ONE POSSIBLE WAY OF TRAINING TEACHERS FOR INQUIRY BASED EDUCATION

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The paper describes the results of a research study aimed at preparation of pre-service primary school teachers for inquiry based mathematics education (IBME). The teaching experiment involved pre-service teachers' work using a variety of techniques (with strictly formulated questions and open problems) in various environments suitable for this approach. We concentrated on pre-service teacher content knowledge and ability to apply the gained knowledge in problem solving.

Keywords: inquiry based mathematics education, pre-service teacher training, subject matter knowledge, knowledge base for teaching

INTRODUCTION

We have been focusing on various issues connected to the process of professionalization of teachers for many years. Contemporary we tried to find the ways of improvement of teachers' knowledge base for teaching (Scherer, Steinbring, 2004) through introducing substantial learning environments (Tichá & Hošpesová, 2011). The issue is closely connected to the question of how to introduce these environments into reality of teaching at schools. This lead us to inquiry based mathematics education (IBME), to the role of the teacher as understood in thus conceived education, and to the requirements on pre-service teacher training. This contribution is concerned mainly with the quality of pre-service teachers' content knowledge, and their ability to apply the gained knowledge in solving process of problems leading to IMBE (requiring discovery, exploratory approach).

INQUIRY BASED MATHEMATICS EDUCATION

IBME in contemporary education raises great interest (for example Canavarro, 2011). It transmits procedures known from real scientific research to everyday school work with pupils. It puts emphasis on independent discovery (a self-discovery), on proper justifying, and on links with everyday reality (i.e. on a practical desirability).

> "Inquiry based mathematics education refers to an education which does not present mathematics to pupils and students as a ready-built structure to appropriate. Rather it offers them the opportunity to experience how mathematical knowledge is developed through personal and collective attempts at answering questions emerging in a diversity of fields, from observation of nature as well as the mathematics field itself, ..." (Artique, Baptist, Dillon, Harlen and Léna 2011, p. 10).

The concept of this approach is far from new. Let us recollect here for example the concept of *guided rediscovery,* anchored in the concept of genetic style of teaching governed mainly by psychological reasons and the concept of guided discovery. It built on characteristics of genetic style of teaching of Brunner (1966), Wittmann (1974), Freudenthal (1973).

> “H. Freudenthal expressed his opinion on guided discovery and genetic style of teaching on a number of occasions. He characterizes the genetic principle as “rediscovery”, i.e. such teaching of mathematics, in which discoveries stand for what they really are, i.e. for discoveries. Discovery takes place even in a modest pupils’ rediscovery by non-prescribed procedure... Freudenthal adds the attribute guided to the concept of “rediscovery”. Although this attribute should be self-evident, it is not useless because of left-wing didacticians who refuse to understand and interpret rediscovery as a process in which pupils should discover everything on their own.” (Vyšín, 1976, 584-585).

As we want to focus on creation of future teachers´ knowledge base for teaching in IBME, it is worthwhile to recollect at this point also Brousseau’s concept of a didactical situation (explanation in endnote 1) and *a*-didactical situation (Brousseau, 1997). “In *a*-didactical situation the educators enable the student(s) to acquire new knowledge in the learning processes without any explicit intervention from them” (Brousseau in Novotná, Hošpesová, 2012, p. 282). This does not necessarily imply that the student (solver) must discover the new knowledge on his own, but IBME can become a specific type of *a*-didactical situation. This makes us ask what the educator’s role in situations of discovery is. Brousseau distinguishes several phases in an *a*-didactical situation that can be used in IBME:

> “Situation of action – its result is an anticipated (implicit) model, strategy, initial tactic
>
> Situation of formulation – its result is a clear formulation of conditions under which the situation will function
>
> Situation of validation – its result is verification of functionality (or non-functionality) of the model” (Brousseau in Novotná, Hošpesová, 2012, p. 282)

The educator’s intervention in these phases will be of different nature. Let us presume that a discovery is initiated by the educator using a problem that opens opportunities for collaborative orientation. We can expect the educator to intervene in the first phase only if the pupils/students are not active, if they do not look for solutions. In contrast the educator’s role in the second and the third phases is crucial. This role is far from traditional: the educator does not explain, produce illustrative examples and exercises. He poses questions, asks for clearer explanations, for assessment of validity of argumentation (e.g. questions: Why do you think so? Could you explain it more clearly? How do you mean this? Are you sure it is so?). In mathematics, this is of specific meaning, as the nature of mathematical knowledge is also specific.

In our study the students were in dual role. They solved the problem (the role of the student) and at the same time, they are supposed to think as the teachers and analyze the didactic potential of the solved problem (the role of educator). The aim of this paper is to find out how this dual role influenced the process of problem solving.

TEACHING EXPERIMENT, ITS PARTICIPANTS, METHODOLOGY

Participants

The teaching experiment was carried out with two groups of participants. We used similar framework steps for both groups.

The first group consisted of pre-service primary school teachers in the second half of their studies attending the course of Didactics of Mathematics (in total all 63 registered students). The students completed in their previous study several courses of mathematics, which focused on the theoretical basis for teaching mathematics for primary school level. Partly we solved problems in learning environments suitable for implementation of IBME. Students analyzed their mathematical content, discuss the possibility of putting them into practice. We tried it show how the traditional subject matter could be enriched through IBME.

The other group consisted of students who had already finished their course of Didactics of Mathematics but wanted to amplify their knowledge by attending an optional subject with the aim of connecting the knowledge gained in the seminars of mathematics and didactics of mathematics in isolated topics and to gain a full, comprehensive view of primary mathematics education. They do not focus specifically on IBME.

Teaching experiment and research tool

Our approach will be illustrated on the problem "Who hits 50". The assignment of this problem was inspired by a paper of Scherer and Steinbring (2004). The problem was posed as follows (the original German wording of the problem is in the endnote 2):

The following rules for calculations hold in the scheme in Fig. 1:

- You can choose arbitrarily the start number (initial - given in an oval) and the number you are adding (addend - given in the circle).
- You gradually get the numbers in the other four squares by adding the "addend".
- You get the "target number" by adding all the numbers in the five squares in a row.
- Which natural numbers do you have to choose as "initial" and "addend" to get the "target" number 50?

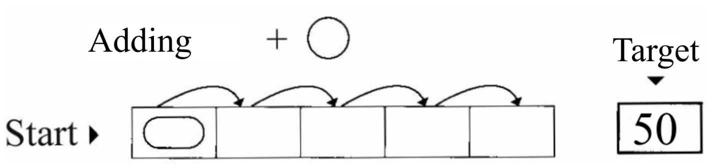


Fig. 1 The scheme for the problem "Who hits 50"

The students were introduced to the environment and assigned the problem. However, they were not told that the problem focused on searching for patterns, coherences and relations and their properties. They were expected to find that out on their own.

The procedure:

1. The participants solved an unknown problem/set of problems in order to get familiar with the environment.
2. a) They were asked questions and assigned problems in the form: “What will happen if the target number is 60, 90? What will happen if there are 6 boxes?” etc.

   b) The students were asked to think over the aim of the assigned problem. They were asked to pose other tasks and questions.
3. This group work was followed by a discussion in which various aspects of the problem were analyzed (mathematical content, number of solutions, possible obstacles for the pupils, different models and representations, possible solving methods, possible extensions of the problem).

In the background of the experiment was our conviction that our students should first get hands-on experience with discovery in these environments, i.e. try to solve problems similar to those they will assign to their prospective pupils (number walls, race to twenty, etc.), which should be followed by a discussion on the didactical aspects.

The students in both groups were first asked to solve the assigned problem, answer the question.

The students in **the first group** then worked in groups of four on modifications of the assigned problem. They were looking for answers to questions aiming at developing their subject matter knowledge and were solving the assigned tasks:

- What relations could you observe in the scheme?
- Propose other target numbers and find several solutions.
- Solve the problem with a different scheme: e.g. a different number of squares; a different target number; 6 squares and target number 60.
- Articulate conclusions on the relationships between the target number and the shape of the scheme.

In **the other group** the students worked in pairs. The work in seminar concentrated mainly on students´ ability to elaborate didactically the subject matter. The students were asked to think over the following questions (and look at the situation in a different perspective):

- What task was given? Why? (Which concept, solving method is being developed?)
- Why did we assign this problem? What is the benefit of this problem? What may become a source of difficulty, problems?

Later, in a whole-class discussion, they came to the conclusion that this was about regularities and dependencies, the students were asked to:

- Think of other questions and tasks leading to discovery of regularities.
- Assess whether this was a suitable and stimulating task.

In the end students in both groups were asked to pose other questions and problems in this environment.

Data collection and analysis

When analyzing the students' work we were inspired by the method of grounded theory (Strauss, Corbin, 1998). We classified the students' written productions and sorted them with respect to their characteristic features. We gradually identified and formulated emergent phenomena, and interpreted them.

FINDINGS AND DISCUSSION

We realized several phenomena. The students differed considerably in:

- ways how to find the solution,
- ability to describe what they saw and which representations they were using,
- which regularities and dependencies they uncovered,
- what type of problems they posed.

How did the students proceed when looking for the solution?

All students managed to find several solutions to the problem. The students were not satisfied with one solution of the problem; they tried to find all solutions. But the most frequently used strategy was the trial and error strategy. The initial numbers were most often chosen at random and some of the groups of students did not try to proceed in a systemic way. This was the case of looking for the "first" as well as the following solutions (that is pairs of starting and adding numbers). In consequence, some groups of students were not sure in the concluding discussion on the number of solutions whether they managed to find all of them. Moreover, they could not tell how to make sure that all the solutions be found. Trial and error strategy is the natural way of solving this type of problems unless the solver knows the procedure and is undoubtedly appropriate in case of primary school pupils (although Scherer & Steinbring (2004, p 68) showed that some of German four-graders were able to proceed in a systematic way).

Students also did not attempt to apply algebra. The exception was a group in which two students who originally studied mathematics worked (see their solution in endnote 3). The question is whether their approach was not caused by the student tendency to solve the problem in a way understandable to their future pupils. Rather, it seems that they were not able to use the knowledge they have acquired in previous studies, although in the seminar they regularly worked like this.

What representations did the students use?

Previous findings are related to the modes of representation, which students used. Despite the attention paid to different modes of representation, translation among them, and transformation within them (Lesh et al., 1987) the students probably neglected and underestimated the role of iconic and pictorial representations in concept construction and tend to overuse verbal representation. They for example wrote:

> The sum of the start and the final numbers is the same number as the sum of numbers in the 2nd and the 4th squares. In case of the target number 50 it is 20. This means that the number in the middle is always the same number. In this case 10. And as there are five squares, the "target number" must be divisible by five. If there are 6 boxes, the "target number" must be divisible by 6.

Visual representation (Fig. 3 for target number 90, Fig. 4 for 6 boxes) accompanying the verbal description of the calculation was used only rarely.

> We divide the final number by the number of boxes (squares) (5) and get the middle number. The sum of the first and the fifth numbers and of the second and the fourth numbers is twice as large as the middle number.

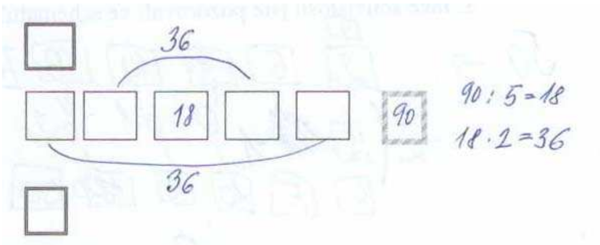


Fig. 3 Visual representation accompanying the verbal description

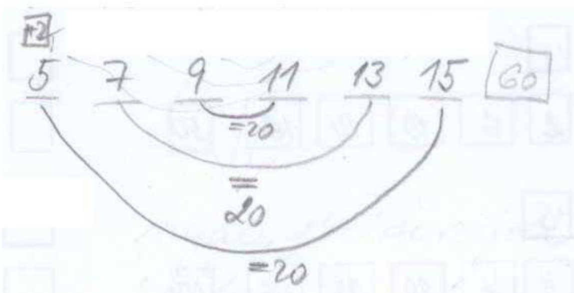


Fig. 4 Visual representation for schema with 6 boxes

In several cases, the visual representation helped us grasp how the students were reasoning (Fig. 5a, b). As important we consider the students´ efforts to seek patterns and coherence. For example Griffith claims that mathematics may be characterized as the search for structures and patterns that bring order and simplicity to our universe. Moreover, it is the discovered patterns and coherence that give mathematics its power (Griffith, 2000).

> An odd number of numbers means that the middle number is always the same and the sum of the outside numbers is double the middle number. An even number of numbers means that the two numbers in the middle are three times smaller than the final number.

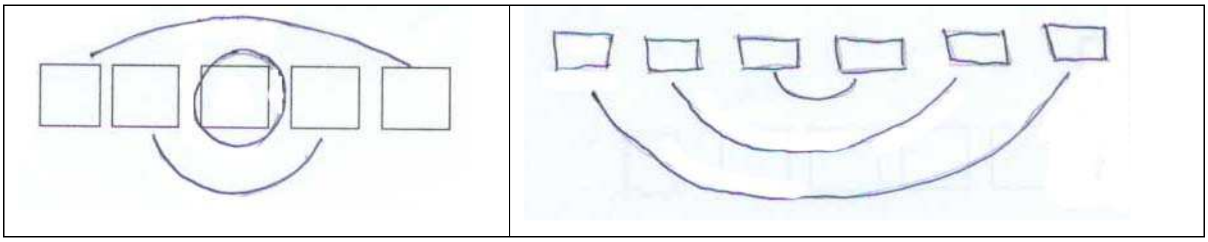


Fig. 5 Visualisations of statements mentioned above

The joint discussion made most students realize the “usefulness of making illustrations”. However, only some of them were able to grasp the table that followed these visualizations (Fig. 6).

x – 2a	x – a	x	x + a	x + 2a

Fig. 6 Visualisation through table

What regularities and coherence did the students notice?

The students usually described their calculation; they stated what they had been doing and how they had proceeded. They mostly recognized that the final number must be divisible by the number of squares but rarely did they verify or justify this conclusion. Their explanations are not always easy to understand. For example

> 50/5 = 10, 10 must be in the middle because we divide 50 into five parts (10, 10, 10, 10, 10) and then the 10 in the middle is left while I always subtract something from the tens on the one side and then I have to add something to the tens on the other side.

> 50/5 = 10 ... to the third. We divide the final number by the number of boxes, the result of which will be in the middle box. We subtract a selected number in the upper box to the left and add it to the right.

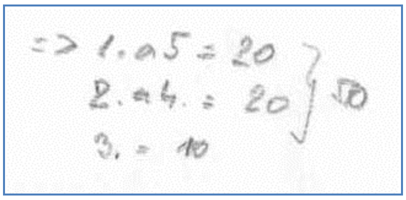


Fig. 7 Students’ solution

We can see that there is number 10 in the 3rd box. We calculated 10 as 50/5 = 10.

These observations also included some interesting remarks that were consequently discussed in the joint reflection. For example:

The sum of the pair of outside numbers is 20.

The addend and the number in the second square are together 10.

If the addend is larger by 1, the initial number is smaller by 2. (This was observed in cases that the students' procedure of looking for other solutions was systematic.)

The greater the initial number is, is the smaller the addend is.

Posed questions and problems

The students' interest in problem posing was raised in case of both groups by the following tasks: Why did we assign this problem to the children, what were our intentions? What kind of problem would you recommend as suitable for assignment to children in the given situation? Why? With what intention?

Let us present a few questions posed by the students:

Which least number can there be in the middle?

What would happen if the middle number were 8?

What would change?

What least number can the target number be?

Can the start number be zero? Can the addend be zero?

What will happen if the target number is odd?

In general, the students tried to pose such questions and problems that could be answered or solved unequivocally.

The students in the second group also considered how to broaden the solution in case that the problem was not limited only to natural numbers. In other words how this environment could be used for motivation of negative or even rational numbers.

CONCLUSIONS

In this study, we dealt with the problem how to prepare future teachers for IBME. We assumed that the solution of appropriate problems and connecting it with didactic questions shows the students how important their knowledge of mathematics is and how they can use it in school practice. Our starting point was in accordance with other authors, for example Lamon stated: "... facilitating teacher understanding using the same questions and activities that may be used with children is one way to help teachers build the comfort and confidence they need to begin talking to children about complex ideas." (Lamon, 2006, xiv)

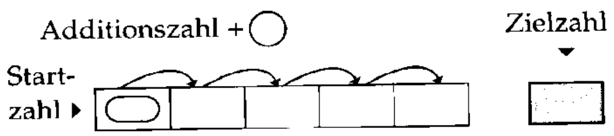
One of the key problems was the students' failure to realize the connection of what they learned before in mathematics courses to the needs of school practice. This fact was in our study manifested by students' preference of arithmetic. Algebra as a tool for solution was used only by those students who were proficient enough, and who are persuaded that they are able to work quickly and without errors. Other students tended to stereotypical procedures. They regarded the use of visual aids, illustrative examples as manifestation of lower level of knowledge. It seems that they perceived as substantial only the questions of WHAT? and HOW? to teach. The question WHY? did not interest them so much. Such belief is in our opinion unsuitable, inadequate, especially in case of primary school teachers (in the Czech Republic teachers of 6-11 years old pupils).

The question is how to change this opinion, how to develop the ability to visualize, explain using visualization, how to show the students that thorough study of mathematics is important for teachers; how to change students' understanding of the role of reasoning in education (not only mathematical) and the related generalization, looking for similarities and differences, discovery of regularities, patterns and coherences. It is also the question of making the students aware of the dangers of badly-founded statements (hypotheses, proclamations) and of when it is the right time for drawing conclusions. The *joint discussion and reflection* of solutions of problems characteristic for IBME offers, in our opinion, good grounds for this.

END NOTES

1. A system in which the teacher, student(s), milieu and restrictions necessary for creation of a piece of mathematical knowledge interact "to teach somebody something". The educator "organizes a plan of action which illuminates his/her intention to modify some knowledge or bring about its creation in another actor, a student, for example, and which permits him/her to express himself/herself in actions" (Brousseau & Sarrazy, 2002).

2. The scheme of the problem "Wer trifft die 50" in the original (Scherer, Steinbring, 2004, p. 65).



3. The students formulated and solved an equation:

$$x_1 + x_1 + a + (x_1 + a) + a + (x_1 + a) + a + a + (x_1 + a) + a + a + a = 50$$
$$5x_1 + 10a = 50$$
$$5(x_1 + 2a) = 50$$
$$x_1 + 2a = 10$$

and systematically proceeded in its solution:

10, 10 + 0, 10 + 0, 10 + 0, 10 + 0 = 50

8, 8 + 1, 9 + 1, 10 + 1, 11 + 1 = 50

6, 6 + 2, 8 + 2, 10 + 2, 12 + 2 = 50 , …

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