

GRAPHIC CALCULATOR USE IN PRIMARY SCHOOLS: AN EXAMPLE OF AN INSTRUMENTAL ACTION SCHEME

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This paper presents an empirically based case study design within a sociocultural theoretical framework. The research aimed to describe the implementation of a graphic calculator in a fifth grade primary school class when four students were engaged in mathematical activities performing tasks and challenges given by the researcher. The students' activities associated with the process of appropriation of a technological artifact played a prominent role in the data analysis. The distinction between artifact and instrument through the instrumental approach was in focus (Rabardel, 2002). The schemes of instrumented action were of special interest (Drijvers and Gravemeijer, 2004). The case study concerns students' discovery of the function key exponentiation with the inscribed symbol $^$ at the key button, and an analysis of exponentiation by schemes of instrumented action.

Keywords: *graphic calculator, exponentiation, instrumented action scheme*

INTRODUCTION

This case study is about students-initiated calculator activity. A student named Signe calls for assistance about her findings concerning the exponentiation¹ function key on the keypad. The study is part of a larger research project where data was collected from task-based video interviews with students undertaken by the researcher. The focus of the research described herein was implementation of an advanced graphic calculator in a fifth grade primary school class and characterization of its use. The choice of research strategy can be described as a case study design. The study is presented in a sociocultural theoretical framework where a graphic calculator is a sociocultural semiotic artifact. Artifact properties, such as the function keys on the keypad of the graphic calculator were variables considered in this study. The instrument-mediated activity approach was in focus. The theoretical framework referred to as instrumentation (Trouche, 2003) associated with research in technology educational environments is applied as the lens for analysis of the data.

The students in the class were accustomed to using a four-function calculator. Advanced graphic calculators are usually not part of teaching mathematics at the primary school level. The purpose of introducing the advanced graphic calculator was to investigate the longitudinal aspects of students' appropriation of the artifact. A task portfolio was designed for the project. The students' spontaneous initiated

¹ The mathematical operation exponentiation has not been introduced to the students at this academic level, and the associated function key was new to them.

exploration activities of the exponentiation function sustained by the graphic calculator, was not part of the task portfolio or the project agenda.

THEORETICAL FRAMEWORK

The main theoretical perspective that frames our investigation is the instrumental approach associated with research in technology educational environments. Instrument genesis is suitable to analyze processes described as appropriation by which an (digital) artifact becomes an instrument in the hands of a student, in terms of transforming an available artifact into an instrument to become a cognitive aid for the student to solve a given task or challenge. The process of appropriation of an artifact to become an instrument, a tool for the competent user is further elaborated by (Trouche, 2003). The instrumental approach with the human/computer interaction and human use of tools constitutes two main processes of instrument mediated activity. The process directed from the artifact towards the human agent is named instrumentation. The tools convey, shape and transform the human agency and the subject adapts to its constraints and affordances. The other process, instrumentalization, is directed from the human agent towards the artifact which includes stages such as familiarization with the instrument, mastering the instrument and adoption of the instrument to one's own personal, specific needs. (Rabardel, 2002) drew on Wertsch's key construct of mediated activity (Wertsch, 1998). Students mediated calculator activity transferred to individuals-appropriate-technological-artifacts-in-cultural-practice constitutes the unit of analysis.

METHODOLOGY

The choice of research strategy in this study can be described as a case study design on two levels. On the first level, there were 23 students in a fifth grade (students aged 11-12) primary school class. The first level formed a backdrop for the next level, the selection of four students. There were two boys and two girls placed into two small groups, one of each gender. The choice of the two boys and two girls was partly based on pre-tests, achievement tests from a pilot study involving a representative sample of the class, but also on the students' motivation as the two groups of friends wished to participate in this study that lasts over two years. We meant that the latter criterion was a good argument for the participants not losing interest during the period of data collection. The second level of the case study consisted of the two respective groups, and was a continuation and an elaboration of the study where the entire class constituted the background. In this study, each student had a handheld calculator at his or her disposal. The sophisticated icon-driven graphic calculator Casio fx9750G PLUS was introduced by the researcher including a 21-character x 8-line display. The mediated artifacts, among them the graphic calculator, were available and were included in their mathematical activities. The data collected and analyzed in this research study was from video-taped interviews when each of the four students were in work sections working

individually, and when they were collaborating in small groups engaged in mathematical activities, challenges and tasks provided by the researcher.

ANALYSIS OF THE TRANSCRIPTION

The analysis of the transcript below is an example of students' appropriation of the artifact function key for exponentiation, a part of the graphic calculator, described by the instrument genesis process.

The built-in artifact exponentiation with its function key \wedge

The term artifact in the case of calculator is material, an example of a physical or touchable object but also a semiotic artifact. Less obviously, algorithms implemented on a computer or on a calculator, in a sociocultural perspective, are regarded as material artifacts. Algorithms are created and used in mathematics and can be programmed into computing devices. Through their materialization in entities in written and in spoken form, algorithms do not exist without socio-historical invented signs. A graphic calculator is a complex technological digital artifact, which constitutes of a collection of artifacts. For example, if one considers the entire software package as a single artifact, a function for calculating the arithmetic mean or another for solving quadratic equations are both built-in algorithms. Each of these examples is regarded as a single artifact. The physical artifacts such as the screen display, the keyboard or a single key on the keypad is each part of the artifact graphic calculator.

The artifacts in focus in this case are the students' appropriation of the exponential application software within the graphic calculator and the associated exponentiation function key with the inscribed symbol \wedge at the key button on the keyboard of the graphic calculator. The two artifacts can be considered as a consisting unit; a part of the software package which the agent is communicating with via the keystroke \wedge for exponentiation in the context of a keystroke sequence $9 \wedge 2$.

Transcript of students' exploration of the exponentiation key \wedge

The following excerpt is a transcript of a video clip of an interview of the students named Hilde, Kate, Signe and the researcher. The video clip is part of a comprehensive video material. The episode is initiated by Signe who enthusiastically requests the researcher's for attention about her findings concerning a new function key for exponentiation with the inscribed symbol \wedge at the key button on the keyboard of her graphic calculator. Signe held out her hand with the graphic calculator towards researcher so he could take a closer look at the screen display.

The circumstances gave the researcher a golden opportunity to put aside the day's interview guide program and instead concentrate on Signe's initiative and encourage her to continue the exploration process by letting her lead the other students on the way. Furthermore, in the interview, the emphasis is on the students, on their own, to take the lead and not be guided or intervention by the researcher. In this context,

concerning the role of the researcher as an observer, the researcher saw the necessity of balancing roles between being observer-as-participant and participant-as-observer avoiding unnecessary intrusion or interruption.

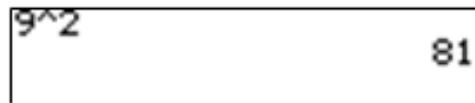


Figure 1: Signe's calculator display

- 112 Researcher: Tell me Signe what do you have on your screen display?
- 113 Signe: It is, eh, 9 (Pause) 2 up and it is equal to 81
- 114 Researcher: Could you show Tom and Hilde the calculator's screen display?
- 115 Signe: Have you learned this yet? (Signe is speaking to Tom and Hilde while she is holding her graphic calculator in a position such that each of the students is able to look at the screen display.)
- 116 Tom: I want to see it! What? Tell me, how did you do it? (Tom expresses astonishment at seeing the syntax shown at the screen)
- 117 Hilde: What does the number 2 mean? (Hilde gets a short glimpse of the screen display of Signe's graphic calculator).
- 118 Tom: I do not understand why it is 81 when you are taking 9.
- 119 Signe: Yes, because, right?, you take 9 times 9 (...)
- 120 Tom: Wow, I made it! (Tom bursts out a "wow" signaling succeeded in typing in the keystroke sequence 9^2 and calculated 81 that Signe just recently has shown him.)
- 121 Researcher: What does the digit 2 mean?
- 122 Hilde: Yes, times 2.
- 123 Signe: No, 9 times 9. (Signe is making hand gestures to emphasize her answer through visualizing the arithmetic calculation by drawing in the air with her index finger the symbol 9, then the multiplication sign and a 9.)
- 124 Hilde: How do I get the digit 2? Ok! (Signe is pointing with her index finger at the keypad and screen display to show Hilde the keys to be pressed that will generate 9^2)
- 125 Tom: There we learned something. This is our secret.
- 126 Signe: Yes, it is our secret.
- 127 Tom: I'm not sure it is a very big secret. (Tom chuckles while talking)
- 128 Researcher: What does the digit 2 mean?
- 129 Signe: What the digit 2 is? It goes like two times up in the brains of the calculator.
- 130 Researcher: Do you understand what Signe means?
- 131 Tom: Yes, it goes twice
- 132 Signe: Two nines, it is the same as 9 times 9, only that we use the digit 2 to go little faster.

Signe's initiated exploration task is associated with the identification of the keystroke sequence displayed with the syntax 9^2 and the students' experimental circumstances, the discovery of the mathematical identity $9^2 = 9 \cdot 9$. In text line 112, she has been able to make use of the function key for exponentiation on the keypad and she has managed to calculate $9^2 = 81$ appearing on her screen display. Researcher seized the opportunity by asking her to pronounce what is displayed on her screen. Signe answered in next the text line by reading aloud off the screen display and uttered the syntax "It is, eh, 9 (...) 2 up and it is equal to 81." Signe pronounced the digit 9 and the digit 2 and then "up", which could mean her visual perception the digit 2 is located somewhat up in its position compared to digit 9. She uttered that the answer to the arithmetic calculation is 81. The reason for the researcher for asking Signe to show Tom and Hilde her screen display, was, hopefully, to initiate a dialogue between the students about the new sign $^$ and what the syntax 9^2 mathematically represents.

Making mathematical meaning of syntax for exponentiation

In text line 115, a dialogue is initiated by Signe's appeal to Tom and Hilde for attention by letting them see the syntax displayed on her calculator screen. Signe's utterance suggests she believed that the other two students did not have any experience of using this function key or that they didn't know about its existence and what it mathematically represents. She said this with an overwhelmingly proud smile to underscore her achievement. After Tom and Hilde cast a glance at Signe's screen, Tom appeared to be surprised (text line 116) over the syntax configuration visualized on Signe's screen. Hilde's utterance (text line 117) is associated with the visual appearance of the text string characters. Her focus is on the digit 2 and attention to the relative positions of the signs in the punctuation of the exponential notation (calculator syntax). Tom's utterance (text line 118) is interpreted that he did not understand how Signe has managed to calculate 81 when she typed in 9. Tom is trying to make sense of Signe's calculator activities and his utterance concerned the mathematical content, the exponentiation. Signe chose to sit down next to Tom on the couch so they both could look at each others' calculator screens. Each of them was holding a graphic calculator in hand while Signe showed Tom the keystroke sequence and how to navigate and manoeuvre among the keystrokes to get the mathematical expression and the arithmetic calculation $9^2 = 81$. In text line 119, Signe answered that it is the multiplication 9 times 9 that gives the answer 81, a result Tom most likely is familiar with. But Signe's answer may not be what he asked for, that was, the role of the digit 2 in the arithmetic calculation. Tom gave an outburst of a "wow" (text line 120), after following guidance and instruction from Signe, signaling that he succeeded in navigating and manoeuvring on the keypad and executed the arithmetic calculation by the use of function key for exponentiation. Researcher intervened in the students' dialogue with a control question to clarify Tom's and Hilde's perception of what the digit 2 in the calculator syntax

mathematically to them represents. The question (text line 121) was asked after Signe had given Tom and Hilde her verbal explanation and clarification by gesturing which consisted of keystroke sequence and the related mathematical content of the calculator syntax. Hilde's response to the researcher's question was that the digit 2 stands for the number of nines. However, Signe uttered a definite "no" of what role the digit 2 is playing in the arithmetic calculation, and she corrected Hilde's statement by saying it means 9 times 9. Signe's utterance (text line 123) was simultaneously accompanied by hand gestures visualizing the arithmetic calculation. Hilde was trying to make sense of Signe's calculator activities and asked Signe about the role of the digit 2. Signe responded by showing Hilde the keystroke sequence, which constitutes $9^2 = 81$. Hilde's response (text line 124) after being guided by Signe was to outburst the "ok!" to confirm she had managed to type in the keystroke sequence for exponentiation.

In a humorous tone (text lines 125-127), Tom and Signe proposed that they keep their findings as a secret from the other classmates. The discovery of the exponentiation button and the mathematical operator in the case of 9 raised to the power of 2 equals 81 is their little secret. The students were preoccupied with the fact that they, on their own, without the teacher's help, had learned about a new function key on the keypad on the calculator. The students had managed to give the syntax 9^2 mathematical meaning via the identity $9^2 = 9 \cdot 9 = 81$. This is an example of personification of knowledge by making it their own, a characteristic feature of the process of appropriation. They reflected upon the knowledge they had acquired during the dialogue. Our claim is that the students, in this context, possessed knowledge at a level of a meta-perspective.

Finally, Signe (text line 132) exhibited a pragmatic attitude to the use of the mathematical operation exponentiation. She probably connected prior knowledge about (repeated) multiplication and saw the opportunity to express the mathematical identity. Signe utilized, in this case, the advantage of exponentiation as a useful and a feasible calculation tool that is faster, more efficient and practical for an agent to perform, rather than performing calculations as repeated multiplication by the use of the calculator. According to Artigue (2002), Signe emphasized on the pragmatic value of technique by focusing on their productive potential like cost (time-consuming aspects), efficiency, and field of validity.

Asymmetry in the students' roles in the investigation

In the episode there was an asymmetry in the different roles of the three participating students which is crystallized in the events that unfolded through their respective calculator activities reflected in the dialogue. Signe undertook the role of a mentor and acted as a supervisor to Hilde and Tom. Signe advised the other two students about the function key $^$ and the role the function key $^$ plays in the keystroke sequence, and regarding on how they could navigate and manoeuvre between the

keys on the keyboard to express the arithmetic calculation exponentiation in the case of 9^2 . Signe was the mediator between Tom and Hilde, relaying her interpretation of how the displayed syntax should be understood and mathematically linking the operator exponentiation to the students' previously knowledge of the arithmetical operation (repeated) multiplication as 9 times 9. Signe answered the other students' questions. Signe explained and clarified misunderstandings like the digit 2 in the screen display syntax meant the multiplication 9 times 9 and not 2 times 9 as Hilde had presumed. Signe had the opportunity to guide the other two students and bring them into her knowledge, and which turned out to be correctly understood, her perception and understanding about how the graphic calculator operates with respect to calculating 9 to the power of 2 equals 81. On the other hand, Tom and Hilde were novice users and unfamiliar with the new function key \wedge . Tom and Hilde were acting and participating in the role of learners that expressed their wishes towards Signe to be guided about the new function key and the associated syntax of the graphical calculator. They had questions about what the built-in application could perform and what the arithmetic operation represented mathematically with respect to how it operates on numbers. Hilde and Tom were asking Signe information about the new function key \wedge , its place and role in the displayed keystroke sequence and about syntax that enabled Signe to execute the arithmetic calculation. They sought assistance from Signe with their challenge of decoding the mathematical content of the syntax.

Anthropomorphic description of the calculator

It is interesting that Signe used a metaphor to explain and to make sense of the arithmetic calculation by attributing the artifact graphic calculator, which does not have human properties, human properties and qualities. This can for instance be seen in the text lines 129 and 132 where she uttered "it goes two times up in the brain of the calculator" and "the digit 2 to go little faster". Signe's explanation on how the calculator operates is associated with the human body as a reference where the brain is located within the head. The human brain can perform (mental) arithmetic operations on numbers. The characteristic feature can be compared and transferred to the calculator, which is equipped with a built-in brain with the objective to perform arithmetic calculations. Like in this case with Signe who ascribed the calculator human characteristics, empirical investigations give accounts of examples in which people provide anthropomorphic descriptions of phenomena related to computing devices (Morewedge, Preston, & Wegner, 2007).

Signe's initiated exploration activity associated with the discovery of a new function key and identification of the keystroke sequence that appeared on the calculator syntax 9^2 , which, to the students, corresponded to a new arithmetic calculation. The mathematical operation was exponentiation for positive integers in the case of the mathematical identity. Based on the students' activity directed towards exploring the mathematical operation exponentiation sustained by the graphic calculator, the

observations of the unfolding activity was analyzed and interpreted in light of the instrumental approach by the scheme of instrumented action.

Components of instrumented action scheme for the exploration of the mathematical operation exponentiation with the graphic calculator

- (1) Knowing where to find the function key \wedge for exponentiation on the keyboard of the graphic calculator.
- (2) Knowing the existence of the command with the inscription \wedge can be used to express exponentiation (as repeated multiplication)
- (3) Remembering and being able to type in the keystroke sequence and corresponding displayed syntax $9 \wedge 2$ with the aim to perform the arithmetic calculation $9 \cdot 9 = 81$.
- (4) Realizing that $9 \wedge 2$ can be substituted by the mathematical operation (repeated) multiplication, that is, recognizing the mathematical identity^{identity}
 $9 \wedge 2 = 9 \cdot 9$
- (5) Being able to interpret the result $9 \wedge 2 = 81$
- (6) Attempts to generalize by a change to a different value for the chosen base 9 and exponent 2.

The bullets (1), (2) and (3) concern aspects of observations of students' concrete gestures², restricted to techniques that the students carried out with the artifact graphic calculator. The technical skills, what the instrumental genesis of instrumented action requires of the student, is to be aware of its existence and the location of the function key exponentiation \wedge on the keyboard of the artifact graphic calculator. Further, the student should be able to navigate and to manoeuvre among keys on the keyboard to type in the keystroke sequence. Navigating: by determining the position and course in the sense of orientation among the keypad to locate the keys to be used in the keystroke sequence to express potency $9 \wedge 2$. Manoeuvring: i.e. make manoeuvres by managing the coordination and the steering of the gestures that constitute the movement of the fingers on the keyboard in the agent's performance of keystroke sequence to execute the command exponentiation. Finally, by pressing the exe-button for the execution of the command exponentiation, the student is mobilizing the tool graphic calculator to perform the arithmetic calculation. The described technical gestures of the students' working with the graphic calculator consisting of a combination of keystrokes, is an example of a usage scheme directly related to the artifact and oriented towards calculator management. According to (Trouche, 2003, p. 789), "We shall call gesture a student's elementary behaviour that may be observed, component of a scheme." The usage scheme (Rabardel, 2002), the operational manipulation of the tool, constitutes

² According to Trouche (2003), a gesture is the observable part of usage scheme.

both by observable gestures and knowledge involved in the making of the gesture. The observable appearance of students' gestures on the keypad of the calculator is not to be considered in isolation, because the gesture requires knowledge from each student to carry out the task which is mobilizing, and utilization of the function key for exponentiation and recognition of the identity as repeated multiplication by the identity. The enclosed transcript illustrates that Signe, Tom and Hilde not only were keen to know about the keystroke sequence to perform the arithmetic calculation, but they also were interested in the ulterior mathematical content to make meaningful use of the tool. My claim is that the mental process associated with the individual mental developing schemes of Signe, Tom and Hilde, incorporates mathematical thinking when each of them were trying to attribute mathematical meaning to the technique of typing in the syntax. This can be seen as instrumented action scheme because of the conceptual aspects or conceptual components in the instrumental genesis. The scheme of instrumented action is oriented by the action itself. The students were not questioned, nor they did get into operationalizing the function key $^$ with other bases than base 9 and other exponents than exponent 2. What would Signe and the other students have said if for example the digit 2 in the syntax was replaced with the digit 3 or the digit 5, and whether they would have been able to calculate 9^3 or 9^5 and mathematically explain the arithmetic calculation? Based on Signe's statement in text line 132, "two nines, it is the same as 9 times 9, only that we use the digit 2 to go a little faster" and in text line 129 "two times up in the brains of the calculator," indicates that Signe mathematically interpreted the exponent 2 in the exponentiation as the number of times the digit 9 shall be multiplied by itself. The screen displays not only the syntax, but also the corresponding answer 81 to the arithmetic problem, which gave Signe the opportunity to connect prior knowledge to the multiplication 9 times 9. The exponentiation corresponded to repeated multiplication and the mathematical identity. This is an indication that Signe, for example, could have answered that 9^3 was the same as $9 \cdot 9 \cdot 9$ and that the exponent digit 3 then told her "it goes three times up in the brain of the calculator." However, this is to draw the conclusion too far and remains a hypothesis. Bullet 6 is incorporated and implemented as a component in the instrument action scheme if the students, through a task design, were given the opportunity and challenged to use other bases and exponents and to make generalizations. The bullets (4), (5) and (6) require of Signe, Tom and Hilde to sharpen and extend the conception of multiplication for positive integers to repeated multiplication, and in a longitudinal aspect, relate it to the new syntax and the function type exponentiation a^b . The example illustrates the technical gesture involved in using the artifact graphic calculator interplay and interacts with mathematical thinking. Technical gestures and conceptual aspects are intertwined and co-developed. The three first bullets on the component list have a more technical character, while the three last on the list have more conceptual characteristics.

CONCLUSION

The process of instrumental genesis is suitable to analyze the process of appropriation, the transformation of an artifact into an instrument in the hands of a user. Instrumentation schemes distinguish between utilization schemes and instrument action schemes. The former schemes concern activity directly linked to the artifact and functions as building blocks for the more global integrated schemes of instrumented action related towards the (calculator) with the aim of carrying out a given task (Trouche, 2003). Mental schemes describe both technical and conceptual characteristics mediated by an artifact. In this case study we described an instrument action scheme concerning student-initiated exploration of the exponentiation key with the inscribed symbol \wedge on the key button on the keypad of the graphing calculator. The component of the instrument action scheme was the theoretical lens to analyze observation of the student's gestures (techniques) when they mobilised the calculator by navigating and manoeuvring among keystrokes to perform the task $9^2 = 81$. The instrument action scheme also offers the researcher the opportunity to analyse the conceptual elements, the mathematical concepts that are involved in students' activities when they are performing the task or challenge. This case study illustrates technical and conceptual aspects of the students' exploration activities concerning exponentiation, in terms of dialectical relationship, co-development and co-emergence.

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