COUNTING ON OBJECTS IN MATHEMATICAL LEARNING PROCESSES. NETWORK THEORY AND NETWORKING THEORIES.

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Latour's actor network theory proposes a sociology of objects, accepting objects and things as participants in the course of action. Mathematics education has to deal with all sorts of objects, didactical tools and manipulatives, diagrams and signs. Mathematical learning appears to be closely connected to objects. Latour's approach is fascinating and irritating and provokes the research question, if and respectively how actor network theory can be a fruitful background theory to get a better understanding about the role objects play in mathematical learning processes. How is it possible to do research in mathematics education respecting Latour's perspective on social interaction? This paper outlines how a change of paradigm might be implemented through local integration of theories.

EXPOSITION: NETWORK THEORY AND NETWORKING THEORIES

On the one hand there is an empirical phenomenon: In the mathematics classroom all sorts of things, objects and visualisations are offered to improve mathematical learning and understanding. However, those objects prove to be resistant. They often neither function nor work the way teachers or learners expect or intend them to. On the other hand there is a theory: Introducing the Actor Network Theory (ANT) Latour reassembles the social. He develops a sociology of objects, accepting objects as participants in the course of action. (Latour, 2005)

Studying Latour's sociological approach is fascinating and provoking. But is ANT a suitable background theory in the field of mathematics education? (How) is it possible to do *empirical* research in mathematics education referring to ANT? Even more precisely: (How) Can we investigate the role objects play in mathematical learning processes, if we adapt Latour's perspective? What ways of acting can be empirically reconstructed, how do objects *act* in the mathematics classroom? This article performs the change in perspectives and focuses objects in the mathematics classroom through sociological lenses.

Latour himself does not suggest any methods of empirical analysis. Exploring appropriate heuristics means firstly to take additional approaches into consideration and secondly to connect them with ANT. In this paper, the networking of theories on the background level is the basis for the development of a local integration of theories (see Bikner-Ahsbahs & Prediger, 2010). Closely connected is the discussion and compilation of appropriate heuristics.

In this article, four approaches come into interaction: Latour's ANT, Goffman's Participation Framework, Sack's Turn-Taking system and Toulmin's Model of Argumentation (Latour, 2005; Goffman, 1981; Sacks, 1996; Toulmin, 2003). Footing on their networking level, two methods of reconstructing the ways objects take effect in learning processes are introduced. The first analytic approach meets the sequential character of interactional processes: A systematic analysis of turn-partaking is implemented relying on own works (Fetzer, 2007, 2009, 2010, 2013), Goffman's participation framework (Goffman, 1981) and Sacks's turn-taking system (Sacks, 1996). The second analytic approach comes up with the lasting quality of objects: The functional orientated concept of argumentation analysis referring to Toulmin (2003) is adapted to trace objects' marks in mathematical learning processes. Aiming at the function of single actions, this analytic approach offers the opportunity to escape the sequence of interaction.

ACTOR NETWORK THEORY: AN 'ANT' IN THE RESEARCHER'S EAR

Latour's actor network theory ANT is a radical change of perspectives proposing a sociology of objects. He recommends a broader understanding of agency as well as action and extends the list of actors assembled as participants fundamentally.

"Any thing that does modify a state of affairs by making a difference is an actor." (Latour, 2005, p. 71). All actors, human or not, are "participants in the course of action" (ibid., p. 71). "Objects too have agency" (ibid., p. 63), and appear associable with one another, but only momentarily. They assemble as actor entities one moment and combine in new associations the next minute. Accepting objects as participants in the course of action, Latour gives in the idea of stable and pre-defined associations and actor-entities.

Looking through Latour's sociological lenses, not only the traditional understanding of agency has to be re-defined, but also the notion of action has to be re-thought. Objects participate in the course of action and take effect. But apparently their mode of action is different from the way human participants contribute to the social interaction.

Latour's approach fascinates and provokes. It puts not a bug, but an ANT in my mathematic researcher's ear. It triggers re-thinking of traditional ideas of the role of visualisations, manipulatives and other objects in mathematical learning processes. In this article, I introduce a local integration of theories in order to develop a new piece of synthesised theory on the role objects play in mathematical learning processes.

NETWORKING THEORIES: TRACING OBJECTS IN MATHEMATICAL LEARNING PROCESSES

Tracing objects' marks in the course of mathematical learning processes referring to ANT means to substantiate the notion of action empirically. How do objects act or

take effect in the course of action? This research issue entails the question, how objects' actions can be empirically traced and observed. What analytic methods prove to be suitable for reconstructing non-human actions? Below, two approaches to explore object's traces systematically are developed. Both are founded on micro-ethnographic research.

Micro-ethnographic approaches to classroom investigation like the method of interaction analysis help to reconstruct the development of interactional processes (see Fetzer, 2007; Krummheuer/Naujok, 1999). Interaction analysis is based on conversation analysis (Sacks, 1996; ten Have, 1999) and reveals, how the sequential organization of interaction is constituted. Accordingly interaction analysis is bound to be a sequential analysis. Every single *action* is interpreted extensively in the sequence of emergence. To investigate the aspect of *inter*-action, every single action is understood as a "turn" (Sacks, 1996) on a previous action. Turn-by-turn the emergence of the course of action is reconstructed. Traditionally, interaction analysis captures human actors as participants of an interactional process and investigates their actions. Own works on a micro-ethnographic approach to an object-orientated analysis of classroom interaction prove theoretically as well as empirically, that this interactionistic tool is a suitable and powerful basis for analysing the *networking* of all sorts of actors (Fetzer, 2009, 2010, 2013). Both approaches introduced below to trace objects' participating in the course of mathematical learning processes are methodologically based on an object-integrating analysis of classroom interaction referring to Fetzer (2009, 2010, and 2013).

Participation Framework

Goffman's "participation framework" (1981) provides "an essential background for interaction analysis" (ibid., p. 3). His approach offers the chance to distinguish between different forms of participating in "moments of talks" (ibid., p. 313). Right from the start Goffman tells hearing and speaking apart from the social slot in which these activities usually occur. "When a word is spoken, all those who happen to be in the perceptual range of the event will have some sort of participation status relative to it." (ibid., p. 3). Some might have the official status of participants. As "ratified participants" (ibid., p. 130) they may be listening or not be listening. Others might not be official participants, but still be following the encounter closely in the status of "eavesdropping" or "overhearing" (ibid., p. 132). Goffman discriminates ratified and non-ratified participants on a phenomenological basis. Besides, he introduces the status of "bystanders" (ibid., p. 132). Those not ratified participants find themselves in visual and aural range of the social encounter. The crucial aspect of this participative status is the fact, that their access to the moment of talk is perceptible by the official participants. Perceiving them as someone having the opportunity to follow the social encounter, the ratified participants assign them the status of bystanders. Thus, a bystander's role is determined in the *interactional* process.

Goffman's participation framework does pay no special attention to objects. Nevertheless, it proves to be a fruitful basis for investigating object-actors participating in the course of action empirically. Below, three connecting points are outlined. First, empirical research on the basis of ANT has to face the rapid change of networking and the unstable boundaries of associations. How is this flood of potential associations manageable? Who respectively what has to be considered as a participant in the course of action? Goffman takes a micro perspective and suggests focusing on "moments of talks" (ibid., p. 131) concerning the framework of participation. That combines well with an object-integrating analysis of classroom interaction. Investigating *moments of networking* in the sequence of emergence may capture the intermittent existences and permanent changes in assembling appropriately. Accordingly, ratified participants as well as bystanders are understood as participants at a certain moment of networking. Second, Goffman stresses, that not sound alone is at issue in social encounters, but also *other ways of perception* as sight or touch (ibid., p. 129f.). Opening the perceptual variety of interaction he clears the way for a wider range of observable actions and participating actors. Object's actions might rather be seen, felt or otherwise perceived than heard. Third, the introduction of the *bystander's role* is promising in the context of object's agency. Objects might be in the perceptual reach of ratified participants as potential actors. Assigned as bystanders, they might come into play, associate with other actors and take effect in the interactional process.

Turn-Taking System

Trying to differentiate empirically the way objects participate in learning processes remains unaccustomed. Nevertheless, own works on the development of an object-integrating analysis of classroom interaction (Fetzer, 2009, 2010, 2013) approve, that objects' contributions to learning processes become accountable in the process of interweaving. As soon as object-actors assemble with other actors they enter the course of action. Their traces render perceivable and can be captured by analysis. *Turn by turn* it can be reconstructed, how objects participate in the emergence of social reality. A closer look on the theoretical basis of the sequential organisation of conversation is a matter of consequence. Referring to Sacks's approach to conversation analysis, conversation is a coordinational problem (Sacks, 1996, Vl. II, p. 32). A basic challenge is to preserve "one party at a time" (ibid., p. 32), namely that any time there is at least one, but no more than one participant speaking. In matters of the "order of speakers" (ibid., p. 32; p. 521), it is decided on the next speaker or the next action, but not on the speakers or actions afterwards. Sacks refers to this sequential organisation of speaker change recurs as the "turn-taking system" (ibid., p. 524). He specifies several techniques of speaker-selection. Some of them correspond to the *next* speaker: "Current speaker selects next actor/speaker" (ibid., p. 524). Others are connected to the *current* speaker: "Next speaker may self-select himself." At this point Sacks's approach to conversation analysis may be connected

to an interactionistic approach: Inter-action is based on mutual exchange. Actions are related to each other as turns. The current speaker may select the next actor. However, this turn-"distribution" (ibid., p. 533ff.) needs to be understood by the designated next speaker. Solely in this case he/she/it may either accept this distribution or refuse to pick up the offered turn. Eventually actors may simple take over the turn. Sacks does not specify in *turn-distribution* and *turn-partaking*. Indeed, this determination proves to be continuative and fruitful when combining conversational and interactionistic approaches in order to grasp objects' traces analytically.

Whenever participants in interactional processes change their status and become active actors, their current action can be interpreted as a turn on previous actions. The question arises, who or what provoked or initialised the change of participation status. Investigating the way objects participate in the course of action, especially a second aspect turns out to be crucial. Moments of networking and changes in participation status permit to reconstruct the *previous role* of the current active actor. Methodically, an "*analysis of turn-partaking*" is implemented (Fetzer, 2007, p. 126 ff.). This method of analysis stood the empirical test earlier in reconstructing actions, that are observable only indirectly (Fetzer, 2007, 2009). Turn-partaking and turn-distribution are linked very closely when investigating object-actors (Fetzer, 2013). In order to differentiate the notion of action in the context of objects' participating empirically, sequential analysis benefits from this strong connection. It is reconstructed backwards or indirectly how objects take effect in the course of learning. When a human actor is operating as a turn on an object, it can be deduced, that the object must have been kind of active before. It must have 'told' the human actor something. The object must have *made the offer to partake the next turn*. As a consequence, the human actor, namely a learning child, brings an active return to the previous objects action.

Empirical research on the basis of this analysis of turn-partaking confirmed, that several ways of taking over offered turns by object participants can be reconstructed. Sometimes human actors accept directly-offered turns. Sometimes humans pick up unspecific offers to take over the next turn (see Fetzer, 2013). In particular this method of analysis facilitates the reconstruction of the ways, in which objects take effect in social interactions and learning processes (see Fetzer, 2013).

Theory of Argumentation

Objects have a lasting quality. Following Latour, objects and things render more durable the constantly shifting interactions (Latour, 2005, p. 68). A book might be standing disregarded on a shelf for weeks, some-thing written on the board might remain unchanged a whole school morning, a bunch of manipulatives may lie on a desk untouched for minutes. Due to their (potential) durability, objects may take effect spreading place and time. Those moments of networking when objects take part actively might be temporarily delayed. Objects may overcome temporal bounds

and limits. This fact brings up a methodological issue. Investigating objects' traces exclusively on the basis of a sequential analysis appears to be too short-handed. In addition, a second approach has to be implemented, that breaks up the narrow boundaries of sequential emergence. Referring to a sociological perspective on learning, mathematical learning processes emerge predominantly in (collective) argumentations (see Miller 1986; Krummheuer/Fetzer 2005). Consequently, investigating the role objects play in mathematical learning processes means to focus argumentative processes. In my research, I refer to the theory of argumentation and the "Toulmin Model" to capture objects' traces in mathematical learning processes analytically (Toulmin, 1958/2003).

Based on Toulmin's approach, arguments show a specific structure. The pattern of an argument has certain constituent elements, namely data, conclusion and warrant. These three functional categories are the core of an argument. The conclusion is the claim that needs to be established. When it is challenged, it has to be proven justifiable. The data is our personal knowledge, the facts we appeal to as a foundation for the claim. It is the ground we produce as support for the original assertion. It is the answer to the challenge: "What have you got to go on?" (ibid., p. 90). The shortest possible argumentation would be: Data *D* is the basis *so* the conclusion *C* can be established.

> "We already have, therefore, one distinction to start with: between claim or conclusion whose merits we are seeking to establish and the facts we appeal to as foundation for the claim – what I shall refer to as our data." (ibid., p. 90).

No amount of facts may establish any conclusion. There needs to be a connection of data and conclusion on another level.

> "Our task is no longer to strengthen the ground on which our argument is constructed, but is rather to show that, taking these data as a starting point, the step to the original claim or conclusion is an appropriate and legitimate one. At this point, therefore, what are needed are general, hypothetic statements, which can act as bridges, and authorize the sort of step to which our particular argument commits us." (ibid., p. 91).

These connecting links are warrants (ibid., p. 91ff.). They indicate the bearing on the conclusion on the data already produced and answer the question "How do you get there?" (p. 91). You can get from *D* to *C since* the warrant *W*.

Toulmin's analytical model is a methodological tool to reconstruct the *function* a certain action fulfils within the argument (see Kopperschmidt, 1989). It focuses on verbal as well as non-verbal actions. It is not restricted to analyse questions in dispute, but is open to all sorts of argumentative processes. Even implicit parts of an argument might be captured by analysis, as empirical research proves (Fetzer, 2007; Meyer, 2007; Schwarzkopf, 2000). Eventually, the Toulmin model connects well with the idea of tracing objects agency.

IMPACT ON RESEARCH AND PRACTICE: COUNTING ON OBJECTS IN THE MATHEMATICS CLASSROOM

Latour put an ANT in my researcher's ear, to re-think the way objects participate and take effect in the emergence of mathematical learning processes. Especially in primary mathematics education it is an expedient effort to change perspectives and explore approaches to "follow" the object-actors (Latour, 2005, p. 12, 156) and their traces. In this article the theoretical basis for empirical analysis on objects' participation in learning processes is outlined. Based on a micro-ethnographic approach to empirical research on mathematical learning processes, Latour's ANT, Goffman's Participation Framework, Sack's Turn-Taking system and Toulmin's Model of Argumentation are networked on the background level.

Actually, this local integration of theories proves to be a theoretical basis that renders empirical analysis well possible. Investigating how objects participate in the course of action leads to the development of a new piece of synthesised theory on the role objects play in mathematical learning processes. Several forms of object-participation can be reconstructed in the development of social learning processes. However, due to space restrictions, no examples of empirical analysis are given here. To learn more about the implementation of analysing objects traces see Fetzer (2013). Below, empirical results on the methodological level as well as on the theoretical level are abstracted. They outline the impact, this new approach might have on research and practice.

Methodological level

An object's opportunity to become an active participant in the course of action is strongly connected to human-actors interpretations and perceptions. It is only in moments of networking, that their acting becomes observable. When inter-acting with other actors, objects traces render perceivable. Accordingly, the basic idea of analysis is to grasp the acting of objects *indirectly*. The *analysis of turn-partaking* is the core of investigation. As soon as a human-actor takes over the turn offered by an object-actor, the current action (human) allows concluding on the previous action (object). In other words: The way students or teachers act as a turn on an object suggests how objects participate. What has the object 'told' them, when it was the object's turn?

The analysis of turn-partaking captures the *sequential* emergence of mathematical learning processes. However, one dominant feature of objects and things is their durability. As a consequence, the second methodological approach to investigate objects traces is the *Toulmin model* on argumentation. This method of analysis aims at the *function* single actions fulfil within an argument. Both tools of analysis prove to be empirically successful. They combine well in order to differentiate empirically the notion of 'action' in the context of objects.

Theoretical level

Objects take effect in the social learning process in a different way than students or teachers do. They may hold different *status of participation*. On the one hand, ratified actors might allocate them the role of *bystanders*. This is the case, if active actors perceive the object-actor as some-thing in the perceptual reach of the interactional process, but not directly involved in the course of action. On the other hand, object-actors may hold the role of *ratified participants*. In this participation status the other participants in the social learning process accept them as participants that take effect in the course of action.

The status of participation of an object is no stable allocation. An object-actor might be perceived as a bystander one moment and become a ratified participant the next minute. These changes of the participation-status from the bystander to ratified participant are triggered by other participants of the social encounter. The transition might be either initiated by human actors or by ratified active object-participants. Different conditions of emergence may be reconstructed concerning the change of participation-status:

- Objects become ratified participants, if the *process of problem solving stagnates* within the current ratified participants. Stretching the group of ratified participants and opening it to an object, which has held the bystander-status so far, often restarts the interactive learning process. These conditions for changing participative roles in case of stagnation emerge predominantly in group working phases, when teacher's interventions are minimal.
- Objects change their participative status, if *teachers* call students' attention to the turn an object offers.
- As soon as *objects* turn to be ratified participants, they often initialise the change of participative status of further objects. This might happen at the beginning of a problem solving process, if the task (an object-actor in the role of ratified participant) claims integrating manipulatives or visualisations (object-actors in the status of bystanders).

Empirical analysis provides a *differentiation of the notion of action* in the context of objects and offers answers to the following questions: How can actions be described, that are performed by object-actors? In other words: How do objects act, and what do things do in the course of action?

- As bystanders, *objects hold unspecific offers in readiness* to take over the next turn. Other participants might *pick up* this unspecific offer and get active in the interactive process.
- Taking the role of ratified participants, *objects partake turns*. At the same time they *make an offer to take over the next turn*. This specific offer addresses other participants. Its stimulative nature may vary from *volunteering* to

provoking to take over the next turn. Following, participants *accept* these volunteered or provoked offers to take over the next turn and get active.

- Objects *take part in the emergence of interactive processes*. They might contribute to the ongoing interaction especially if the problem solving process stagnates. Sometimes object-actors take over the role of a student's partner in interaction. In this case, they might not only contribute to the emergence, but *give the direction to the course of action*.
- As ratified participants *objects take over elements of an argument*. Playing the role of *data*, they clarify what there is to start from. In other cases they contribute to the conclusion. If objects take over the warrant, they legitimate the conclusion. It can be reconstructed, that objects particularly take over data and warrant. Accordingly, objects play a central role in mathematical argumentation and learning processes.

The ANT in my ear triggers a change of perspectives and shows objects through sociological lenses in a different light. Accepting objects as participants in the course of action and following the idea of objects having agency raises a scientific discussion and results in networking theories. This article shows that empirical research on the basis of Latour's approach is possible. It gives an example of how local integration of theories can lead to the development of a new piece of synthesized theory, a theory of objects participating in mathematical learning processes. Besides, the empirical results have impact on the practice of mathematics education. (For more detailed information on that aspect see Fetzer (2013)). I found out that objects may influence learning processes especially if they take over the role of ratified participants. As a consequence, our didactical efforts should aim on the following points: We should try to get objects out of the bystander-status. As mentioned above, conditions are good in group works, when students integrate objects as participants in a stagnating solving process. In other situations teachers might call students' attention to the turn an object offers. Once objects turned their status and became ratified participants, we should try to extend this participative constellation. And last but not least, objects play an important role in mathematical arguments. The empirical analyses revealed that objects participate in argumentative processes by taking over data and/or warrants. Thus they contribute to 'clearer' and 'deeper' argumentations. The more explicit and the more complex an argumentation is, the easier it becomes to follow the argumentative line and the more mathematics there is to learn.

Objects leave their traces in the emergence of social learning processes and take part in the course of action. We should count on objects in mathematical learning processes!

REFERENCES

Bikner-Ahsbahs, A. & Prediger, S. (2010): Networking of theories – an approach for exploiting the diversity of theoretical approaches. In B. Sriraman & L. English (Eds.): *Theories of mathematics education: Seeking New Frontiers*, Series Advances in Mathematics Education, Springer, New York, (pp.483-506).

Fetzer, M. (2007). *Interaktion am Werk: Eine Interaktionstheorie fachlichen Lernens, entwickelt am Beispiel von Schreibanlässen im Mathematikunterricht der Grundschule*. Bad Heilbrunn: Verlag Julius Klinkhardt.

Fetzer, M. (2009). Objects as Participants in Classroom Interaction. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education*, (pp. 974-983).

Fetzer, M. (2010). Reassembling the Social Classroom: Mathematikunterricht in einer Welt der Dinge. In B. Brandt, M. Fetzer & M. Schütte (Eds.), *Auf den Spuren Interpretativer Unterrichtsforschung in der Mathematikdidaktik* (pp. 267-290). Münster: Waxmann.

Fetzer, M. (2013). Mit Objekten rechnen: Mit Latour auf den Spuren von Materialien im Mathematikunterricht. In T. Alkemeyer, H. Kalthoff & M. Rieger-Ladich (Eds.), *Bildungspraktiken. Körper - Räume - Artefakte*. Weilerswist, in press.

Goffman, E. (1981). *Forms of Talk*. Philadelphia: University of Philadelphia Press.

Have ten, P. (1999). *Doing Conversation Analysis*. London: Sage.

Kopperschmidt, J. (1989). *Methodik der Argumentationsanalyse*. Stuttgart - Bad Cannstatt: Frommann – Holzboog.

Krummheuer, G. & Fetzer, M. (2005). *Der Alltag im Mathematikunterricht. Beobachten, Verstehen, Gestalten.* München: Spektrum Akademischer Verlag.

Krummheuer, G. & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung*. Opladen: Leske + Budrich.

Latour, B. (2005). *Reassembling the Social. An Introduction to Actor-Network-Theory*. Oxford: University Press.

Meyer, M. (2007). *Entdecken und Begründen im Mathematikunterricht: Von der Abduktion zum Argument.* Hildesheim: Franzbecker.

Miller, M. (1986). *Kollektive Lernprozesse - Studien zur Grundlegung einer soziologischen Lerntheorie*. Frankfurt am Main: Suhrkamp Verlag.

Sacks, H. (1996). *Lectures on Conversation.* Cornwall: Blackwell Publishers.

Schwarzkopf, R. (2000). *Argumentationsprozesse im Mathematikunterricht*. Hildesheim: Franzbecker.

Toulmin, S. (2003). *The Uses of Argument*. Cambridge: University Press.