BRIDGING DIAGNOSIS AND LEARNING OF ELEMENTARY ALGEBRA USING TECHNOLOGIES

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This paper presents research developed in the multidisciplinary PépiMep project (supported by the Ile de France region) which consists in transferring diagnosis and differentiation resources into the Sésamath platform, very much used by mathematics teachers in middle school in France. The research is based on the potentialities of the diagnosis software Pépite, which establishes an individual cognitive profile of the students in elementary algebra. We designed an interface to allow teachers to generate automatically exercises for differentiated instruction courses adapted to the learning needs of various groups of their classes.

Keywords: diagnosis and learning, differentiated instruction course in algebra, ICT, elementary algebra.

INTRODUCTION

Effectively helping students in the classroom with appropriate learning material is a difficult task for teachers. To make every student progress, they need detailed diagnosis about individual students' learning. But, simultaneously, teachers also need to manage the whole classroom by proposing differentiated activities adapted to groups of learners with close competences in algebra or who require the same teaching strategy. Our hypothesis is that software tools can help teachers to do that. This paper addresses theme 2, "students' learning with technologies", of the Working Group 15 and the question of the impact of using technologies on students' learning in the field of elementary algebra at the end of compulsory education in France (16 years).

Our research concerns the development and the use of online resources for diagnosis and differentiated learning. It takes place into the *Pépite* and *Lingot* projects, two multidisciplinary projects in Information and Communication Technology (ICT) (Delozanne and al. 2010). Based on a multidimensional analysis of algebraic activity (Grugeon 1997), the *Pépite* software is a diagnosis tool which generates automatic multi-criteria assessments of students' competences in school algebra (Delozanne and al. 2008). Furthermore, it also shows the principal characteristics of students' activity in algebra which constitutes their cognitive profile.

This paper deals with the transfer and the integration of the diagnosis *Pépite* within the online databank *LaboMep*, developed by the French maths teachers association *Sésamath*. The success of the *LaboMep* platform shows that such online resources may answer the teachers' needs. More precisely, from the potentialities of the diagnosis software *Pépite*, which establishes individual cognitive profiles of the students in elementary algebra, our goal is to design an interface to automatically index exercises for differentiated instruction courses in algebra adapted to the learning needs of various groups of a class. We deal with two research questions. On the mathematics education side: how can we automatically generate exercises of differentiated instruction courses proposed to students according to their diagnosis assessment about the learning objectives aimed at by the teacher? On the collaborative work side: how can the collaborative work between IT specialists and mathematics education researchers make possible the creation of an operational model to index a database of exercises that allows you to automatically produce differentiated instruction courses adapted to students' learning needs?

After clarifying some theoretical and methodological elements for the diagnosis, we will discuss the design of differentiated instruction courses. We will explain the principle of indexing exercises databases that allow for the automatic generation of instruction courses.

THEORETICAL AND METHODOLOGICAL ELEMENTS

Assessment

Diagnosis assessment is an important part of teachers' practice. But this term is used to refer to several types of assessment. Ketterlin-Geller and Yavonoff (2009) identify two practices within diagnosis assessment that use students' procedures and errors as their basis for analysis: analyses of students' answers to specialized tests and cognitive diagnosis assessments using standardized and psychometric models. In both cases, the objective is a local study of students' misconceptions. The diagnosis developed through the *Lingot* project aims to create an overall and multidimensional analysis of students' knowledge and abilities in algebra. It doesn't use psychometric models. It doesn't rely more on the conversation theory whose fundamental idea is that learning occurs through conversations between students which serve to make knowledge explicit (Scott 2001). It is based on an epistemological study of elementary algebra from cognitive and anthropological approaches which enables to predict students' learning needs.

Assessment and epistemological references

Grugeon (1997) defined a model of algebraic competence at the end of compulsory education. It is the foundation of an epistemological reference to guide the design of an appropriate diagnosis (Artigue and al. 2001). This approach allows one to categorize tasks for a diagnosis test – problems of generalisation and proof, traditional arithmetical problems, problems where algebra appears as a modelling tool, algebraic and functional problems – and to structure the different aspects of the multidimensional analysis of students' activities in elementary algebra.

From an international synthesis of research related to the learning of algebra, Kieran (2007) proposed the GTG model of conceptualizing algebraic activities which differentiates three complementary aspects: (1) *Generative activities* involve the production of expressions or formulas or equations or identities (2) *Transformational activities* involve the usage of transformational rules (factorizing, expansion of products, rules for solving equations and inequalities, etc.) (3) *Global/meta-level activities* involve the mobilization and usage of the algebraic tool to solve different types of problems (modeling, generalization, proof). This model will be used to create student working groups composed of students who can work on tasks with the same learning goals.

Assessment and Anthropological Theory of the Didactic (ATD)

The ATD theory takes into account the institutional context of education. It proposes an epistemological model in which all human activity is to accomplish a task of some type of tasks with some technique. The technology of the technique is intended to provide justification for the technique and the praxeology's theory supposed to justify the technology itself. We postulate that assessment should locate the personal relationship of students with algebra in solving diagnosis tasks, and technological elements involved in their resolution compared to those expected, taking into account praxeologies to teach (curriculum, textbooks) and praxeologies taught (teaching practice) (Chevallard 1999). The linking between epistemological references and institutional praxeologies in algebra allows one to identify learning needs often ignored by the institution and often implicit in the curriculum and textbooks (Bosch, Fonseca and Gascon 2004, Castela 2008). For Chevallard, "assessment will focus, by practical necessity, for each student, on a sample of all types of tasks constituting referred praxeological organizations". Diagnosis tasks are thus characterized by a type of tasks, the complexity of algebraic objects involved, the level of involvement of tasks in the resolution.

Collaborative Work

An iterative process between educational researchers, computer scientists, teachers and trainers allowed to design and to test prototypes that implement the diagnosis in order to favor its evolution. There were four iterations to test the different versions of diagnosis (El-Kechaï and al. 2011). Our research approach is a bottom-up one informed by educational theory and field studies. In previous work (iteration 1), we started from a paper and pencil diagnosis tool grounded on mathematical educational research and empirical studies. Then (iteration 2), we automated it in a first prototype, also called *Pépite,* and tested by dozens of teachers and hundreds of students in different school settings (Delozanne and al. 2005). In more recent work (iteration 3), we implemented *Pépinière* that generalizes the first tool to create a framework for authoring similar diagnosis tools, offering configurable parameters and options (Delozanne and al. 2008). From 2010 (iteration 4), with the *PépiMep* project, we deployed the *Pépite* diagnosis tool on the *LaboMep* platform developed by *Sésamath*. For each iteration, we started with a didactic model and we defined a formal model for the implementation of the prototype. After, we tested prototypes with teachers or researchers (coding exercises, terms used in the interface to present the assessment, interface to index exercises) and, if necessary, we proposed an evolution of the prototype. Particularly, we tested the interface with teachers to adapt the terms used (type of tasks) with those of the curriculum and the mathematics textbooks (capacity).

FROM ASSESSMENT TO DIFFERENTIATED INSTRUCTION COURSES

We now present the principal elements taken into account to define the differentiated instruction courses related to the diagnosis assessment which supports the indexation of exercises.

Assessment and stereotypes

The diagnosis test included in the *Pépite* software implemented in *LaboMep* is composed of 10 diagnosis tasks (27 items), which cover the range of algebraic problems: exercises involving creating mathematical representations of problems in order to generalize, create a model, complete a proof, or write an appropriate equation (7 items); exercises covering techniques of algebraic calculation (8 items); or exercises in recognition (19 items). The diagnosis tasks may be multiple-choice or open-ended questions (Grugeon and al. 2012). Responses to the items of the diagnosis test are not only analyzed in terms of success or failure and mistakes. They are also coded according to properties and justifications used repeatedly, corresponding to institutionally recognized technologies, which highlight a coherent set of techniques (correct or incorrect) built within the institution. The main characteristics of a student's cognitive profile, considered relatively to its grade level, are automatically calculated by *Pépite* through a transversal analysis that codes the student's responses to the 10 diagnosis tasks.

The above model provides a description of the cognitive profile of each student. However, teachers need to use the diagnosis to form groups of students who require the same proposals of teaching to manage the whole classroom. We defined cognitive stereotypes in elementary algebra (Delozanne and al. 2010) as sets of equivalent profiles that can be considered close enough that students can work with the same learning goal tasks. The stereotypes model has three components: Usage of Algebra for solving problems (coded UA); flexibility in translating different types of representations (geometric figures, graphical representations, natural language) into algebraic expressions and vice versa (coded TA); ability and adaptability in the various uses of algebraic calculations (coded CA). For each of the three components, a scale with different technological levels has been identified, along with appropriate criteria for each level (Delozanne and al. 2005). For example, for the component CA, we distinguished three technological levels according to the types of manipulation and associated justifications: (CA level 1) expected technology taking into account the structure of expressions and their equivalence, (CA level 2) technology only supported on syntax rules, (CA level 3) technology without operational priority leading to concatenation rules and false linearity.

Stereotypes and differentiated instruction course

We postulate that stereotype definition in elementary algebra is an important step in comparison with categories (good, average, low) commonly used by teachers regarding the design and the implementation of differentiation strategies. Indeed, *PépiMep* automatically calculates groups of students who have close profiles in algebra (Grugeon and al. 2012). Figure 1 shows (on the left) the cognitive profile of a 9th grade student with (on the right) the groups of students (A strong, B or C weak).

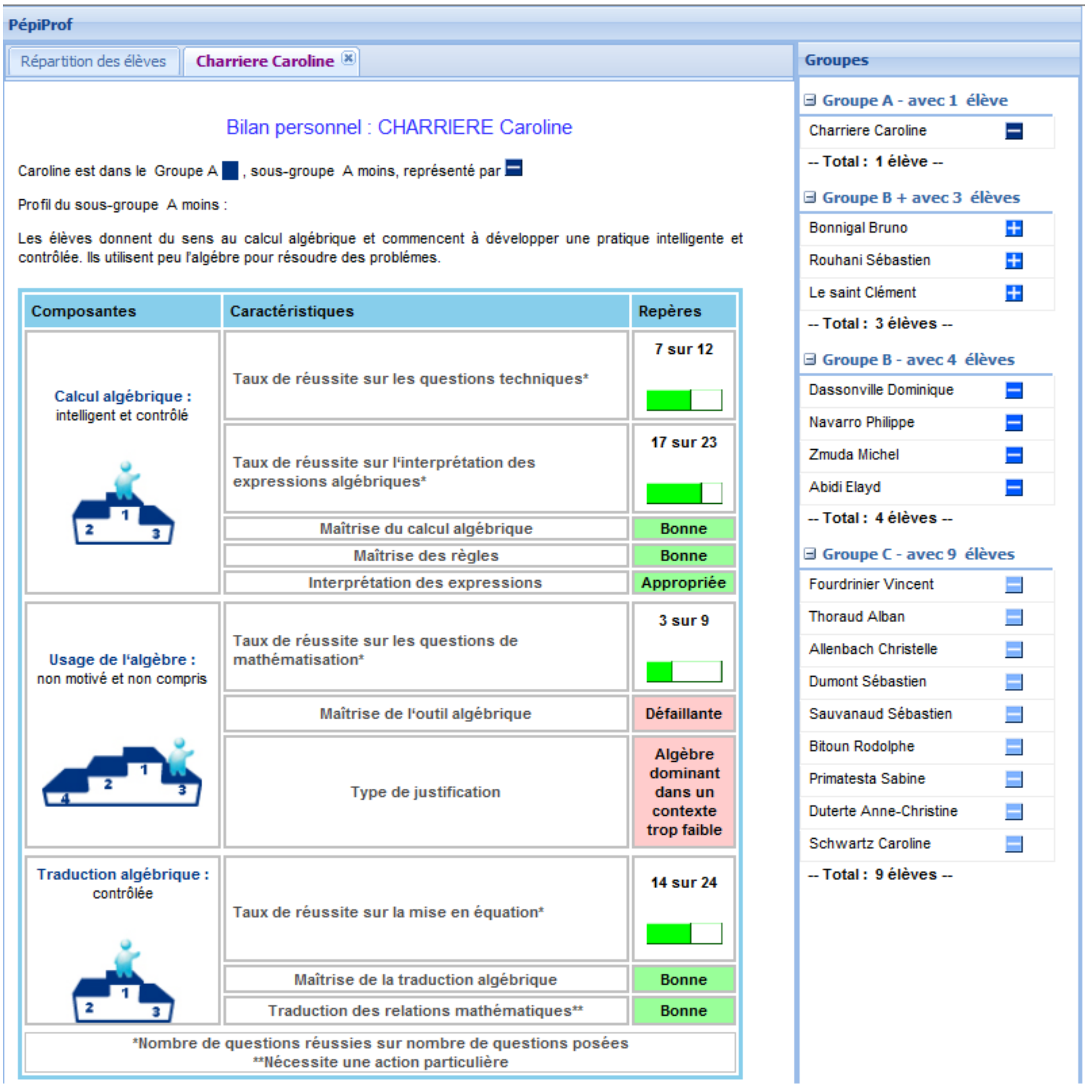


Figure 1: Personal cognitive profile of a 9th grade student

This model allows one to identify target learning needs often ignored by the institution. These are essential common learning issues to work on by all the students of a classroom: particularly, the need to produce general expressions to prove the equivalence of calculation programs, the dialectic numerical / algebraic, the double aspect, procedural / structural of an object, the equivalence of expressions (Pilet 2012). To define a differentiated instruction course, for a given mathematical topic, at a grade level, at a time of education, we identified issues of common learning for the class (presented above), i.e., the tasks involving types of tasks to work on. For a given purpose, we assigned tasks to each group, tasks that are associated with variables related to the technological level involved. An example of a differentiated instruction course is proposed in Grugeon (2012). This design also supports the indexation of the exercise database.

INDEXATION OF AN EXERCISE DATABASE

Our indexation is structured by capacity. This choice can appear in opposition with the ATD theory briefly exposed previously, which organizes assessment like the personal relationship of students with algebra in solving diagnosis tasks, and technological elements involved in their resolution. This choice from the necessity to take into account the teachers' working context: they need exercises categorized by capacities in accordance with the curriculum and the mathematics textbooks. A capacity refers to a kind of task to which is attached the targeted technology.

We developed an interface for the data capture relative to every exercise of the exercise database. For each exercise, our indexation takes into account: identification parameters (identifying in the database, title of the exercise), the school level for which the exercise is intended (7th grade, 8th grade, 9th grade and 10th grade), the targeted capacity (detailed below), the mathematical domain concerned (literal calculation), the input and output objects (numbers, algebraic expressions…), the task complexity (detailed below), the input and output frame.

More precisely, the targeted capacity is composed of three elements: relevant components (UA, TA or CA), main capacities for each component (see table 1), elementary capacities for each main capacity (see table 2). The task complexity, as in a PISA test, is related to the level of proposed tasks described by Castela (2008): EL elementary, CS conceptual simple, MP multi-step, CX complex.

Component UA 0 – Conjecture that calculation programs or literal expressions are equal or not 1 – Produce a literal expression or formula for solving a problem 2 – Put in equation and solve a problem 3 – Demonstrate (calculation rules, properties, identities) or prove that calculation programs or literal expressions are equal or not 4 – Expressing a variable according to a formula in another
Component TA 5 – Translate a literal expression, a calculation program… 6 – Graph a function (linear or affine), the solutions of inequality or of system 7 – Recognize an object 8 – Read on a graph

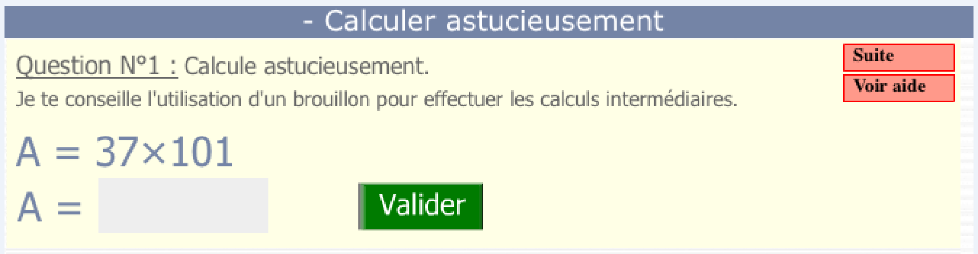
Component CA
9 – Calculate
10 – Test equality
11 – Reduce a simple polynomial expression
12 – Develop a simple polynomial expression
13 - Factorize a simple polynomial expression
14 – Know the remarkable identities
15 – Transform equalities
16 – Recognize the structure
17 – Choose the most suitable form of an expression
18 – Solve equality or inequality or system
19 – Determine the algebraic expression of a linear or affine function from data
20 – Identify a calculation error and correct

Table 1: Main capacities table

9 – **Calculate**	9.1calculate the value of a literal expression giving numerical values in the variables
	9.2 calculate a numerical expression using a rewrite
	9.3 calculate a numerical expression by using the remarkable identities
	9.4 calculate the result of a calculation program for a number
	9.5 calculate the value of a literal expression knowing a numerical relation linking variables
	9.6 calculate the image of a number by a function

Table 2: Elementary capacities for main capacity 9 “calculate”

For example, exercise 578 (see figure 2) is relevant for component CA.



Calculate cleverly.

I recommend you the use of a draft to make the intermediate calculations.

Validate

Figure 2: Exercise 578

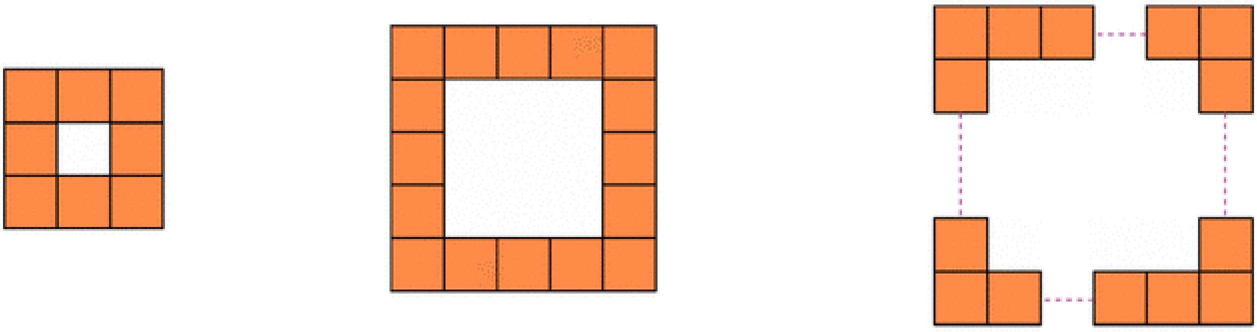
Three main capacities are worked on in this exercise: 9 calculate (9.1calculate the value of a literal expression giving numerical values in the variables - 9.2 calculate a numerical expression using a rewrite *(for example 101=100+1)*); 12 develop a simple polynomial expression (12.1 develop an expression by using the simple distributivity of multiplication on addition). In particular, the question of rewriting the expression convened in capacity 9.2, is not often highlighted in class to carry out the calculation successfully.

EXAMPLE OF AUTOMATIC GENERATION OF A DIFFERENTIATED INSTRUCTION COURSE

The automatic generation of differentiated instruction courses uses *LaboMep* exercises indexed by main capacities and elementary capacities. From the common learning objective selected by the teacher for all students in the class, a list of capacities to work on is proposed. In addition, the teacher proposes differentiated exercises for each group by changing the choice of expressions. We now present an instruction course having for objective: working on the role of algebra to solve generalization problems. This instruction course includes hidden capacities corresponding to learning needs ignored by the institution. More precisely, if generalization problems are discussed in classes, equivalence of calculation programs is very merely worked on.

This instruction course leans on the lined square exercise (see figure 3), a generalization problem, which aims at proving that several calculation programs are equivalent. This problem consists in establishing algebraic expressions that allow one to calculate the number of square units colored with a figure built on the model below, whatever the number of square on the side of the white square is. Writing algebraic expressions involves operating priorities. It includes several steps: determine the number of squares colored for definite values of the number of squares on the side of the square, produce a mathematical expression, compare calculation programs.

We consider a white square surrounded with square units colored as on figures below. The objective is to calculate the number of square units colored.



1) If the white square has a side of 3 units, calculate the number of square units colored.
2) Same question with the white square with sides 4 units.
3) Same question with the white square with sides 8 units.
4) Same question with the white square with sides 100 units. To help you, indicates at first the process of used calculation.
5) Write a formula which gives the number of square units colored according to the number of square units on the side of the white square.
6) Compare your formula with those found by your classmates. What can you say about these formulae?

Figure 3: The lined square exercise

In this exercise, algebraic expressions are objects with which you can make calculations replacing letters with numbers. How can we show that two calculation programs are equivalent? This is possible using calculation rules that guarantee the equivalence of calculation programs that translate expressions. Algebraic identities, such as simple distributivity, that students need to admit, are involved. The articulation between the procedural and structural aspects of an expression is merely worked on in the classroom. In this instruction course, with the objective “proving that calculation programs are equivalent", students in groups B and C (weak), worked with the lined square exercise which led to first degree expressions: N=4n+4 ; N=4(n+1) ; N=2(n+2)+2n ; N=4(n+2)-4. The students in group A (strong) worked with a similar problem but with a different pattern which involves an expression of the second degree.

CONCLUSION AND FUTURE WORK

We have described research about a diagnosis tool, developed in research laboratories, and transferred to *LaboMep*, an ICT platform mainly used by mathematics teachers at the secondary school level. The diagnosis tool allows the teacher to have, for every student, a very precise profile concerning their skills in elementary algebra. By using an indexation of the considered domain, the software automatically proposes, to groups of students identified as having close profiles, a differentiated instruction course adapted to their competences. This automation of the differentiated instruction course was made possible by the crossed successive enrichment of the underlying didactic model and the IT model. Using ICT leads to rethink the teaching resources not only for the students but also for the teachers, so two Ph. D. projects are in progress: Julia Pilet's thesis is interested in profits for the students while the thesis of Soraya Bedja studies the integration of the software in teaching practices.

Future research consists in honing the role played by capacities. What is the impact of the choice of capacities on the instruction course automatically proposed by the software? What is the role played by capacities in the link with the activity of the teacher in his classroom? Can we specify an ontology in elementary algebra? Indeed, this question of the choice of capacities is at the heart of the problem of the transferability of our research in a wider mathematical domain, even in other mathematical domains.

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