PRE-SERVICE AND IN-SERVICE TEACHERS' VIEWS ON THE LEARNING POTENTIAL OF TASKS – DOES SPECIFIC CONTENT KNOWLEDGE MATTER?

Anika Dreher, Sebastian Kuntze

Ludwigsburg University of Education, Germany

This study examines views of pre-service and in-service mathematics teachers on the learning potential of tasks and interrelations of such views with relevant content knowledge. Focusing on the role of representations for learning and the content domain of fractions, the paper hence aims at connecting different sub-aspects of professional teacher knowledge. The results indicate that the learning potential of problems focusing on a conversion of representations is hardly acknowledged in comparison to tasks requiring only a calculation on a numerical-symbolical representational level and giving a rather unhelpful pictorial representation. However, there is a tendency that teachers with higher content knowledge scores rate the learning potential of the first type tasks comparatively higher.

INTRODUCTION

Epistemological views related to tasks are expected to play a key role when mathematics teachers select or create problems for the classroom. Hence, professional knowledge related to overarching aspects of pedagogical content knowledge (PCK) – such as the idea of using multiple representations – should also be examined on the level of task-related views. However, other components of professional knowledge might have an influence on task-related views, such as domain-specific content knowledge (CK), but unfortunately there are still very few studies making such links. Consequently, this paper focuses on views of pre- and in-service teachers regarding the use of representations in tasks. The selected problems stem from the content domain of fractions, in which the teachers have also been assessed in a CK test on representations of fractions.

The results indicate that there is task-specific variation in the views about the learning potential of the tasks presented to the teachers. Nevertheless, the analysis yielded two types of tasks, in line with the theoretical design of the corresponding questionnaire unit: tasks making use of the learning potential of changing between representations and tasks with rather unhelpful pictorial representations. In-service academic-track secondary teachers rated the learning potential of type 1 tasks higher than pre-service teachers. Moreover, teachers with higher CK scores tended to acknowledge the learning potential of those tasks comparatively more.

The following second section gives a brief overview of the theoretical background, which leads to the research interest of this study presented in the third section. We will then describe the methods and design in the fourth section, present results in the fifth section, and conclude with a discussion in the sixth section.

THEORETICAL BACKGROUND

National standards in many countries emphasize the importance of dealing with multiple representations for mathematical learning. In the case of the German standards for the mathematics classroom "using mathematical representations" is stated as one out of six core aspects of mathematical competency - in particular, recognizing interrelations between different representations and changing between them is stressed explicitly (KMK, 2004). There are very good reasons for such an emphasis of multiple representations in the mathematics classroom: Representations play a major role in all kinds of mathematical activities, since the perception of mathematical objects is dependent on representations (Duval, 2006). We take the notion *representation* to mean something which stands for something else – in this case for an 'invisible' mathematical object (cf. Goldin & Shteingold, 2001). In particular, *pictorial* representations are illustrations, diagrams or sketches. Since usually a single representation makes visible only some aspects of the corresponding object, multiple representations complementing each other are needed for getting hold of it (Gagatsis & Shiakalli, 2004) in the sense of developing a rich conceptual understanding. Hence, the ability to recognize a mathematical object behind its different representations and to use them flexibly is a key for successful mathematical thinking and problem solving (i.e. Lesh, Post, & Behr, 1987; Duval, 2006; Gagatsis & Shiakalli, 2004). Consequently, reflecting and discussing interrelations and conversions between different representations should be part of the mathematics classroom in order to foster the students' ability to use multiple representations flexibly. In particular, tasks focusing on conversions from one mode of representation to another (and back), which promote insight into their interrelations, can make an important contribution to students' understanding (Duval, 2006). Therefore, the awareness of the importance of dealing with multiple representations and also of the difficulties which come with them for learners, as well as knowledge about how to foster their abilities in making use of multiple representations are important aspects of PCK.

Against this background the question arises as to what extent teachers are aware of the learning potential of tasks focusing on conversions of representations. The results of a prior study about pre-service teachers' views on pictorial representations in tasks indicate that many pre-service teachers tended to overemphasize the motivational aspect of pictorial representations and hardly saw the learning potential of such pictorial representations which enable students to take an additional approach to mathematical concepts. (Dreher & Kuntze, 2012; cf. also Dreher, 2012). In this prior study, the teachers were asked about their views regarding the pictorial representations. A follow-up question was how these views may impact on the learning potential of a task as a whole in the eyes of the teachers, i.e. how they see the potential of the learning opportunity which is set by the task. In addition, there has been a need for including data from in-service teachers in order to get insight into the role of classroom experience. Both of these follow-up research interests have been taken up in the study reported here.

Views about the learning potential of tasks are considered to be individual, conviction-like and in the first place restricted to the particular case of the task considered. However, when looking at types of tasks and investigating the views of teachers related to such types, the data may give insight into whether teachers are able to ‘see the difference’, i.e. to realize opportunities for conceptual learning associated with the types of tasks. In this sense, task-related views also reflect components of PCK on a level beyond the specific case of a particular task.

As a theoretical background for analyses on these two layers, this study uses a multi-layer model of professional knowledge (cf. Figure 1), which combines the spectrum between knowledge and beliefs (e.g. Pajares, 1992), the professional knowledge domains by Shulman (1986; cf. also Ball, Thames & Phelps, 2008) – with levels of globality respectively situatedness (cf. Törner, 2002; Kuntze, 2012).

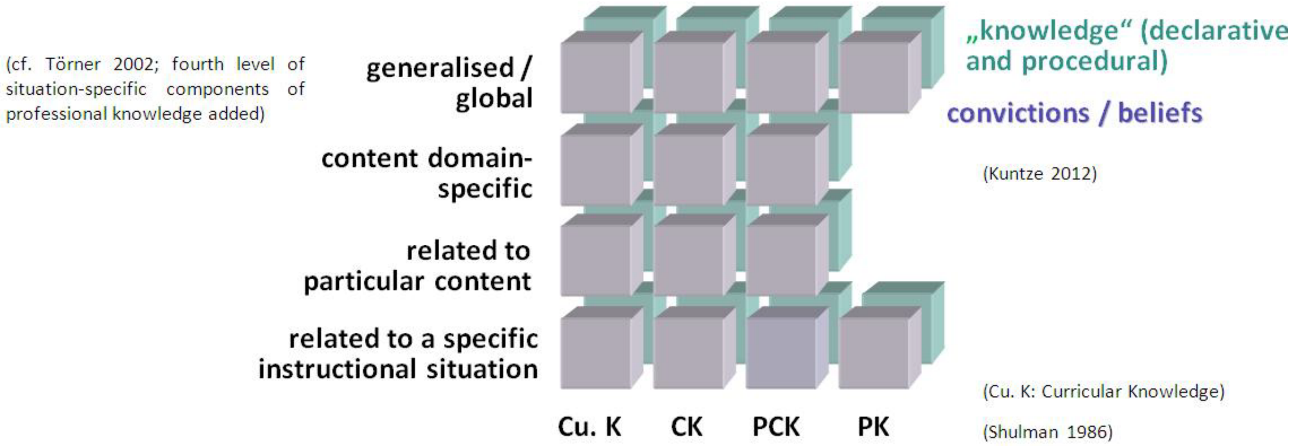


Figure 1: multi-layer model of professional knowledge (Kuntze, 2012)

According to this model, task-related views can be described as basically content-specific convictions in the domain of PCK (Kuntze, 2011). As argued above, there is the possibility of going up one level of globality and making a bridge to the knowledge side, if the data affords looking empirically at types of tasks.

In the case of the present study, the tasks are situated in the content domain of fractions with an emphasis on representations of fractions and on operating with fractions. The PCK component mentioned above is hence bound to this content domain. This aspect calls for including content domain-specific CK, in order to be able to answer the question whether CK is sufficient for PCK related to the use of representations in the task types and/or which role CK plays for PCK in that content domain. Exploring such links between content-specific convictions (related to tasks) and relevant content knowledge could provide useful information for designing effective professional development activities.

RESEARCH INTEREST

Given the significance of professional teacher knowledge related to the idea of using multiple representations (cf. Kuntze et al. 2011) in particular related to the design of learning opportunities provided by problems, the previous section highlights that task-related views and corresponding PCK and CK components are in the center of interest. Moreover, empirical insight into relationships between these components of professional knowledge is needed. In our study, we hence concentrate on the following research questions:

- Which task-specific views relevant for the use of representations do mathematics teachers hold? In particular, how do they evaluate the learning potential of types of problems which make use of multiple representations in different ways?
- Are there differences in the views between groups of teachers with different qualification levels?
- Is content knowledge interrelated with such task-specific views?

DESIGN AND METHODS

For answering these research questions, a questionnaire was administered to 219 pre-service teachers (183 female, 26 male, 10 without data) and 83 in-service teachers, of which 58 were teaching at academic track secondary schools (23 female, 32 male, 3 without data) and 25 at secondary schools for lower attaining students (15 female, 10 male). The pre-service teachers were on average 20.7 years (SD = 2.5) old and at the beginning of their first semester of teacher education. The teachers at academic track secondary schools resp. at secondary schools for lower attaining students were on average 41.5 (SD = 12.3) resp. 39.9 (SD = 11.3) years old and had been teaching mathematics since 13.6 (SD =12.3) respectively since 10.8 (SD = 9.5) years.

Corresponding to the first two research questions for this study, the participants were asked to evaluate the learning potential of six fraction problems by means of multiple-choice items. The teachers could express their approval or disagreement concerning these items on a four-point Likert scale. They were told that the problems

Make up a situation or a word problem which is suitable for the calculation $3 \div \frac{1}{4}$ and then use it to solve the calculation.	Do you know what $\frac{1}{2} \cdot \frac{1}{4}$ is? You can use the pictures below to help:  

Figure 2: Samples for tasks of type 1 (left) and of type 2 (right)

were designed for an exercise about fractions in school year six. Three of these tasks are about carrying out a conversion of representations, whereas solving the other three tasks means just calculating an addition or a multiplication of fractions on a numerical-symbolical representational level. The pictorial representations which are given in the problems of the second type are rather not helpful for the solution, since they can't illustrate the operation needed to carry out the calculation. Some of them may even be misleading. Samples for both kinds of tasks are shown in Figure 2.

In order to find answers to our third research question, a further section of the questionnaire was included in the analyses: A test on specific CK about dealing flexibly with multiple representations for fractions and their operations. As in the sample item shown in Figure 3, given (incorrect) conversions between different forms of representations had to be checked and corrected or a conversion had to be carried out.

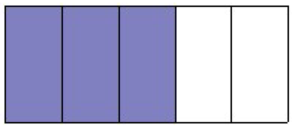
Please change the diagram, if necessary, so that $\frac{3}{5}$ of $\frac{1}{4}$ is shaded. Otherwise just tick the box on the right-hand side.		

Figure 3: Sample item of the CK test

RESULTS

We start with the results concerning the first two research questions, namely the teachers' evaluation of the learning potential of the six tasks given in the questionnaire. The design of this questionnaire section could be confirmed by a factor analysis: For each task there is a single reliable four-item scale (Cronbach's α range from 0.72 to 0.87) about its learning potential with respect to its use of representations. A sample item of these scales is: "The way in which representations are used in this problem aids students' understanding." Figure 4 shows the means and standard errors of these six scales for all three subsamples.

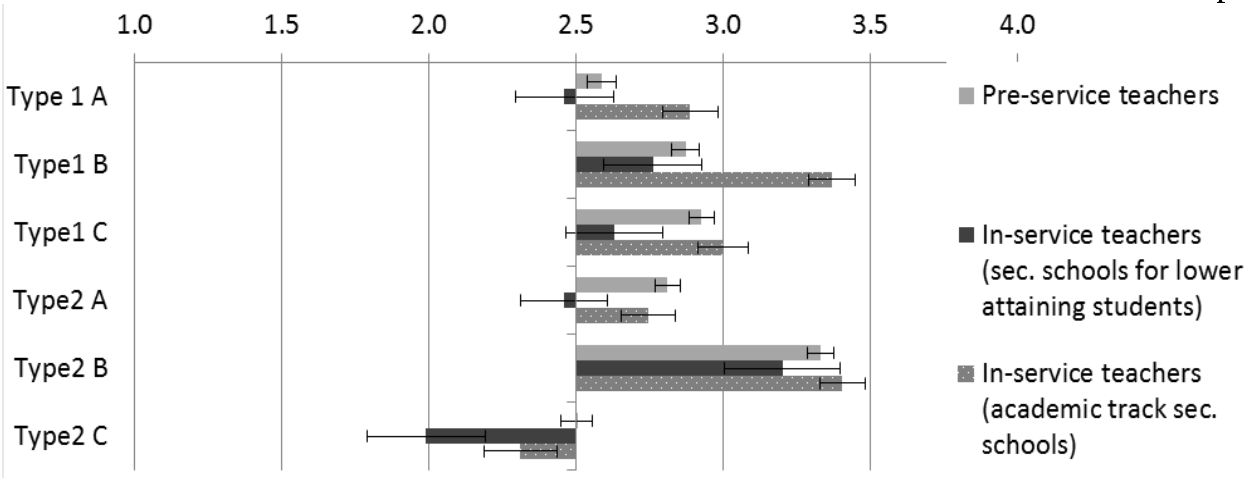


Figure 4: Evaluations of the task-specific learning potential

The value 1 means strong disagreement, whereas the value 4 stands for strong approval. Looking for differences between the means of the subsamples yields mainly that the in-service teachers at academic track secondary schools have evaluated the learning potential of the first two type 1 problems much higher than the other participants.

Comparing the evaluations of the different tasks creates the impression that the views, which were expressed here, are very task-specific. The uniqueness of each problem seems to be predominant over their classification into two types according to their use of representations: The tasks with the most positive and the most negative ratings both belong to the second type (calculation tasks with rather unhelpful pictorial representations). Nevertheless, the theoretical classification of the tasks underlying their creation can be reconstructed empirically from the teachers' evaluations of their learning potential:

Rotated Component Matrix[a]						
Component	Type1_A	Type1_B	Type1_C	Type2_C	Type2_A	Type2_B
1	.719	.696	.659			
2				.786	.628	.610
Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization.						
a. Rotation converged in 3 iterations.						

Figure 5: Factor analysis (48.9 % of variance explained)

Carrying out a factor analysis with the six scales about the learning potentials of the six problems yields two "meta-scales" linked to the two types of tasks (cf. Figure 5), where both scales are reliable with $\alpha = 0.79$. Having now scales corresponding to the learning potential of two types of tasks which make use of multiple representations in different ways, it is worthwhile comparing the aggregated evaluations of the subsamples once again. The means (and standard errors) in Figure

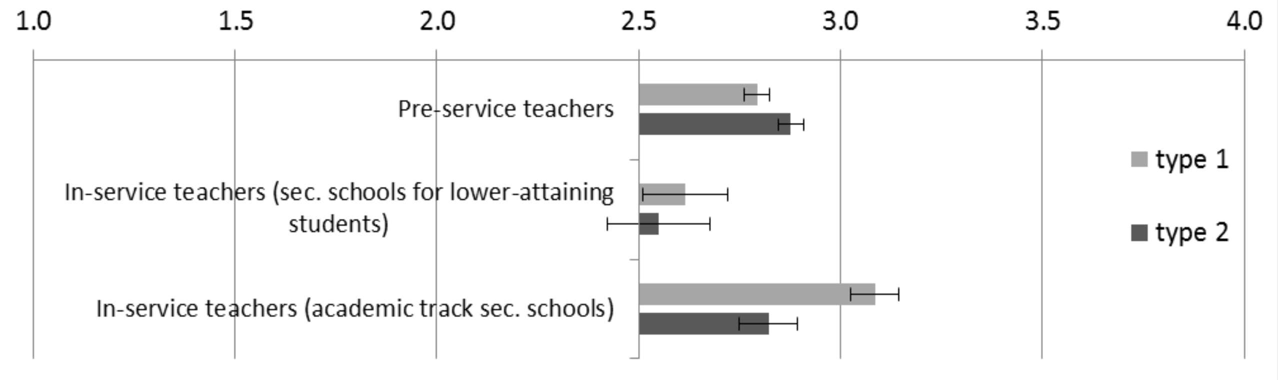


Figure 6: Evaluations of the learning potential regarding the two types of tasks

6 show an interesting result concerning the subsamples of this study: While the pre-service teachers' rating of the learning potential is higher for type 2 tasks than for type 1 tasks ($T=2.121$, $df=218$, $p<.05$, $d=0.18$), the pattern might be reversed for the in-service teachers at secondary schools for lower-attaining students (not significant)

and is completely reversed for the in-service teachers at academic track secondary schools (T=3.015, df=57, p<.01 d=0.53). Focusing on the views about the learning potential regarding tasks of the first type, a comparison between the subsamples yields that the in-service teachers at academic track secondary schools have given higher ratings than the pre-service teachers (T=4.221, df=275, p<.001, d=0.63) and than their colleagues at secondary schools for lower attaining students (T=4.113, df=81, p<.001, d=0.98). Comparing the sub-samples regarding their view about type 2 tasks on the other hand shows that the pre-service teachers have assigned a higher learning potential than the in-service teachers at secondary school for lower-attaining students (T=2.487, df=26.9, p<.05, d=0.68).

These results give rise to the third research question of this study: Is specific CK interrelated with such views concerning the learning potential of types of tasks as a part of domain-specific PCK? Comparing the evaluations of those participants having a score of at least 50% in the test about specific CK to the rest of the sample might give some insight. Figure 7 shows that the participants with at least 50% CK

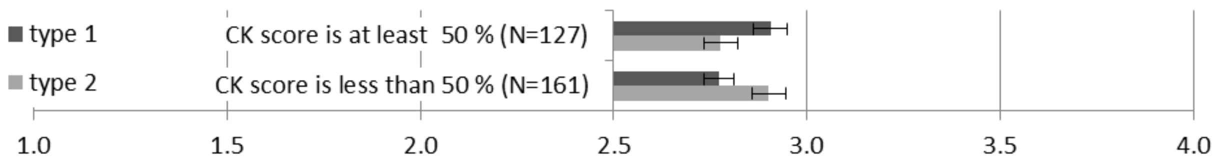


Figure 7: Evaluation of the types of tasks according to CK score

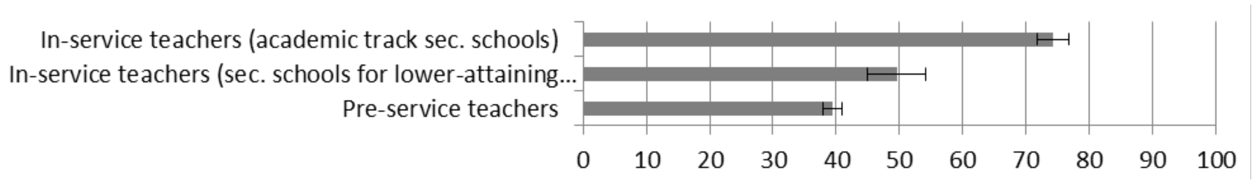


Figure 8: Content-specific CK scores (in percent)

score have assigned a higher learning potential to type 1 tasks than to type 2 tasks (T=2.413, df=126, p<.05, d=0.27), whereas the evaluations of the subsample of teachers with a lower CK score shows the reversed pattern (T=2.564, df=160, p<.05, d=0.29). The rather low effect sizes are not the only reason why this result should be interpreted with care: Another indication for scepticism can be found in the results concerning the CK scores shown in Figure 8. The mean scores of the three subsamples in the test on specific CK are very distinct: The in-service teachers at secondary schools for lower-attaining students have on average scored higher than the pre-service teachers (T= 2.196, df=229, p<.05, d=0.47), but lower than their colleagues at academic track secondary schools (T=5.111, df=80, p<.001, d=1.23). Thus, the division into groups according to CK scores as done above leads to a rather uneven distribution of the subsamples, so that most of the in-service teachers can be found in the group with higher CK scores, whereas the majority of the pre-service teachers belongs to the other group.

Subsample 1: Pre-service teachers

Subsample 2: In-service teachers at academic track secondary schools

Subsample 3: In-service teachers at secondary schools for lower-attaining stud.

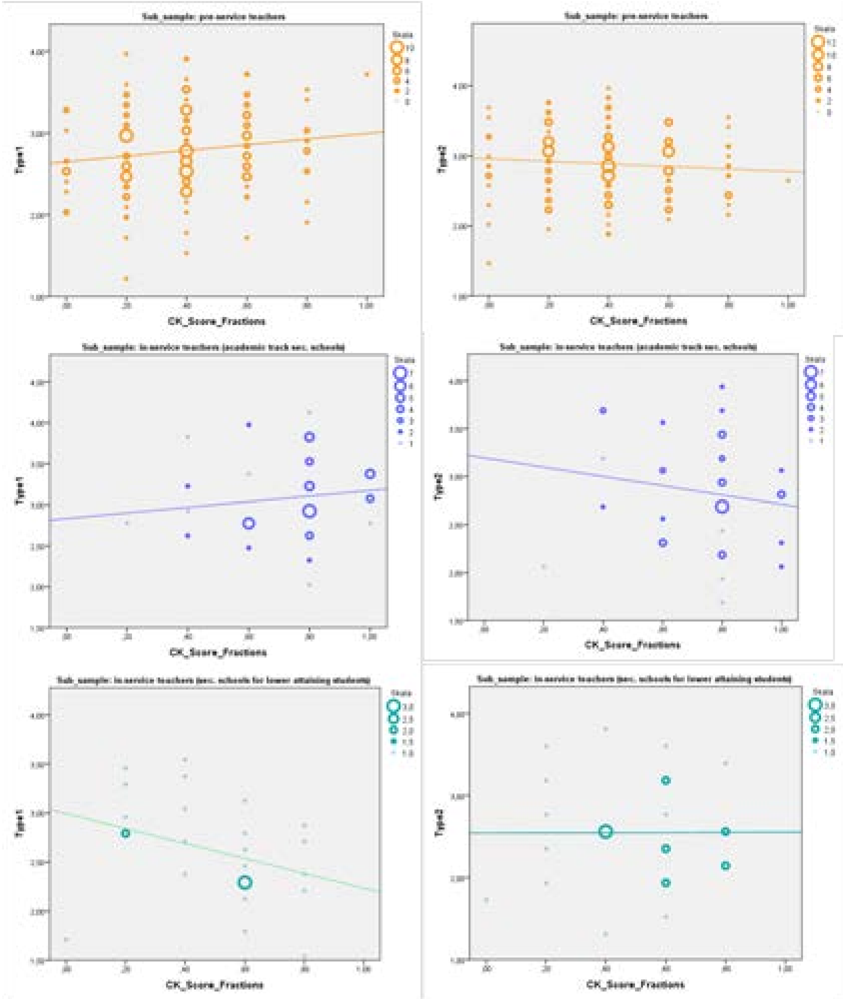


Figure 9: Content-specific CK scores and task-specific views according to subsamples

However, since CK may not be the only aspect in which pre-service teachers differ from in-service teachers, it doesn't necessarily have to be the specific CK that is decisive for the distinct task-specific views of the two groups. Hence, in order to investigate possible interrelations between specific CK and the evaluations of different types of tasks, one actually has to look at each subsample separately. This is done in Figure 9, which shows scatter plots according to subsamples. Regarding the pre-service teachers and the in-service teachers for academic track secondary schools in the sample, a relationship is visible to some extent, whereas it cannot be found with respect to teachers working at secondary schools for lower-attaining students.

DISCUSSION AND CONCLUSIONS

The interplay between situatedness and globality is a challenge for teachers in their everyday work: e.g. specific and individual classroom situations with learners, snapshots of learning processes seen through the lens of interactions in the classroom on the one hand and the rather non-individual contents, the relatively 'stable' and 'global' mathematical knowledge catalogue on the other hand may be seen in contrast to each other, and the teacher's role is to bridge this gap in order to support the students’ (individual) learning. This contrast is in a way ‘mirrored’ in the teachers’ professional knowledge. Consequently, research about professional knowledge on different levels of globality respectively situatedness can help to describe interdependencies between these two poles.

The present study aims at explaining task-specific views through more general characteristics of the tasks which are linked to the way they make use of representations. Hence, the overarching idea of using multiple representations is reflected in the tasks (to different degrees). Against this background, one of the major results of this study is that the overarching idea can explain task-related views even to the extent that they form scales according to task types.

As knowledge about using multiple representations in teaching and learning situations can be considered as PCK, this empirical structure makes the analysis of task-specific views to an indicator of domain-specific PCK. The results suggest that the subsamples differ with respect to this aspect of PCK.

However, even if these subsamples also differ with respect to their content-specific CK, CK differences are not sufficient for explaining differences in task-related views, as the study shows for the example of using multiple representations in the content domain of fractions. Beyond a base of CK, domain-specific PCK appears as a professional knowledge component of its own right.

A key follow-up question concerns the further exploration of such PCK and its structure. Corresponding evidence may come from an analysis of other questionnaire sections, which is currently being carried out.

ACKNOWLEDGEMENTS

The data gathering phase of this study has been supported in the framework of the project ABCmaths which was funded with support from the European Commission (503215-LLP-1-2009-1-DE-COMENIUS-CMP). This publication reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

We further acknowledge the support by research funds of Ludwigsburg University of Education (project La viDa-M) during the data analysis phase.

REFERENCES

Ball, D. L. (1993). Halves, pieces, and twoths: Constructing representational contexts in teaching fractions. In T. Carpenter, E. Fennema, & T. Romberg, (Eds.), *Rational numbers: An integration of research* (pp. 157–196). Hillsdale, NJ: Erlbaum.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.

Dreher, A. (2012) Vorstellungen von Lehramtsstudierenden zum Nutzen vielfältiger Darstellungen im Mathematikunterricht. *Beiträge zum Mathematikunterricht 2012*. Münster: WTM-Verlag.

Dreher, A., & Kuntze, S. (2012). Pre-service teachers' views about pictorial representations in tasks. Proceedings of the 12th Int. Congress on Mathematical Education. Topic Study Group 27 (pp. 5705-5713). Seoul, Korea COEX.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, 61, 103–131.

Gagatsis, A., & Shiakalli, M. (2004). Translation ability from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology*, 24(5), 645–657.

Goldin, G., & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The role of representation in school mathematics* (pp. 1–23). Boston, Virginia: NCTM.

Kultusministerkonferenz (KMK). (2003). *Bildungsstandards im Fach Mathematik für den Mittleren Schulabschluss*. [retrieved on 13.09.2012 from http://www.kmk.org/].

Kuntze, S. (2011). In-Service and Prospective Teachers' Views about Modelling Tasks in the Mathematics Classroom – Results of a Quantitative Empirical Study. In G. Kaiser et al. (Eds.), *Trends in Teaching and Learning of Mathematical Modelling* (pp. 279-288) Dordrecht: Springer

Kuntze, S. (2012). Pedagogical content beliefs: global, content domain-related and situation-specific components. *Educ. Studies in Mathematics*, 79(2), 273–292.

Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H.-S. Winbourne, P. (2011). Development of pre-service teachers' knowledge related to big ideas in mathematics. In B. Ubuz (Ed.), Proc. of the 35th Conf. of the Int. Group for the Psychology of Math. Education, Vol. 3 (pp. 105-112). Ankara, Turkey: PME.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Hillsdale, NJ: Lawrence Erlbaum.

Pajares, F. M. (1992). Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct. *Review of Educational Research*, 62(3), 307-332.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.

Törner, G. (2002). Mathematical Beliefs – A Search for a Common Ground. In G. Leder, E. Pehkonen, & G. Törner (Eds.). *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 73–94). Dordrecht: Kluwer.