ENHANCING MATHEMATICS STUDENT TEACHERS' CONTENT KNOWLEDGE: CONVERSION BETWEEN SEMIOTIC REPRESENTATIONS

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An innovative course in mathematics teacher education is currently running at University of Agder. One of its aims is to enhance student teachers' content knowledge and to develop awareness of the specificity of mathematics as a subject-matter. Results from the research presented in this paper show student teachers' reflections as they engage with mathematical tasks addressing the transition between semiotic registers. Implications for teacher education programs are discussed.

INTRODUCTION

How do we, as mathematics teacher educators, face the challenge of enhancing student teachers' content knowledge? And how do student teachers evaluate their own engagement with mathematical tasks addressing the idea of conversion between semiotic representations? In order to address these questions, I present the main aspects of an innovative course in teacher education currently running at the University of Agder (UiA), Norway. The aims of the course are to strength mathematics teacher education at UiA by making explicit the link between theory (results from research in mathematics education) and teaching practice, and to facilitate the transition from being a student teacher to becoming an in-service teacher (Grevholm, 2003, 2010). In addition the course aims at bringing the specificity of mathematics as a subject-matter to the fore by adapting and designing tasks as a means to enhance student teachers' awareness of the importance and the relevance of the use of semiotic representations (Duval, 1995, 2006). This course is based on a research project called "Inquiry-based mathematics teacher education: preparing for life-long learning", (the IBMTE project from now) and it is elaborated in collaboration with Barbro Grevholm. The ideas developed in our research project are rooted in our earlier research in teacher education (Grevholm, Berg, and Johnsen, 2006) where the idea of inquiry plays a central role. It was during spring 2011 that we initiated the IBMTE project and it is currently running at UiA.

In this article my focus is on student teachers' reflections on mathematical tasks which were designed as a means to develop both their awareness of the specificity of mathematics as a subject-matter, and to engage in reflecting on the use of semiotic representations. Based on this approach the research questions addressed in this paper are: What is the nature of student teachers' reflections as they engaged with a task designed to encourage exploring the transition between different semiotic representations? And do student teachers recognise the importance of using different representations? The structure of the paper is as follows: First I present the rationale of the IBMTE project emphasising the role played by the ideas of "inquiry" and "transition between semiotic representations" in mathematics teacher education, considering a perspective both from student teachers' content knowledge and didactical knowledge (Durand-Guerrier & Winsløw, 2005). Then I turn to analysing student teachers' reflections after engaging with a mathematical task and discussing the relevance and meaning of the transition between semiotic representations. The data presented in this article are from student teachers' answers to a questionnaire and to an interview conducted with two students. I conclude by discussing possible development of and implications of this research for mathematics teacher education. Especially I argue for recognising the importance of developing student teachers' awareness of both the relevance and the use of the transition between different semiotic representations.

THE MAIN ASPECTS OF THE IBMTE PROJECT

Rationale for the IBMTE project

Our research project emerges from the recognition that teachers and teaching plays a central role in creating rich learning opportunities for pupils (Jaworski, 2006), and therefore we consider that it is important that student teachers experience a rich and stimulating learning environment. From studies in teacher education it is well documented that the gap between theory and practice often creates a reality-chock for new educated teachers (Grevholm, 2003, 2010). Therefore we aim at designing a course that could enable new educated teachers to see how they could engage in a continuous learning process, as in-service teachers, and use tools like "inquiry cycle at three levels" as a means to engage in life-long learning while recognising the importance of results emerging from research in mathematics education. In Norway there is a demand for developing teacher education programs which are research based. However the consequences and implications of this claim are not clear. The IBMTE project and the innovative course in mathematics teacher education emerging from this project are to be considered as an attempt to address this challenge. Finally we are convinced that as we bring the idea of *inquiry* to the fore, we introduce the dimension of reflection at three levels, as explain below, an aspect which we consider as central to the education of future mathematics teachers.

Inquiry cycle at three levels

The idea of inquiry is crucial in the IBMTE project and it refers to a cyclical process consisting of asking questions, recognising problems, exploring, investigating, seeking answers and solutions and thereby engaging in an inquiry cycle (Berg, 2011b, 2011c, in press). This process is understood at three levels: at the first level *inquiry in mathematics* as the student teachers engage in mathematical tasks; at the second level *inquiry in teaching mathematics* as student teachers engage in looking critically and reflecting on their own teaching practice during their practice period; and at the third level *inquiry in student teachers' own reflections* as they engage in looking critically at their own development as future mathematics teachers. Our aim is that student teachers recognise inquiry as being a useful tool and gradually move into considering “inquiry as stance” (Cochran-Smith & Lylte, 1999) or “inquiry as a way of being” in practice (Jaworski, 2006, 2008). We consider as important to offer student teachers’ opportunities to engage in inquiry cycle as a means to raise their awareness, as future teachers, of the specificity of mathematics as a subject matter.

The specificity of mathematics as a subject-matter

What is the specificity of mathematics as a subject-matter and what are the consequences of this specificity for learners of mathematics? On the contrary to physics, chemistry or biology where the objects of study are accessible either by perception or by instruments, the mathematical objects like numbers, functions, geometrical shapes or vectors are not directly accessible. The only possibility to have access to these objects and to perform some operations on them (calculations) is to introduce signs and semiotic representations (Duval, 1995, 2006). As a consequence learners need to use signs, diagrams, figures, and notations which stand for and represent the mathematical object at study. More generally his approach emphasises the role played by visual representations in the learning of mathematics (Arcavi, 2003). For example, according to Duval (2006), the algebraic expression of a function, a table of the different values of the function, a text describing a situation, and a graph are four different representations of the mathematical idea of function (see Figure 1). Other representations are possible, as a correspondence between sets.

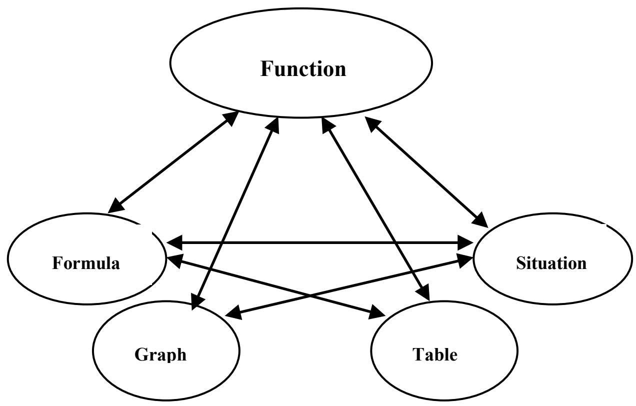

Figure 1: Distinguishing the mathematical object “function” from its representations (Berg, in press)

Duval (2006) introduces the distinction between the concepts of *treatment* and *conversion* where the former is defined as “transformations of representations that happen within the same register” while the later refers to “transformations of representation that consist of changing a register without changing the objects being denoted” (p.111-112). Furthermore the nature of the treatments which can be carried out within a register are depending of the possibilities of semiotic transformations and these are specific to the used register. Conversion refers to “a representation transformation which is more complex than treatment because any change of register first requires recognition of the same represented object between two representations whose contents have very often nothing common" (p.112). For example the process of solving an equation belongs to treatment while the transition from the algebraic notation of a function to its graph brings the necessity of using two different semiotic registers and illustrates the notion of conversion. According to Duval (2006) the teaching and learning of mathematics witnesses a lack of recognition of the specificity of mathematics as a subject-matter and he argues that

> Changing representation register is the threshold of mathematical comprehension for learners at each stage of the curriculum. It depends on coordination of several representation registers and it is only in mathematics that such a register coordination is strongly needed. Is this basic requirement really taken into account? Too often, investigations focus on what the right representations are or what the most accessible register would be in order to make students truly understand and use some particular mathematical knowledge. With such concern of this type teaching goes no further than a surface level. … The true challenge of mathematics education is first to develop the ability to change representation register (p.128).

As teacher educators we need to take Duval's claim seriously into account and to avoid that student teachers engage in future in teaching mathematics at a surface level and without recognising its specificity. This acknowledgment brings with it clear demands to teacher education as to create new opportunities for student to engage in situations where they are confronted with different semiotic representations and need to move between different registers. Furthermore I see Duval's distinction between treatment and conversion as related to the syntactic and semantic aspects of algebra. *Syntactic level* refers to the organisation and transformation of symbols following specific rules of manipulation, while the *semantic level* addresses the meaning endorsed by the symbols and by the expressions (Berg, 2009; Drouhard & Teppo, 2004; Puig & Rojano, 2004).Thereby the process of treatment is deeply related to the syntactic aspect of the expression at study (symbols manipulation), while the process of conversion is rooted in the ability of addressing the semantic aspect (developing awareness of the meaning of symbols, expressions and figures). I argue that bringing to the fore the specificity of mathematics as a subject-matter, as explained above, has deep implications for mathematics teachers' education. More specifically the design of activities aiming at developing student teachers' awareness of the importance of semiotic representations in the learning and teaching of mathematics is central as a means of enhancing their content knowledge and didactical knowledge (Durand-Guerrier & Winsløw, 2005).

Methodological considerations and research setting

The methodology within the IBMTE project follows a developmental research approach. This implies that Barbro Grevholm and I, we engage in studying, documenting and researching student teachers' development and, at the same time, we recognise that our research activity contributes to that development (Goodchild, 2008). Concerning methods used in this research, the data was collected as part of the first teaching period with student teachers during fall semester. The semester is organised as follows: student have first one week with teaching at the university and then three weeks with teaching practice in different schools, both primary and secondary, around Kristiansand. It was during the first teaching period that I decided to introduce a task related to the process of conversion between different semiotic representations as a means to address the specificity of mathematics as a subject-matter and to address Duval's claim concerning developing the ability to change representation register. Inspired by a task called "multiplied representations" (Swan, 2006), I designed an activity consisting of putting together cards representing the same mathematical idea, but presented in different ways (algebraic notation, sentence with words, geometric shape). I consider that this task offers a rich opportunity to discuss and compare, for example, the meaning of $3n^2$ compared to $(3n)^2$. Thereby student teachers need to consider not only the syntactic aspect of algebraic notation (calculation and manipulation of expressions) but also the semantic aspect, which corresponds to the meaning of these expressions (Berg, 2009; Drouhard & Teppo, 2004; Puig & Rojano, 2004). I present the following three corresponding cards as a complete and coherent set in Figure 2:

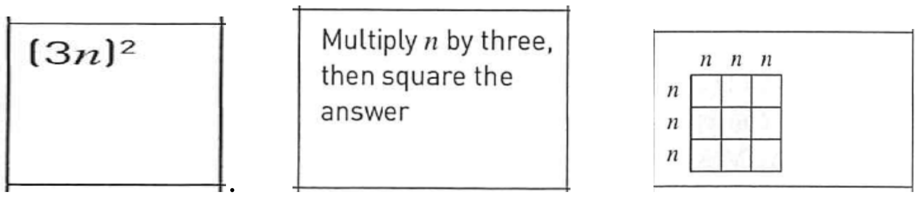

Figure 2: Example of the multiple representations task (Swan, 2006)

In addition I decided to give students some cards which were empty and where they had to draw the corresponding geometrical shape, to find the corresponding algebraic expression, or to formulate an adequate sentence with words in order to have a complete and coherent set of cards. This activity formed a basis from which I could introduce Duval's theory and emphasise results emerging from his research (importance of distinguishing a mathematical concept from its representation and developing the ability to move between several representations). The task was proposed to 52 student teachers and they were sitting in groups of three or four during the activity. I could follow how the task stimulated a lot of discussions as they tried to find corresponding cards and to make sets addressing the same mathematical idea. Right after the activity I distributed a questionnaire where I encouraged the student teachers to reflect on how they experienced this task and if they could see it as useful to introduce in their own teaching. In addition I present excerpts from an interview conducted at the end of the semester where 4 student teachers who volunteered to reflect on their engagement with the task. In the following section I present an analysis of both student teachers' answers to the questionnaire and reflections during the interview.

Conversion between semiotic representations: Student teachers' reflections

Results of the analysis of data show that students' answers to the questionnaire can be grouped in three categories: those who are very positive to the task (39), those who think it was a nice task (11) and some few who find it not interesting and difficult (2). In the following I present results from the analysis according to these three categories, and results from the interview with two student teachers.

Analysis of student teachers' answers from the first group

Answers from that group are very positive and the students offer the following explanations

Student 1: I think this task was very funny because we get different perspectives within the same task. Get to look at the algebraic expressions in another way, what we actually did. It is easy to see the difference between $3n^2$ and $(3n)^2$ here. The cards with text help us about how to read these expressions. To draw was a nice exercise. It was not so difficult to find the cards which belong to each other, it was more challenging to draw [the corresponding geometrical shapes]. We could test if we understood what is happening

Student 8: Interesting because we had to justify why we think it was correct, and sometimes we saw that it [the drawings] were not correct. It was unusual and difficult to see how the answer should be, especially when we had to draw [a geometrical shape].

Student 19: Very exciting. Here we need basic understanding of what a sentence, a formula or a drawing means and represents. We get a larger overview over the connection [between these representations] and also the fact that these are the same even if they look different.

Student 15: I think it was a very interesting task because we really had to think hard to be able to draw the missing cards or to write the corresponding text. It was in fact challenging.

Results of the analysis indicate that the following aspects are in common in many student teachers' answers from this first group. Most of them report on the fact that the transition between algebraic expression and the corresponding text is not so difficult, but the challenge comes from finding or drawing the corresponding geometrical shape. In addition many characterise the task as offering a very nice opportunity to develop understanding of an algebraic expression. The aim of the task is to recognise that the cards belonging to a set, as in Figure 2, represent the same mathematical idea. Thereby these cards offer an illustration of the different semiotic representations as an algebraic expression, a sentence written with words and a geometrical shape. The focus is on recognising the transition between these semiotic representations or on the process of conversion (Duval, 1995, 2006). Furthermore, it seems that most of the student teachers have less difficulty in establishing a link between an algebraic expression and a text than linking these to a geometrical shape.

This recognition emphasises the lack of seeing mathematics as a coherent subject-matter and not only as consisting as blocs (algebra, geometry, statistics) with no connections between each other. Most of the students use the expression "developing understanding" and my interpretation is that the task encourages them to explore the semantic aspect of algebra (Berg, 2009; Drouhard & Teppo, 2004; Puig & Rojano, 2004) as represented on the cards. Thereby, the focus is now on *the meaning* of the algebraic expression and not only on the computational aspect (syntactic aspect of algebra) of the expression and it seems that this element is new for these students.

Analysis of student teachers' answers from the second group

In this group the student were also positive to the task but it is possible to see in their answers that the challenges they met

> Student 2: I think it was ok to put together the corresponding cards, but when we had to write [the algebraic expression] or to draw [the geometrical shape] it became difficult. A lot of confusion.

> Student 13: I think it was nice to see the connection between a figure, a text and an expression. It helps with understanding and one can do not make mistakes because one can see that the expression/text/figure do not fit together.

> Student 42: I think the task was ok. It was challenging, especially because it is a while since I have working with this.

> Student 49: It was ok, but the problem is that my knowledge in mathematics is not quite good yet.

In this group it seems that the student teachers still agree about the usefulness of the task as offering an opportunity to see connections between different semiotic representations. At the same time they recognise the challenges they meet while engaging with it and report on "a lot of confusion", "challenging". One of them acknowledge the lack of having a robust knowledge in mathematics and my interpretation is that since the task does not address usual questions involving calculations and manipulations of expressions (treatment related to the syntactic aspect of algeba) students get confused and fail in developing understanding of the meaning of the different expressions (semantic aspect of algebra). Thereby they are not able to see that the same mathematical idea is illustrated through the different representations (conversion) and to group the cards in coherent sets.

Analysis of excerpts from the interview

I chose this short excerpt as it offers deeper insights into the reason why student teachers found this task both interesting and difficult

> Student B: It was a nice task, but it was difficult.

> Student A: Yes, and I realised very clearly what was difficult for me. Because I had difficulties with the geometrical shape, where you could see how big it [the

expression] was. That part was a real challenge for me. I think it was difficult. For me algebra is so easy, I can see it at once, but this geometrical shape, it's all Greek to me. Then I could see clearly where my limits are….

Student B: Maybe we are at that level [draw a horizontal line with her hand], the level of formula and we do not manage to get down to the practical level.

Student A: …. yes, but it was also because I think, well I mean, for me this was unnecessary to have that kind of representation.

According to these two students the difficulties appear when they tried to establish a link between the proposed geometrical shape and the two other representations. It seems that, for these two students, a geometrical shape belongs to another level, "the practical level", and stands in contrast to a more abstract level "the level of formula". Thereby they could not see the necessity of introducing this kind of representation in the task and it provoked difficulties rather than bringing deeper understanding of the meaning of the algebraic expressions (semantic aspect). These insights are important for us, as teacher educators, as we aim at enhancing student teachers' content knowledge and I will comment on this aspect further in the last section.

Analysis of student teachers' answers from the third group

Results of the analysis shows that there are only two students in this group

Student 21: Difficult, don't remember basic knowledge in mathematics.

Student 44: Quite boring and too difficult.

It seems that these two students agree about the level of difficulty of the task, but unfortunately they do not offer deeper considerations about the nature of these difficulties. My interpretation is that the way the task is formulated is unusual for them and due to a lack of mathematical content knowledge they were not able to engage with it. It is also interesting that student 21 explains his answer by referring to the lack of "remembering basic knowledge in mathematics". Here it seems that mathematical knowledge is understood as a process of memorising.

Discussion and conclusion

My aim with this article is to present how student teachers evaluate their engagement with a mathematical task aimed at illustrating the conversion between different semiotic representations. The analysis of student teachers' answers both to a questionnaire and during an interview shows that most of the students recognise the task as interesting but challenging. More particularly they express difficulties at two levels: firstly in recognising geometrical shapes as useful representations and secondly in deciding which geometrical shape corresponds to the two other representations. In addition, results seem to indicate that a conversion between an expression with words (a sentence) and an algebraic expression is understood as a process between entities at the same level of abstraction, while a conversion involving a geometrical shape introduces a level of different nature, "the practical level". I consider these results as interesting as it brings to the fore a demand for us, as teacher educators, to justify the use of several semiotic registers and to make explicit the important relation between algebraic expressions and geometrical shapes. As mentioned above, this activity created a lot of discussion from which Duval's theory was introduced. Another challenge appeared as new terminology (conversion, treatment, semantic and syntactic aspects) was introduced to student teachers.

One of the aims of our IBMTE project is to enhance student teachers' content knowledge and didactical knowledge by making explicit the link between theory and teaching practice. This is done through bringing to the fore the specificity of mathematics as a subject-matter and emphasising the need for using semiotic representations as a means to address mathematical objects at study. Especially the notions of treatment and conversion are central to the learning and teaching of mathematics and the research presented in this article illustrates how student teachers engage with a task designed in order to encourage students to explore the transition between different representations. I argue that in order to develop deeper and relational understanding of mathematical objects, and thereby to address both the semantic and the syntactic aspect of algebra, it is necessary to offer student teachers rich learning opportunities where the specificity of mathematics as a subject-matter is brought to the fore and discussed. According to Duval (2006) changing representation register constitutes the true challenge of teacher education and I consider that his claim needs to be seriously addressed by teacher educators. The research presented in this article is an attempt to face this challenge.

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