PRE-SERVICE ELEMENTARY TEACHERS' PROCEDURAL KNOWLEDGE OF THE GREATEST COMMON FACTOR AND LEAST COMMON MULTIPLE

Jeffrey A. McLean

Syracuse University

Pre-service elementary teachers in the United States demonstrated on a survey and follow-up interviews what I categorized as either superficial procedural knowledge or deep procedural knowledge of the greatest common factor (GCF) and least common multiple (LCM). Those exhibiting deep procedural knowledge varied the processes used to find the GCF and LCM based on the numbers' representation. Those exhibiting superficial procedural knowledge followed the same procedure regardless of the numbers' representation. Statistically significant evidence suggested that pre-service elementary teachers who define the GCF (and LCM) through the relationships between two or more numbers and their GCF (and LCM) also demonstrate deep procedural knowledge of the GCF (and LCM).

REVIEW OF LITERATURE

Zazkis and Campbell (1996) investigated pre-service elementary teachers' understanding of elementary number theory concepts and determined that their respondents showed a disposition towards procedural thinking, even when they displayed a conceptual understanding of the topic. Hiebert and Lefevre (1986) characterize procedural knowledge as memorization of facts and algorithms used to solve mathematical tasks (1986). Star (2005) later argued for a reconceptualization of procedural knowledge and asserted that a distinction must be made between those learners with superficial and deep procedural knowledge: "There are subtle interactions among the problem's characteristics, one's knowledge of procedures, and one's problem-solving goals that might lead a solver to implement a particular series of procedural actions" (p. 409). If a solver possesses a superficial knowledge of the procedures s/he may fall back on the known standard procedure to solve the problem, regardless if it is the most efficient process. If the solver instead possesses deep procedural knowledge, s/he may use various techniques to produce a solution that best matches the form of the problem.

FINDINGS

Through use of a survey instrument and interviews, I assessed 48 pre-service elementary teachers' knowledge of the GCF and the LCM. The respondents demonstrated what I categorized as either superficial procedural knowledge or deep procedural knowledge. Items on the survey asked the participants to determine the GCF and LCM of two numbers given various representations of the numbers, such as the numbers' prime factorizations, lists of each number's factors, or lists containing the first 10 multiples of the numbers. The participants classified as demonstrating superficial procedural knowledge of the GCF and LCM applied the same procedure to each survey item regardless of the numbers' representation. Those exhibiting deep procedural knowledge varied the processes used to determine the GCF and LCM and applied more efficient methods based on the numbers' representation. For instance, a survey item asked respondents to find the GCF of two numbers that were represented by lists containing all of their factors. Those that I classified as demonstrating superficial procedural knowledge did not utilize this list, found the prime factorization for each number, and then used these prime factorizations to determine the GCF. Those that I classified as demonstrating deep procedural knowledge exhibited an understanding that the representation of two numbers as lists of their factors is transparent with respect to their GCF, and determined the GCF by finding the largest factor shared on each list of factors.

The survey also asked the participants to define the GCF and LCM. The responses fell into the following two categories: (1) those describing the relationship between two or more numbers and their GCF and LCM, and (2) those detailing a process that can determine the GCF and LCM for two or more whole numbers involving the numbers' prime factorizations. Using Fisher's exact test, I compared the relationship between the respondents' definitions of the GCF and LCM with the form of procedural knowledge that they demonstrated. The data revealed statistically significant evidence ($p=0.03$) suggesting that pre-service elementary teachers who define the GCF correctly through relationships also exhibit deep procedural knowledge of the GCF. Similarly, the data revealed statistically significant evidence ($p=0.04$) suggesting that pre-service elementary teachers who define the LCM correctly through relationships also exhibit deep procedural knowledge of the LCM.

POSTER FORMAT

In the poster I will explain my methodology, display the survey instrument with typical participant responses, and discuss my rational for classifying these responses as demonstrating superficial or deep procedural knowledge.

REFERENCES

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36(5)*, 404-411.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Zazkis, R., & Campbell, S. R. (1996). Divisibility and multiplicative structure of natural numbers: Preservice teachers’ understanding. *Journal for Research in Mathematics Education*, *27(5)*, 540-563.