USING DYNAMIC SOFTWARE TO FOSTER PROSPECTIVE TEACHERS' PROBLEM SOLVING INQUIRY APPROACH

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Mathematical tasks are essential components that help prospective teachers to develop mathematics and didactic knowledge. What type of reasoning do problem solvers exhibit when they use dynamic software to solve textbook tasks? We document the extent to which the use of the tool can offer prospective teachers the opportunity to construct and explore a task's dynamic model where visual, empirical and geometric reasoning complement and enhance formal approaches.

INTRODUCTION

Teachers take several important decisions during the preparation and development of their mathematical lessons. These involve the selection of problems, the introduction and discussions of concepts, ways to organize or structure learning activities, and ways to answer, evaluate, and orientate students' comments and participation. How do teachers support and carry out choices and decisions related to the framing and development of a mathematical lesson? Schoenfeld (2011) proposes a framework to characterize and interpret ways in which people in different domains engage in, and develop practices associated with such domains or fields. "People's decisions making in well practiced, knowledge-intensive domains can be fully characterized as a function of their orientations, resources, and goals" (p. 182). Kahneman (2011) identifies two systems to account for or explain the decisions and choices that people make: "System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control. System 2 allocates attention to the effortful mental activities that demand it, including complex computations. The operations of System 2 are often associated with the subjective experience of agency, choice and concentration (p. 20). Hence, it becomes important to document the extent to which the opportunities and experiences that prospective teachers develop and encounter in their education influence the development of their practices.

How do teachers construct their orientations or beliefs, dispositions, values and resources to pursue their goals? What is the role of teachers' initial preparation and experience to achieve instructional goals that are consistent with mathematical practices? To delve into the teachers' preparation implies recognizing that there are multiple paths or programs and traditions to prepare prospective teachers around the world. In some cases, the faculty of education and the mathematics departments jointly share the responsibility to prepare teachers; other programs are part of a school or institution (*e.g.*, normal schools) dedicated exclusively to the education of teachers. Both teaching models recognize the need and importance for teachers to develop the mathematical and didactic knowledge that can help them structure and implement proper conditions for students to learn the subject. However, the extent to which prospective and in-service teachers develop the mathematical sophistication needed to structure a sound mathematical lesson, to interpret students' ideas or comments and guide their learning has recently been questioned and is generating an ongoing debate.

Even (2011) states that the assumption that "advanced mathematics studies would enhance teachers' knowledge of mathematics, which in turn will contribute to the quality of classroom instruction" needs to be reexamined in terms of what it means for teachers to have adequate subject-matter knowledge to become an expert teacher and how a teacher can develop and use that knowledge in his or her teaching.

Many practicing teachers, for different reasons, have not learned some of the content they are now required to teach, or they have not learned it in ways that enable them to teach what is now required. … Teachers need support if the goal of mathematical proficiency for all is to be reached. The demands this makes on teacher educators and the enterprise of teacher education are substantial, and often under-appreciated (Adler, et al., 2005, p. 361).

To shed light on the role of advanced mathematical knowledge in teachers' classroom decisions, Zazkis and Mamolo (2011) provide examples where teachers' awareness of that knowledge becomes useful to orientate the development of a lesson. They use the construct *horizon knowledge* to refer to "teachers' advanced mathematical knowledge which allows them a "higher" stance and broader view of the horizon with respect to specific features of the subject itself (inner horizon) and with respect to the major disciplinary ideas and structures…occupying the world in which the object exists (outer horizon)" (p. 10). For instance, they describe a class event where Mrs. White asked her Grade 3 students to count the number of triangles formed by drawing segments from each vertex of a regular pentagon to the other vertices. When students provided their answers (32, 27), Mrs. White noticed that both were wrong and her recognition was based on using a mathematical result she had studied in her university course of algebra. Thus, Zazkis and Mamolo suggest that it is important for teachers to take advanced mathematics courses in order to widen their mathematical horizon and to use that knowledge during the development of their teaching practices.

In Santos-Trigo and Camacho-Machín (2010) we show mathematical thinking features during the implementation of problem solving approaches in teaching scenarios where teachers systematically use diverse computational tools. In this process, we also identify and discuss concepts and processes associated with the different approaches consistently shown by the students when they think of the problem in terms of the tools employed. The discussion leads us to identify components of a framework that teachers can use to structure and guide their students' use of several tools in problem solving approaches. The framework was presented in terms of episodes that are extensions of the phases used by Polya (1945) to explain problem solving behaviour.

We argue that the ways in which prospective teachers study mathematics courses play a crucial role in developing the resources and contents to be used in their teaching. In other words, it is not sufficient for prospective teachers to take advanced courses; but they also need to reflect on ways to connect mathematical results with other problems or situations. In this context, the systematic use of diverse digital tools can provide prospective teachers with the opportunity to enhance and extend problem-solving approaches that involve the use of pen and paper. For instance, with the use of dynamic software they can construct models of mathematical problems where objects or elements can be moved within the model to observe patterns of parameters that emerge as a result of moving those objects. For example, the task of "finding the locus of points whose distance to a fixed point is the same as the distance to a given line" can be initially approached by drawing a line L and a fixed point F (Figure 2). Then, later, a perpendicular line to L at point A on L and the perpendicular bisector of segment AF are drawn. This perpendicular bisector intersects the perpendicular to L at P. What is the locus of point P when point A moves along line L? Since P is on the perpendicular bisector, then d(P, A) = d(P, F) and this means that the locus is a parabola with focus point F and directrix L.

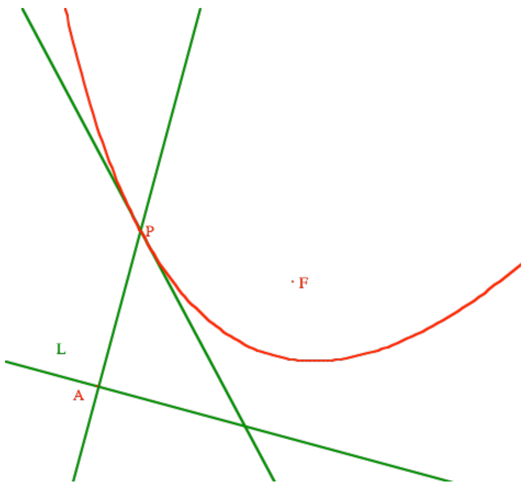


Figure 2: What is the locus of point P when point A is moved along line L?

It is obvious that using a dynamic approach to represent and deal with this problem differs from an analytic approach where the problem statement needs to be represented and operated algebraically. The software approach involves exploring the meaning and relations of the perpendicularity concept and the perpendicular bisector to the conditions of the problem statement in order to construct a dynamic model of the problem. Can this approach be applied to other families of problems? What kind of learning opportunities can this dynamic approach offer to prospective teachers and students? Our goal is to analyze salient features of mathematical processes involved in approaching textbook problems through the use of dynamic software. We argue that the use of the tool offers opportunities for prospective and current high school teachers to transform some routine problems into a set of activities that fosters mathematical reflection and connections between concepts. This activity is useful for exploring, extending and discussing mathematical concepts in problem solving situations.

THE CONTEXT, PARTICIPANTS AND METHODS

Our research group formed of mathematics educators, mathematicians and high school teachers, has focused its agenda on analysing the types of reform that teachers' educational programs need to consider in order to incorporate the systematic use of digital technology to develop both mathematics and didactical knowledge. Our initial literature review revealed that there is scant information on the characterization of ways of reasoning that subjects or learners can construct as a result of using a particular tool and how the use of several tools can help them enhance their problem solving approaches.

Our point of departure was to work on mathematics problems that appear in textbooks with the use of dynamic software and discussed, within the group, features of mathematical reasoning that characterize and are consistent with this approach. We focus on contrasting the software approach that involves visualizing and supporting mathematical results through the use of empirical and geometric arguments with the analytic approach that relies on the use of algebra to identify mathematical relations. In this process, the software properties are helpful in visualizing the behaviour of particular parameters or relations without making the algebraic model explicit. We contend that while the use of the dynamic tool demands that problem solvers think of the problem in terms of properties and their geometric meaning to construct a dynamic model; the analytic approach asks the problem solver to represent and explore the problem through an algebraic model.

In this report, the unit of analysis is the group's work displayed while working on and discussing textbook problems. We do not intend to describe in detail the contribution of each group member to the solution; instead, we focus on what the group as a whole agreed and identified as important ideas associated with the problem solving process. A set of textbook tasks that appears at the end of the unit involving the study of perimeters and areas of triangles is used to illustrate the common mathematical features that emerged during the solution process,. It is important to mention that the initial task was to analyse the list of problems in terms of concepts involved and possible strategies needed to approach each problem. We found that the problems, in general, required finding a particular answer and there was little opportunity for learners to connect the statement with other concepts or problems. Here, we regrouped the problems and identified those that could be extended and connected with key mathematical ideas.

An Initial Prompt

Two textbook problems in which students are asked to find areas and perimeters of particular triangles were slightly changed and posed as:

a. Draw two triangles that share one side AB (base) and whose third vertex lies on a line that is parallel to segment AB. By observing the figure, do those triangles have the same area/perimeter (explain)? With the use of the software, observe what happens to the area and perimeter values of the triangles when the third vertex is moved along line L.

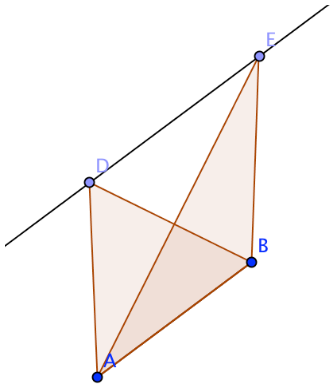


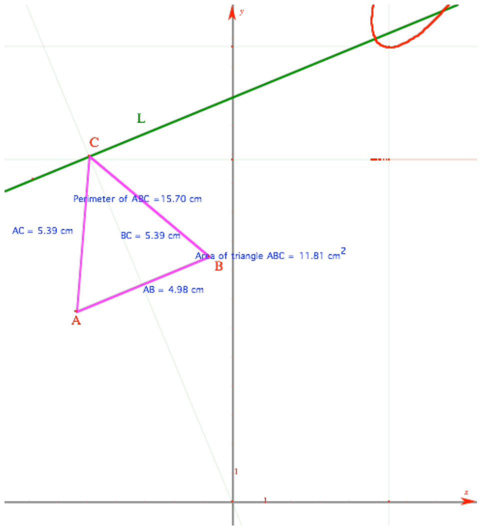
Figure 2: What happens to the triangle's area when point C is moved along line L?

b. Can you construct three triangles that share a common side (base) and also the same perimeter?

The group discussion of the tasks was framed around two problem-solving principles:

- All tasks are conceptualized as opportunities for learners to connect or extend initial statements.
- Solving the tasks involves looking for different ways to represent, explore and solve them and contrasting mathematical qualities associated with the solution process.

For example, during the process of dealing with the first task, the representation shown in Figure 3 was generated and it became a source of introducing several concepts into the discussion.



What happens to the triangle's area and perimeters when point C is moved along line L? How to graph area and perimeter behaviour? Is it possible to identify a position where triangle ABC reaches it minimum perimeter? How can we prove it?

Figure 3: Graphic representation of the perimeter and area of the generated triangles.

We argue that with the use of the dynamic software, learners have an opportunity to quantify particular parameters (side, area, perimeter, angles, etc.) embedded in the representation and observe their behaviours within the dynamic model. The use of the tool also allows the problem solver to graph particular functions or relations without defining the algebraic model explicitly. For example, the graph of the variation of perimeter as a function of the length of side AC and the corresponding perimeter as point C is moved along line L. Conjecture emerged here:

> From all the triangles that are formed by moving point C along line L, the triangle with minimum perimeter is located when point C is the intersection of line L and the perpendicular bisector of AB. That is when triangle ABC is isosceles.

Other concepts that were addressed while discussing this task include:

- The concept of the height of a triangle and the calculation of the area.
- The concept of variation (graph of perimeter) and constant function.
- The concept of perpendicular bisector and its relation to the perimeter variation.
- The concept of infinity (how many triangles can be generated while moving point C along L?)
- The use of the Cartesian system and ways to support conjectures.

The first task also provided the context to address problem b). Using the software, Figures 4 and 5 represent two ways to explore the problem:

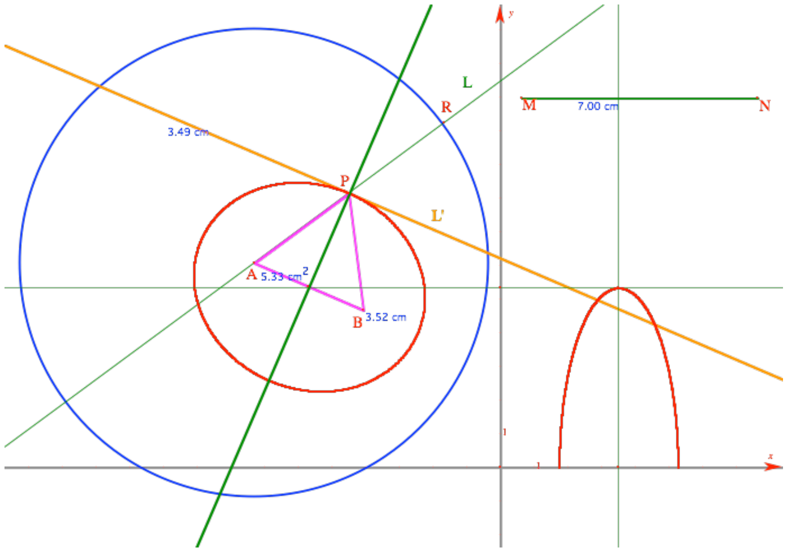


Figure 4: Drawing the condition through the perpendicular bisector

What is the locus of point C when point P is moved along segment MN?

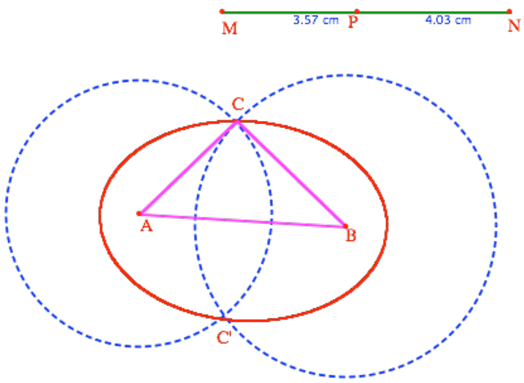


Figure 5: Another way to explore the problem

In Figure 4, segment AB represents the common triangle side and segment MN the sum of the other sides of the triangle, i.e., the perimeter of triangle ABC is the sum of segment AB and segment MN. Point A is the centre of a circle with radius segment MN and AR is a line passing by R (which is any point on that circle). L' is the perpendicular bisector of segment BR which intersects line AR at point P. The locus of point P, when point R is moved along the circle, determines the set of points that are the candidates for locating vertex C to form a family of triangles with a fixed perimeter. Indeed, the locus is an ellipse since PB = PR (definition of perpendicular bisector) and AR is the radius of a circle. Figure 5 represents another way of exploring the same problem: Segment AB is the common side and MN is the sum of the other two sides of the triangle. P is a point on segment MN. Two circles are drawn: One with centre point A and radius MP and the other with its centre at point B and radius PN. These circles intersect each other at points C and C'. The locus of point C when point P is moved along segment MN is an ellipse and each point on this locus determines and becomes the third vertex of triangle ABC with a fixed perimeter. Thus, Figures 4 and 5 were two different dynamic models of the problem and the source to discuss the following:

- The relationship between the common side AB and the sum of the other sides (segment MN). That is, when can the triangle be drawn? (the triangle's inequality).
- Definition and properties of the ellipse.
- The area variation of the family of generated triangles. Again, the intersection point of the perpendicular bisector of segment AB and the perpendicular bisector of segment BR determines the vertex C where triangle ABC gets its maximum area (Figure 4).
- Connections that emerge while moving point B out of the circle (Figure 6). It is observed that the locus becomes a hyperbola.

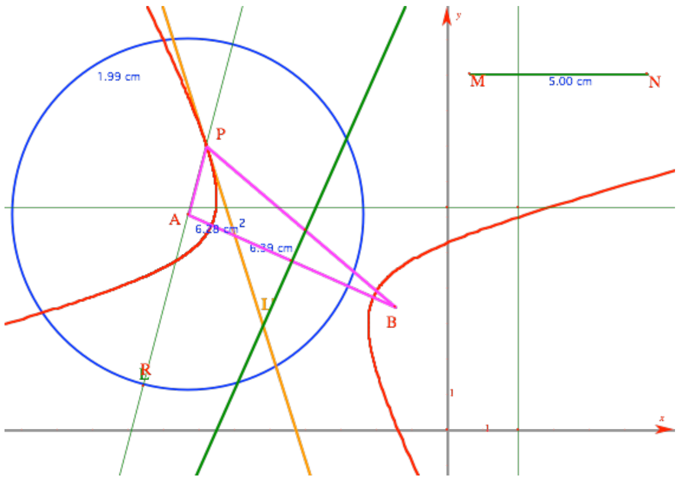


Figure 6: when point B is moved outside the circle, the locus is a hyperbola.

DISCUSSION

We recognize that there might be different ways or programs to develop prospective teachers' mathematical and didactical proficiency; however, all of the aforementioned used mathematical tasks as the vehicle for promoting the development of that competence. In other words, the type of tasks, the ways to frame or structure them, and the opportunities that learners encounter during the problem solving environment are key ingredients to foster the learners' mathematical inquiry and reflection.

We argue that the use of computational technology (dynamic software) can offer learners the possibility of transforming certain routine problems, found in regular textbooks, into a set of tasks where they can exhibit and contrast different ways of reasoning about the problem that involve visual, empirical and formal approaches. In this context, the use of dynamics tools plays an important role for conceptualizing the tasks as an opportunity for learners to engage in an inquiring or inquisitive process that goes beyond reporting a particular solution. For example, the dynamic models of the two tasks become a platform to explore not only different forms of representing emerging relations, but also ways to extend and connect the initial statement of the tasks. In this process, it is possible to generate graphic behaviour of particular relations (perimeter variation and locus of particular objects) without defining the algebraic model. In addition, concepts like perpendicular bisector and loci of points became relevant to explain and justify those relations. In general terms, the use of the tool offers problem solvers the opportunity to graphically examine relations that can later be explored and contrasted algebraically. In this context, the use of the tool complements or extends mathematical reflection that learners engage in when using algebraic approaches.

REMARKS

We argue that the use of computational tools plays a crucial role in extending prospective and practicing high school teachers. Thus, typical tasks found in textbooks offer a point of departure to construct dynamic models or tasks in which problem solvers can identify and explore not only different mathematical routes (contrasted with algebraic approaches) but also possible didactic ways to discuss empirical, visual and graphic approaches. We also contend that prospective teachers should discuss mathematical tasks and the use of computational tools within a community consisting of mathematicians, educators and practicing teachers.

Acknowledgement note

This report is part of wider project that deals with prospective and high school teachers' use of digital tools in extending both mathematics and didactic knowledge. Reference numbers: Conacyt-168543, EDU2011-29328, EDU2009-07298.

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