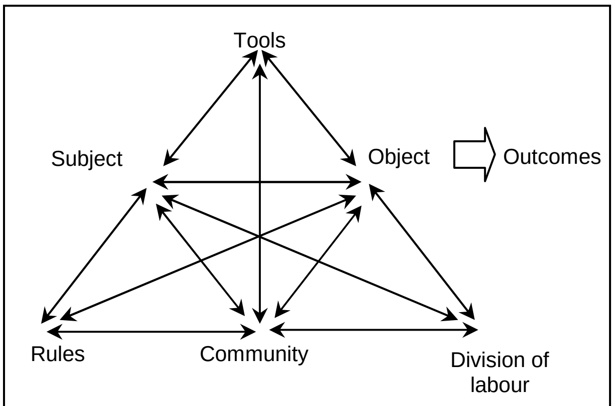

Figure 1: Extended Mediational Triangle

Students' participation in different communities (classroom, school, friends, parents etc.), each with its own rules and division of labour, has an impact on their classroom activity. Teacher's activity, on the other hand, based on the tasks and the tools s/he designs to achieve certain goals, is framed by the rules of the communities he belongs to.

In the present paper EMT is used to analyze the activity of the teacher and the activity of the students and to interpret conflicts and contradictions that the teacher faces in his attempts to support students using technological tools as a means to construct mathematical meaning.

Various attempts have been made to study technology integration into mathematics teaching, especially in the upper secondary school, as well as the particularities of this integration. For example, Biza (2011) investigated Year 12 students' understanding of the concept of tangent in computational contexts. She initially identified students' misconceptions with the concept itself and their difficulty in moving between representational systems. The teaching intervention employed was based on the usage of examples and of dynamic graphs. The classroom discussions analyzed showed limited taken-for-shared mathematics meanings between the teacher and the students as well as conflicts and fluctuations in students' arguments. Furthermore, the teacher's attempt to negotiate the construction of a shared mathematical meaning was not straightforward, fluctuating between orchestrating the classroom discussion, introducing new examples and changing the sequence of the examples.

Kendal and Stacey (2001) studied the teaching of derivatives by two secondary teachers to Year 11 students via a Computer Algebra System. One of the teachers relied predominately on lecturing and demonstrating while the other on exploiting children's ideas emerging during classroom discussions. The analysis of these discussions revealed that teachers' pedagogical choices were compatible with their conceptions about mathematics and its teaching as well as about technology usage for educational purposes. In particular, the first of the teachers exploited technology more and his students used effectively the technological artifacts to solve procedural problems. However, the students in the second class were more capable in dealing with conceptual issues related to derivatives.

Monaghan (2004) looked at how secondary mathematics teachers take advantage of digital technology in teaching. The participating teachers, who made moderate educational use of technology in their regular classes, encountered substantial difficulties in exploiting effectively technology and tended to encourage classroom activity that differed in structure from that employed otherwise. In particular, they showed preference for open-ended tasks, often requiring extensive investigative processes. They also expressed concerns for the nature of the mathematics involved and noticed that the students tended to concentrate on technological details to the expense of mathematics. The author concludes that students' interpretations of the tasks affected the emergent teachers' goals and the design of the teaching sessions.

In Trouche's (2004) study, Year 12 students were invited to deal with a demanding task in computational contexts, involving solving equations, studying function variation, finding limits and studying sequence variation. The students worked in groups and were expected to submit a research report by the end of the session. The researcher claims that the reported success of the activity should be attributed to the expertise of the teacher and to the highly motivated and intelligent students and that it is hard to carry out in "normal" classes due to the demanding instrumental orchestration required. Such an orchestration should allow students to first make sense of the problem, then to explore some special examples and to finally discuss relevant conjectures.

The studies presented above indicate that the usage of technology may facilitate students' classroom participation but it also gives rise to unexpected and unexplored challenges during the mediation process. These challenges might increase instead of decreasing the conflicts and contradictions present in the activity of the teacher and of the pupils, thus weakening and blurring the mathematical meaning construction process. The study presented below is an attempt to investigate how this takes place.

SETTINGS-METHODOLOGY

The study is an action research of a high school teacher with 20 years of teaching experience and his collaboration with four researchers. The teacher had just completed a Masters' program in Mathematics Education and participated initially as a teacher and later as a teacher educator in a number of professional development seminars related to the introduction of digital technologies in mathematics teaching. By returning to school, after a three-year school leave to complete his postgraduate studies, he wished to “implement” innovative ideas and approaches he came across during his studies. To this end, he introduced digital technologies into his teaching, promoted students’ conceptual understanding through different representations and generally encouraged students to make connections across contexts and representations. These approaches were beyond students’ experiences of mathematics teaching, which were textbook-based, over-emphasizing procedures and sophisticated techniques.

During the first months he faced a number of tensions related to students’, colleagues’ and parents’ expectations and thus decided to inquire his teaching and investigate systematically its effectiveness. To this purpose, he established a cycle of planning-implementing-reflecting lessons, which was regularly discussed with the first author of this paper at the planning and reflection phases. Furthermore, central classroom incidents were placed under scrutiny and emerging issues were explored in weekly meetings of all four researchers (mostly through Skype). In the occasion reported here, the teacher wanted to investigate how he could help students to explore algebraic relations and in particular the linear function and its graphical use to solve algebraic equations and inequalities in the context of dynamic environments, such as Geometer’s Sketchpad and Geogebra.

The first author of the paper observed his teaching in a Year 10 class (15-16 years old) for two months (27 students, 12 boys and 15 girls, for 17 teaching periods). The main research question addressed here concerns the ways in which the available technological tools shaped the teacher and the students’ activity and thus the mathematical meaning construction. The data consisted of the transcribed classroom observations and the discussions both at school and at the meetings. Analysis was based on the identification of critical incidents, where contradictions and conflicts emerged and the teacher had to interpret and manage. The EMT was used to analyze the incidents, allowing the interpretation of these contradictions and the deepening of our understanding of what could be characterized as effective teaching management.

RESULTS

In both incidents reported below the students work in pairs or groups of three in the computers’ room with a Geogebra file and a worksheet with technical instructions and mathematical tasks, both prepared by the teacher. The first comes from the first teaching session on linear functions while the second from the 5th and 6th sessions.

Incident 1: Teacher’s management of students’ unexpected responses

The teacher tries to use the possibilities offered by technology in order to allow students to identify many instances of the graph of a linear function. His goal is the students to explore general function properties by linking different representations.

For the needs of the introductory session on linear functions, the teacher prepared a Geogebra file consisting of a kinaesthetic representation of the function with two sliders, one for each parameter of the formula of the function. As the teacher explained just before the lesson: “They will manipulate the object [intuitively], gain a familiarity with this [make sense] and then make an interpretation of the object [typical meaning]”. The use of technology influences his decision, as “it allows testing dynamic changes”.

At the beginning of the session, he gives the students the worksheet and asks them to directly answer the first question: “Use the slider. What is the shape of the graph?” Students work in pairs:

1	S1:	Sir, when you say what is the shape, it is a triangle, that means if we use a specific [she means a specific value of the parameter *a*], it is true.
2	Teacher:	Which one is the graph?
3	S2:	A straight line.
4	Teacher:	The graph is this. Where is the triangle?
5	S1:	Here it is! Isn’t this the triangle? [she points at the triangle defined by the line and the two axis, see a similar inscription at Figure 2]
6	S2:	I can see it too.
7	Teacher:	We are not interested in the axes. What if [they are] hidden? Hide the axes.
8	S2:	Yes, what if … but.
9	Teacher:	Just a minute
10	S1:	Sir, I understood it.
11	Teacher:	If you cover the axis, this is the shape; this is the function graph, you need to refer to this.
12	S3:	What do you mean ‘what shape is the graph’?
13	S4:	Sir, you don’t have good expression.

The environment in which students work combines paper-pencil and computer based activities and includes: the symbolic representation; the kinaesthetic representation; two sliders; and the open-ended question regarding the *shape* of the graph. Students face difficulties in the connection of the representations of the linear function, so a group of them concludes that the shape of the graph is a *triangle* (Figure 2). The teacher did not expect this response from the students. For him, a graph is a geometric expression of an algebraic relationship; therefore, the line on the screen is the *shape of the graph*. However, for the students, a *shape*, in both geometric and algebraic contexts, is the same: does not represent a relationship, but something that should have area. So, they can only see a *triangle*. A linguistic conflict can be identified here between the teacher and the students: the graph of a linear function is a shape for the teacher, whereas students expect to see a geometric shape that has a surface. This interpretation gave rise to a new teaching goal for teacher –students’ understanding of what is *the shape of the graph*. To this aim, he tries to make the mathematical objects transparent to the students through questions and features of the educational software (transcript lines [2], [4], [7]):

> In the first question, I say: "What shape is the graph?" They say: "It is a triangle." I understood that in this case they also saw the axes as part of the graph. So, I hid the axes and said: "Which is the graph?".

The teacher has chosen to negotiate different contexts (geometrical and algebraic) of the concept of linear function, with the intention to link the two, a process that was not easy for students. Finally, after the teacher used the Geogebra tool to hide the axes of the graph, students' responses ("Isn't that the graph?", "It is always a straight line.", "A line?") showed that they began to understand what he referred to.

Incident 2: Teaching goals versus students' explorations

In the second incident, the initial intention of the teacher was to engage students with the investigation of the properties of the linear function (f(x)=*a*x+*b*) , especially regarding the role of *a* and *b*. To this aim he created a Geogebra file with a graphics window for the function graph and a spreadsheet window for the values of *x* and *y* (Figure 3).

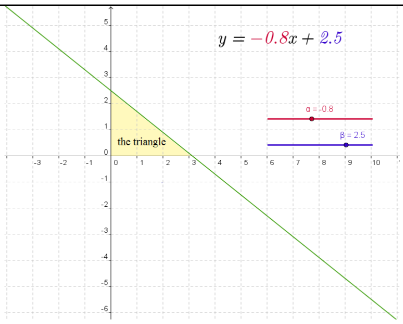

Figure 2: Geogebra file, Incident 1

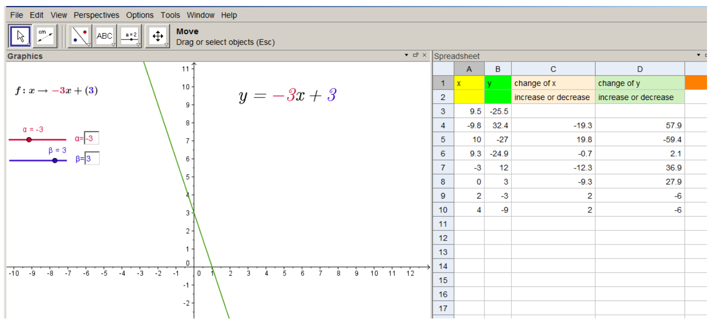

Figure 3: Geogebra file, Incident 2

The teacher has assigned to each group of students different values for *a* or *b*. In the spreadsheet, the first four values of *x* are random numbers, which change with F9 key, whereas the last four values of *x* are fixed and the same for every group. When the incident starts, students have already filled the columns of *y*, *change of x* and *change of y* by using the corresponding values of *x* and they are ready to deal with the task: "List as many observations you can regarding the results in the spreadsheet (try to be analytic in your description)". The teacher expects that some groups will observe that the change of *y* is proportional to the change of *x* and will try to connect the ratio of change of *y* over change of *x* with the parameter *a* or the slope of the graph. Any observation of this type might be helpful to him to introduce students to the monotonicity of functions at a later stage.

During the whole class discussion, a student working in a group with the function *f*(*x*)=0.5*x*-3 notices that the values of *y* have always the same sign with those of *x*.

The teacher reminds the students that, by pressing F9, the values of x change and that x can be any real number and asks them to think “whether for every value of x the value of y=0.5x-3 always have the same sign as x”. Some students agree, others not. He takes up the challenge and attempts to make students investigate the problem, by encouraging them to make connections among different representations of the function and also to link the solutions of equations and inequalities with points of the graph. A student says that for “every negative value of x this will be true”, another argues that “if x=0 then y=-3”. Other students suggest that there are positive values of x, for which y is negative or positive, “all the values of x for which 0.5x<3 or 0.5x>3, respectively”. However, there are students who fail to follow this suggestion. The teacher asks about the graph of the function and draws it roughly on the blackboard, following students’ suggestions regarding the y-intercept and the positive slope of the line. Then, he asks students not participating in the previous conversation to propose positive values of x, for which y>0 or y=0 or y<0, fill a table with the corresponding values of y and plot some corresponding points on the graph.

The initial aim of the teacher was the students to relate the ratio of change of y over change of x with the parameter a or with the slope of the graph. To achieve this, he asks students to work with different values of a and b, in order to observe the pattern in the spreadsheet. However, the open-ended question in the worksheet drives students’ observations to a different direction. By reflecting on his lesson after the session he notes: “The way that the question was posed in the worksheet might have been vague for what I had in mind”. As in the first incident, he takes into account students’ responses – although beyond his aims – and he creates a new teaching goal: to support students to give meaning to the graphical representation of the points (x, y) and relate them to the roots of the corresponding equation and inequality. Nevertheless, some students find it difficult to follow this shift:

> Instead of allowing all students to present their results, I grasped the first opportunity offered by the first student’s response: “if for every value of x the value of y=0.5x-3 will always have the same sign as x” and I invited the class to work on this. I saw it as a good opportunity to discuss issues related to the solution of an equation and an inequality and its relation to the corresponding point on the graph. […] Students found it difficult to shift their attention from the problems they were working on to the specific case […]

Analyzing the two incidents by the EMT

The two incidents exemplify some of the observations across all the sessions offered by the teacher on linear functions. It seems that his *good* pedagogical intentions were challenged by discrepancies between his and his students’ activities.

The EMT below (see Table 1) is used to interpret the context of the teaching/learning activity that took place across the observation.

Many of the teacher’s decisions, choices and rules are deeply influenced by the Master’s community. He strongly believes that conceptual understanding is the key for learning mathematics and he sets particular rules to provide for this:

> After the Master degree, for me, there are no students’ simpleminded answers, which could exist because they would not study enough and, therefore, would not understand. My approaches on how I interpret what they do have certainly changed. I saw and realized how complicated things about maths and the various concepts are.

For this reason, he uses new practices and tools he came across, such as group communication and Geogebra tasks, interprets students’ responses being informed by research literature and follows a different teaching structure from the one suggested by curriculum. Mathematical learning is being pursued by connections between different representations and contexts and the electronic environment and the questions in the worksheet aimed to facilitate students to make these connections.

Subject:	Teacher	Students
Object:	Should enable all students to: • connect symbolic, graphical and tabular representation and the graph of linear function • connect the geometric context and the algebraic context of linear function	• answer the questions of the worksheet.
Tools:	• Geogebra file • worksheet	• Geogebra file • worksheet
Community:	• classroom community • school community • master’s community (researchers, schoolmates, professors) • wider educational community (other textbooks, the internet) • wider social community (relationships with students’ parents)	• classroom community • school community • private mathematics lessons • friends • family • wider social community (team sports etc)
Rules:	• Different agenda from colleagues and curriculum: open-ended activities in agreement with findings of research literature - emphasis on students’ conceptual understanding • group communication • teacher’s requirements • exam requirements • pressures of the curriculum particularly with regard to time management • norms regarding working in mathematics and generally in class • norms regarding working in computer laboratory	• At private mathematics lessons and in other classes at the same school: Solving a great number of exercises for procedural understanding and practice. Teaching according to the book and curriculum structure. • examination requirements • peer pressures
Division of labour:	• planning the worksheet and Geogebra file • organization of students into groups • support for the students’ teamwork • discussion with the whole class at the end of teamwork • consolidation of mathematical meanings	• working in groups • enhance personal meanings concerning the task • contribution to teamwork and the whole class discussion with the teacher

Table 1: EMT describing the context of the teaching/learning activity

However, it seems that the teacher has different objectives and rules from students, whose teaching/learning experience is within the traditional paradigm, which promotes practicing on exercises and developing procedural skills. As a result, although the connections of different contexts and representations are included in the teacher's objectives, these are not visible for the students in advance. In addition, these connections are not included in either students' objectives in practicing procedures of mathematics or school community's practices of teaching and learning. This contradiction of objectives led, as a consequence, to a contradiction between the teacher's expectations and to what the students actually did. Later on, when the teacher noticed that his students did not make these connections (in discussing with the first author after lesson 10), he reflected:

> At the beginning, they had made these tasks with the parameters a and b… but possibly I did not manage to make these connections apparent… To make them apparent or to let them discover for themselves?... I do not know. This is why today they had this difficulty. They worked rather mechanically with the images, not realizing what the main point was.

DISCUSSION

The teacher's teaching goal is students' conceptual understanding via connections, which must be carried out through investigation. These connections are between different representations of functions (symbolical, graphical and tabular) as well as between different mathematical contexts (algebraic and geometrical). He incorporates technology into his everyday teaching and engages students in group-communication, practices that are new both to him and to his students. In class, the students start directly the investigation, without being informed about their teacher's objectives. The latter frequently experiences situations that he does not expect. He is flexible enough to hear students' voices and adapt his lesson plan accordingly. However, the students, being unfamiliar with this style of teaching and expecting a well-defined teaching agenda known in advance, aren't always able to follow him. Moreover, the exploration allowed by the technological tools makes them feel uncertain about the goals of their activity.

It could be argued that in a teaching approach where technological tools are exploited, students' activity becomes less controlled by the teacher. Furthermore, more layers of complication are added concerning the multiplicity of representations, the interaction with the environment and the dynamics of the emerging constructions. As a result, the meanings shaped by the students might frequently diverge from the meanings intended by the teacher. Many unexpected events may emerge from the students' reactions and the teacher needs to be flexible enough to adapt his/her planned goals and act in-the-moment. These actions require deep mathematical and pedagogical content knowledge for his attempts to effectively promote the mathematical meaning under construction. On the other hand, the openness of the situation, although might motivate students to get involved with the tasks, it is possible to be in conflict with other day-to-day classroom and institutional norms and practices that influence mathematical teaching and learning. Thus, the implementation of technology in the mathematics classroom requires careful consideration of teachers' scaffolding practices, which should aim to gradually introduce students into new ways of being involved with the mathematics classroom activity that promote genuine interactive construction of meaning, challenging them to sensitively explore resistances and resolve conflicts on the way.

REFERENCES

Biza, I. (2011). Students' Evolving Meaning About Tangent Line with the Mediation of a Dynamic Geometry Environment and an Instructional Example Space. *Technology, Knowledge and Learning, 16*(2), 125-151.

Cole, M. and Engeström, Y. (1993). A cultural-historical approach to distributed cognition. In Salomon, G. (ed.), *Distributed Cognitions: Psychological and Educational Considerations*. New York: Cambridge University Press.

Engeström, Y. (1998). Reorganising the motivational sphere of classroom culture: an activity-theoretical analysis of planning in a teacher team. In Seeger, F., Voigt, J. and Waschescio, U. (eds.), *The Culture of the Mathematics Classroom*, Cambridge: Cambridge University Press.

Jaworski, B. & Potari, D, (2009). Bridging the macro- and micro-divide: Using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72, 219-236.

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning, 6*, 143-165.

Maracci, M. & Mariotti, A. (in press). Semiotic mediation within an AT frame. *International Journal for Technology in Mathematics Education*. Special issue "Activity theoretic approaches to technology enhanced mathematics learning orchestration".

Mellin Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: D. Reidel.

Monaghan, J. (2004). Teachers' Activities in Technology-Based Mathematics Lessons. *International Journal of Computers for Mathematical Learning, 9*(3), 327-357.

Skott, J. (2010). Shifting the direction of belief research: From beliefs to patterns of participation. In Pinto, M.F. & Kawasaki, T. F. (Eds.). *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol.4, pp. 193-200), Brazil:PME.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281–307.