TEACHERS AS INVESTIGATORS OF STUDENTS' WRITTEN WORK: DOES THIS APPROACH PROVIDE AN OPPORTUNITY FOR PROFESSIONAL DEVELOPMENT?

Sinem Bas[1], M. Gozde Didis[1], A. Kurşat Erbas[1], Bulent Cetinkaya[1], Erdinc Cakiroglu[1], Cengiz Alacacı[2]

Middle East Technical University[1], Istanbul Medeniyet University[2]

The aim of the study was to examine how teachers' investigation of students' written works contributed to their professional development. The research was conducted at a public high school with the participation of six mathematics teachers and their students. The teachers have examined their students' written works as products of their solutions for some modeling problems for 5-week period. The preliminary analyses showed that teachers' collective examinations and interpretations of their students' written works have contributed to their professional development in terms of their subject matter knowledge and pedagogical content knowledge. Moreover, the collaborative learning environment in the study had positive effects on the affective domain regarding teachers' ways of knowing of students' thinking.

INTRODUCTION

This study reports on preliminary findings from a study carried out as a part of a larger research project about mathematical modeling where the primary purpose is to develop pre-service and in-service mathematics teachers' knowledge and skills about using modeling problems in teaching mathematics. The main three components of inservice teacher education dimension of this program were (a) planning a lesson in which a modeling problem was integrated, (b) implementing the problem in the classroom, and (c) investigating/assessing students' works in modeling problems. The focus of this study is on the third component.

More precisely, the purpose of this study was to examine how teachers' investigation of students' written work contribute to their professional development in terms of their subject matter knowledge and pedagogical content knowledge. The following research question guided this study: How does teachers' collective investigation of students' written works contribute to their professional development in terms of subject matter knowledge and knowledge of students' thinking as a sub-component of pedagogical content knowledge?

Teachers' Knowledge of Students' Thinking

It is widely accepted that teacher knowledge comprises three major dimensions: subject matter knowledge (SMK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK) (e.g., see Shulman, 1986). In particular, teacher knowledge of students' mathematical thinking has attracted much interest. In this regard, teachers' knowledge of students' conceptions, difficulties and potential misunderstanding have been the subject of various studies as an important aspect of teachers' pedagogical content knowledge (e.g., Carpenter et al., 1996; Chamberlin, 2002; Gearhart & Saxe, 2004) as it could offer crucial contributions to teachers' professional growth. According to Gearhart and Saxe (2004), knowing what students know is crucial for effective classroom practice. When teachers attend to students' mathematical thinking, they can prepare the instruction with respect to their needs and level of understanding, emphasize important mathematical ideas, understand students' misconceptions and create learning environment to foster students' mathematical ideas (Kulm, Capraro, Capraro, Burghardt, & Ford, 2001 as cited in An, Kulm & Wu, 2004). All these studies highlighted that when teachers attend to and understand their students' thinking, both their instructional practice and students' achievement can benefit (Chamberlin, 2002)

However, although teachers' understanding of students' mathematical thinking is important for teachers to teach mathematics effectively, teachers usually had poor subject matter knowledge and their knowledge was not connected to students' thinking (e.g., Nathan & Koedinger, 2000; Bergqvist, 2005). Ball (1997) indicated that during classroom teaching, attending students' thinking was not easy because teachers often could not find an opportunity to interact with all of their students in their own classrooms. Additionally, teachers have difficulties in identifying students' ways of thinking as students' thinking can change under different circumstances or they cannot always express their thinking although they construct the idea in their mind (Ball, 1997; Chamberlin, 2002). Because of these reasons, it may be difficult for teachers to understand students' mathematical thinking.

All of these are particularly true when teachers attend to students' reasoning in modeling problems. One of the main features of these problems is that they allow students to think about the problem differently and solve it in different ways. Thus, teachers are confronted with different ways of student thinking when they attempt to understand students' reasoning while implementing the problem and assessing students' written works after the implementation. This is quite challenging for teachers (Doerr, 2007).

Professional Development Approaches That Focus on Student Thinking

In reviewing the works on the professional development of mathematics teachers Sowder (2007) contented that one of the main goals of professional development is "developing an understanding of how students thinking about and learn mathematics" (p. 163). The studies that use students' ways of thinking on mathematical concepts as a tool for improving teachers' content knowledge and pedagogical content knowledge are designed based on the fact that attending students' mathematical thinking provides benefits for effective instruction and thus help support increasing students' achievement.

Ball (1997) suggests three approaches to improve teachers' attending to students' thinking. The first approach is "discussing cases of students' thinking" (p.808). In this approach, teachers work together with written cases of episodes of students' thinking, and they investigate and find different interpretations of students' thinking. The second approach is "using of redesigned curriculum materials" (p.808), and the third approach is "investigating artifacts of teaching and learning" (p. 811). The third approach is related to teachers' examination of "unnarrated" students' works and thoughts. Unlike the other two approaches, in the third approach, teachers engage in the artifacts obtained from real classroom settings, such as videotaped classroom lessons, students' written works and drawings on a paper or a chalkboard. Therefore, this approach is more realistic and provides teachers the opportunity to examine actual student products which are not interpreted by somebody else (Ball, 1997). The main idea of this approach can be seen in different professional development programs. For instance, *Cognitively Guided Instruction* (CGI) (Carpenter, Fennema, & Franke, 1996), *Multi-tier Program Development* (Koellner-Clark & Lesh, 2003), and *Integrating Mathematical Assessment* (IMA) (Gearhart & Saxe, 2004) can be considered as well-known examples of such programs. All of these programs acknowledged the importance of teachers' attending to and understanding students' mathematical thinking, although they used different approaches to develop it. For example, CGI was based on using the research-based knowledge of students' ways of thinking about a certain mathematics topic (e.g., addition and subtraction) to enhance teachers' instruction. As an alternative to CGI model, the professional development program of Koellner-Clark & Lesh (2003) situated in Models and Modeling Perspectives was grounded on providing opportunities for teachers to work on their students' works rather than providing research-based knowledge to teachers. According to this professional development approach, changes in teachers' knowledge and ultimately in their views about their teaching is possible only if they engage in situations where their existing knowledge is challenged and thus they experience some kinds of cognitive conflicts. The way of doing this includes activities of giving teachers tasks where they are required to interpret students' mathematical thinking (in modeling problems), and having teachers create conceptual tools (e.g., student thinking sheets, concept maps, etc.) to use in teaching practice (Koellner-Clark & Lesh, 2003). In this study, we adopted the *Multi-tier Program Development* (Koellner-Clark & Lesh, 2003) by having teachers engage in activities in which they investigate and think about students' modeling processes. Our main intention in this design was to foster teachers to think about their knowledge for teaching mathematics.

METHOD

Participants

This research was conducted at a public high school during spring semester of 2010-2011 school years. The school was selected because it was one of the distinguished schools with a well-organized workgroup of teachers in order to meet regularly across the semester. In addition, this school was willing to open their doors to carry out the study.

Six of the mathematics teachers in the school and their students participated in the study. The teachers had more than 8 years of experience and four of them had master degree in mathematics. On average, the teachers had strong mathematical knowledge and their students had quite high achievement level relative to other schools in the district.

Procedures

The teacher investigations lasted four weeks. Before the first week of the investigations, the introductory meeting was conducted with all teachers. In this meeting, the outline of the 5-weeks program was explained, the Student Thinking Sheet (STS) was introduced, and modeling problems that would be implemented in the classrooms were determined with the teachers.

During the 5-weeks period, each week a modeling problem was implemented in two classrooms at the same grade level by two teachers. Students worked on the problems in groups of 3 or 4 students during the two class periods (i.e., 90 minutes).

Before implementing a modeling problem in the classrooms, teachers were asked to solve the problem like a student and to create pre-implementation STS individually according to their predictions. After the modeling problem was implemented by two teachers in their classrooms, student works were collected and copies of them were provided to the teachers. Then, teachers were asked to examine the students' works in depth and to create individually post-implementation STS.

Next, teachers met for the follow-up meetings that lasted about 90 minutes. In these meetings, the teachers evaluated the classroom implementations. However, teachers who implemented the modeling problems shared their opinions about the tasks with other teachers and their experiences during implementation. These deliberations lasted approximately 15-20 minutes. Then, teachers interpreted their students' thinking strategies by the help of the STS. At this stage, teachers sequentially were asked to express verbally their individual written notes on the STS and to share with other teachers. Initially, the teachers, who conducted the classroom implementations, shared their thoughts. During this process, teachers showed examples of students' works while presenting their observations. In this way, teachers discussed their students thinking and collaboratively produced a shared STS which included students' fundamental thinking strategies.

Data Sources

Student Thinking Sheet (STS)

Student Thinking Sheet (STS) is a form designed to help teachers to think about and document students' mathematical thinking. It consists of a two-page document formatted as a table. The first page is divided into rows in which teachers are required to report different solution strategies used by their students in working on modeling problems. The table in the second page includes sections in which teachers are asked to report mathematical concepts, skills and process as well as students' errors and misconceptions for each different solution strategy.

Modeling Problems

For this study, five modeling problems were used to examine the interpretation of teachers' to understand their students' ways of thinking in certain mathematical situations. Modeling problems are non-routine tasks and they differ from the traditional textbook word problems. In each of these modeling problems, students interpret a complex real-world situation and formulate a mathematical description; therefore, what students produce has to go beyond short answers (Lesh & Doerr, 2003).

Weekly Meetings

The aim of the weekly meetings was to help teachers to examine and interpret students' different solution strategies, their misconceptions and errors. Each meeting lasted approximately 90 minutes and was audio and videotaped. In these meetings, the researcher (the first author of the study) had a facilitator role and managed the group discussions. While teachers were sharing their thoughts about students' thinking strategies, the researcher posed some questions like "What do you think about the students' mathematical thinking underlying this strategy?", "What do you think about the effectiveness of this strategy to solve the problem" in order to encourage teachers to think about students' ways of thinking more deeply.

Interviews

For this study, two different types of interviews were carried out. The first type of interviews was conducted with the teachers who implemented modeling problems in their classrooms. These interviews occurred before and after teachers implemented modeling problems in their classrooms and they lasted approximately 20 minutes. In pre-implementation interviews, teachers were asked about their predictions and expectations about students' ways of thinking; e.g., "What are your predictions about solutions students would have?" and "What are your predictions about students' difficulties and errors?" On the other hand, in post-implementation interviews, teachers were asked questions such as “Were there any ways of solutions which you found surprising? If any, what were they? Please explain them briefly” or “Which important mathematical ideas did students reach at the end of the process?

The second type of interviews was conducted individually by all six teachers at the end of the semester and lasted approximately 40 minutes. The aim of the interviews was to inquire into how to examine students’ thinking and work on STS and to see how attending weekly teaching meetings affect teachers’ professional development. During the interviews, teachers were asked questions such as “Could you compare your interpretation of students’ ways of solutions in the first week to the last week” or “How was this experience for you? Each interview was audio-taped and transcribed verbatim.

Data Analysis

The analysis of data was completed in two stages as pre-data analysis and in-depth data analysis. The pre-data analysis of the study has already beendone and the in-depth analysis of data is in progress. For pre-data analysis, all written, audiotaped and videotaped data were examined carefully and organized by the researchers to prepare for analysis. Although students’ written works produced during implementations were not the main source of data, they were used to provide background for the data analysis. Therefore, students’ works were first examined by the researchers to determine the required data. The video recordings of the weekly teachers’ meetings were carefully watched and notes were taken. Next, field notes were looked over; the transcripts of interviews and “Student Thinking Sheets” were reviewed. In this way, the initial codes were determined. In order to construct actual codes and present certain findings of the study, in-depth analysis of data will be conducted.

RESULTS

Pre-analysis of the data indicated that teachers’ collective examinations and interpretations of their students’ written works provided contributions to their professional development both in terms of subject matter knowledge and pedagogical content knowledge. Besides, there were some affective contributions of this type of collaborative learning environment to teachers’ professional development. Despite the fact that finding out probable contributions of a professional development environment to teachers’ knowledge requires a long-term study, the four-week study provided some clues about these contributions as follows.

Contributions to subject matter knowledge:

When teachers were trying to interpret students’ different solution strategies from their written works, they were also trying to understand the mathematical aspects of these strategies. While they discussed the mathematical thinking underlying the strategy, they reveal their own mathematical knowledge and shared it with their colleagues. Especially when teachers detected some errors in students' solutions, they discussed what the mathematical sources of the error were. These discussions provided them with meaningful opportunities to reveal and improve their own subject matter knowledge. For example, the following excerpt shows that teachers discuss deeply on the mathematical concepts such as linear function, geometric series or slope etc. while they are investigating whether students' solution strategy is correct

Fevzi: Then, we accept the elements of the geometric series as linear. Is there such thing?

Huseyin: No no. But actually, this ratio is the slope, isn't it? Can I think of this ratio as the slope?

Fevzi: It is just like the terms of a geometric series.

Huseyin: The ratios that I took their ratios in the geometric series.

Fevzi: The r's

Huseyin: Are the r's slope? Can I think of them as slope?

Fikret: Is there any linearity at there?

Fevzi: So, if x-axis [*represents*] is the number of bounces and y-axis is the height. [*Thus*] Linearity does not hold.

Huseyin: No, then it is not. Is it quadratic, then?

Handan: Yes, it becomes quadratic. When it is linear, it has to decrease in a constant rate, doesn't it? Isn't it a property of a linear function?

Contributions to pedagogical content knowledge:

This study focused on teachers' knowledge of their students' thinking as one main dimension of their PCK. Data indicated that this type of collaborative investigation of students' works had potential to improve teachers' understanding of their students' thinking. Teachers' interpretations of students' written responses to the modeling problems improved noticeably from the first meeting to the last one. For instance, in the first meeting, although there were many different solution approaches in students' work, almost all teachers were not able to see and detect the different solution strategies except for two or three strategies. In the interviews, they explained this situation that they didn't know exactly what the focus of the activity was. Especially, when a student used a strategy different from their expected solution, they tended to ignore this strategy. Some teachers explained the reason of this situation as they were accustomed to assessing students this way in their daily teaching practice. That is, student work often did not get credit if it did not follow teachers' "expected" way of solution. But as meetings proceeded, they tried more to examine and understand different solution approaches even if they were incorrect.

Also, teachers could better see and interpret different solution strategies, mathematical ideas underlying these solutions and errors when they came across with them.

Additionally, the ways in which teachers interpreted and described students' solution approaches changed considerably from the first meeting to the last. For instance, at the beginning, they often looked only at what students did superficially. They did not tend to investigate what the mathematical thinking underlying the solution was, whether this solution was correct or not. As the following excerpt exemplifies, during the first investigation of students' written works teacher Handan just describe students' solution strategy superficially rather than making inference or providing detailed analysis.

> **Handan:** Students obtained a value with trial and error and then they transformed it into a formula to support it. So, at least the students began with the trial and error method but then they formulated it.

As meetings proceeded, teachers focused more often on the underlying thinking processes rather than just looking at what students did. They considerably tried to understand their students' thoughts.

Contributions to attitudes towards students and collaboratively working with a colleague:

Almost all teachers expressed the changes in their attitudes towards their students, especially towards those students who were not successful in standard tests and examinations. The following excerpt illustrated changes in a teacher's point of view towards their students.

> **Huseyin:** Now, I thought of that. These students shut down themselves after an hour. But, they solve whatever they want if they are provided with an appropriate question and the environment. They [*the project activities*] changed my thoughts about these students. Namely, yes, when you provide these children with an appropriate environment and offer them appropriate things there is nothing they would not do. Now, it has increased my respect to these children.

Accordingly, teachers explained that through these close examination of students' work, they knew their students better and they appreciated their different ways of thinking. They also expressed their thoughts and feelings about the collaborative working with their colleagues. Similarly, the data from the whole group discussion in meetings showed the benefits of discussions and exchanging ideas for better interpreting their students' thinking and filling the gaps in their subject matter knowledge.

DISCUSSION

Our findings showed that interpreting students' different and sometimes unusual strategies on thought revealing non-routine tasks were initially not so easy for teachers. This result is compatible with the findings reported by others (e.g., see Koellner-Clark & Lesh, 2003). However, as teachers analyzed students' work over time, they started to appreciate and understand students' solution strategies and also better interpret the mathematical ideas underlying these strategies. Like other professional development programs (e.g., Carpenter, Fennema, & Franke, 1996; Koellner-Clark & Lesh, 2003), these findings indicated that a learning environment in which teachers work collaboratively to analyze and interpret students' works on thought revealing non-routine tasks, provides crucial contributions to not only teachers' subject matter knowledge and pedagogical content knowledge but also to their attitude and beliefs towards teaching mathematics. On the other hand, collaborative work with colleagues on STSs and the nature of the modeling problems are two major factors that positively affect teachers' professional development. Nevertheless, when teachers did not solve the tasks themselves and/or come to the meetings without analyzing students' works carefully, they had difficulties in interpreting students' thinking. Therefore, as consistent with findings of Nathan and Koedinger (2000) and Bergqvist (2005), this study shows that such conditions can create barriers to provide adequate contributions to their professional developments while teachers investigate students' works. In addition, it is also observed that teachers need to spend adequate time to work on students' works for their professional development. In summary, this study contributes to our understanding of the literature on teacher knowledge and changes thereof in that investigations of students' works resulting from non-routine, thought-revealing activities could be a starting point for in-service teachers regarding what they know and think about their students' thinking and change their instructional practices.

NOTES

1. Work reported here is based upon a research project supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under grant number 110K250. Opinions expressed are those of the authors and do not necessarily represent TUBITAK's views.

2. The modeling problems used in this study were developed by the researchers working on this research project.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education, 7*, 145–172.

Bergqvist, T. (2005). How students verify conjectures: Teachers' expectations. *Journal of Mathematics Teacher Education, 8*, 171-191.

Ball, D. L. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In B. Biddle, T. Good, & I. Goodson (Eds.), *International handbook on teachers and teaching* (Vol. II, pp. 679-718). Dordrecht, Netherlands: Kluwer Press.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3-20.

Chamberlin, M. (2002). *Teacher investigation of students work: The evaluation of teacher's social process and interpretations of students' thinking*. Unpublished doctoral dissertation, Purdue University, Indiana, U.S.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. Galbraith, H. Henn, and M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 69-78). New York: Springer.

Gearhart, M., & Saxe, G. B. (2004). When teachers know what students know: Integrating assessment in elementary mathematics teaching. *Theory into Practice, 43*, 304-313.

Koellner-Clark, K., & Lesh, R. (2003). A modeling approach to describe teacher knowledge. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah: Lawrence Erlbaum.

Lesh, R. & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah: Lawrence Erlbaum.

Nathan, M. J., & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction, 18*(2), 209-237.

Ponte, J. P., & Chapman, O. (2002). Pre-service mathematics teachers' knowledge and development. In L. D. English (Eds.), *Handbook of International Research in Mathematics Education* (pp. 223-261). New York, NY: Routledge.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Charlotte, NC: Information Age Publishing.