INSTRUMENTAL GENESIS IN GEOGEBRA BASED BOARD GAME DESIGN

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In this paper I address the use of digital tools (GeoGebra) in open ended design activities, with primary school children. I present results from the research and development project "Creative Digital Mathematics", which aims to use the pupil's development of mathematical board games as a vehicle for teaching skills with GeoGebra, as well as an entrepreneurial attitude towards mathematics. Using the instrumental approach I discuss how open ended transdisciplinary design activities can support instrumental genesis, by considering the extent to which the pupils address mathematical knowledge in their work with GeoGebra and how they relate their work with GeoGebra and mathematics to fellow pupils and real life situations. The results show that pupils' consider development of board games as meaningful mathematical activity, and that they develop skills with GeoGebra, furthermore the pupils considers potential use of their board game by classmates in their design activities.

DIGITAL TOOLS IN PRIMARY EDUCATION

The use of digital technologies, such as symbolic calculators, computer algebra systems and dynamic geometry systems, are changing teaching and learning of mathematics at different levels of the educational system. These tools leads to new didactical possibilities but new challenges emerge as well (Borba & Villarreal, 2005; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Guin, Ruthven, & Trouche, 2005; Kaput & Balacheff, 1996). Most research on the possibilities and pitfalls with digital tools for teaching mathematics deals with students at secondary and tertiary level, but recent research suggest that dynamic geometry tools are relevant in primary level as well (Sinclair & Moss, 2012). Furthermore curricular development in several countries suggests that ICT should be included in the primary level curriculum (for example Norway see, Saabye, 2008). Hence trends in research and curriculum development suggest that it is relevant to investigate possibilities and problems with using dynamic geometry tools in primary education.

Innovation and technological development drives a substantial part of the economy, and the application of mathematical concepts, models and methods to the developing cultural artefacts is hence of increasing importance. This development could call for a more entrepreneurial and attitude to the interplay between mathematics and technology in educational settings. Such entrepreneurial approach has been addressed by relating education more directly to innovative disciplines (Rangnes, 2011; Shaffer, 2006), and, with the use of robotics and programming languages (Resnick, 2012).

In this paper I will investigate the process of instrumental genesis in a situation where pupils in primary and middle school use GeoGebra in an intervention where they design their own mathematical board game. The analysis is based on the instrumental approach (Guin et al., 2005), and focusses on three aspects of the pupils instrumental genesis; the degree to which GeoGebra is appropriated to fulfil the students own need, the pupils use of GeoGebra for epistemic mediations towards mathematical concepts, and whether or not the pupils considers their work with designing games as authentic in the sense that it relates to a future use situation where someone is playing the game.

GAME DESIGN AS A MATHEMATICAL ACTIVITY

The empirical basis of the current report is a pilot project for the project *Creative Digital Mathematics*. The pilot project has been running between March 2011 and November 2011, and been through 2 cycles of design and intervention (grade 5 = age 11 and grade 3 = age 9). In both cases this intervention was the first time that these pupils used powerful mathematical tools in their mathematics class. In each intervention, the pupils have developed their own board game using the tool GeoGebra. GeoGebra is a dynamic mathematical software that provides a close connection between symbolic manipulation, visualisation capabilities, and dynamic changeability of geometrical constructions. In this project GeoGebra is mainly used for its geometric capabilities, and less for dynamic and algebraic capabilities.

The pupils' work is organised by a web based interface, and they start with a few simple drawing tasks, continues to solve a number of mathematical tasks before they start developing their project; a mathematical board game designed in GeoGebra. The teaching material is collaboratively authored by the involved teachers with inspiration and technical assistance by the researcher (Morten). The tasks are simple instructions inviting pupils to use GeoGebra for a number of aesthetic and mathematical activities. A translation of the first and simplest scenario ("the fraction crusher" – designed for grade 5.) can be found at the following url: https://sites.google.com/site/fractioncrusher/fractions, and the second scenario ("the multiplication crusher") can be found in Danish at the following url: https://sites.google.com/site/spilfabrikken/.

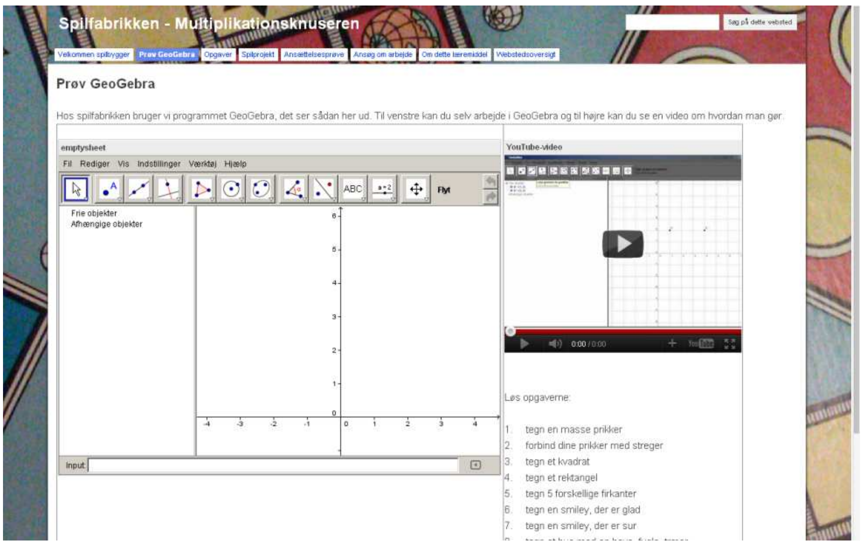

Figure 1: Screenshot from "multiplication crusher" with tasks (text field, bottom left), video-introduction (top left) and an embedded GeoGebra workspace

The board game design activity is an important part of the teaching scenario. The pupils make their visual layout of the board in GeoGebra, they are writing rules for the game, printing the game and trying to play it with their classmates.

THEORETICAL FRAMEWORK

The intervention and research design is guided by two concerns with relates to different theoretical frameworks. The first concern relates to the didactical use of transdisciplinary, open ended and entrepreneurial practices that simulates professional activities, and the second concern deals with the use of digital mathematical tools in the classroom. The analysis that I present are based on the instrumental approach, and relates to the second concern. However I present the theoretical foundation relating to the first concern in short.

Shaffer has developed the concept *epistemic frame* to describe students learning in simulated work life situations. He describes and epistemic frame as a combination of values, knowledge, skills, and identity that people have when they are competent in such a work life situation (Shaffer, 2006). This means that he view students activities in the light of professional skills, knowledge and values, and considers students learning as a result of adopting a certain epistemic frame. Shaffer describes interventions aiming at this kind of learning as epistemic games.

The term *microworld* was first used by Papert (Papert, 1980) to describe how the Logo software could reform primary and lover secondary mathematics education. Papert's approach was to use pupils creative and aesthetic work, in a computer based environment, as a mean to develop their skills in mathematics. Papert suggest that the combination of Logo's focus on computational procedures and geometric (aesthetic) output allows students to learn mathematics in interaction with computers when they are working to obtain their own goals. It is an important part of Papert's approach to teaching with technology that pupils produce (digital) artifacts as part of their learning process.

Kaput and Ballachef (Kaput & Balacheff, 1996) describes a mathematical microworld as a combination of a set of primitive objects and procedures that constitutes a formal system, as well as a domain phenomenology that determines the feedback that students receive from their on screen work. In a GeoGebra environment the primitives are for example geometrical concepts such as lines, polygons and circles, while the domain phenomenology in could relate to the dynamic aspect of geometric constructions as well as the consistent use of multiple representations.

Students appropriation of digital tools for solving mathematical tasks has been described within the instrumental approach to mathematics education (Guin et al., 2005). The instrumental approach builds an activity theory framework that studies computational artifacts as mediating between user and goal (Rabardel & Bourmaud, 2003), and considers use situations as continuation of a given design (or tool). Hence a pupil's goal directed activity is shaped by his use of a tool (this process is often referred to as *instrumentation*) and simultaneously, the goal directed activity of the pupil reshapes the tool (this process is often referred to as *instrumentalization*) (Rabardel & Bourmaud, 2003, page 673). In order to relate the appropriation of tools in goal directed activities to learning of mathematics Luc Truche (Guin et al., 2005, p.149), referring to Vergnaud (1996), introduces the concept of scheme as consisting of both a conceptual and a competence oriented aspect. Hence we can investigate the schemes in students' instrumented activity by studying the conceptual entities and involved competencies. However such cognitive components can be difficult see empirically and hence I will apply two more concepts from the instrumental approach. A distinction between *epistemic mediations* and *pragmatic mediations* (Guin et al., 2005; Rabardel & Bourmaud, 2003). Epistemic mediations relate to knowledge (Rabardel & Bourmaud uses the example of a microscope, and Lagrange (in Guin et al., 2005, chp 5.), refers to experimental uses of computers), and pragmatic mediations relate to action (Rabardel & Bourmaud uses the example of a hammer, Lagrange (in Guin et al., 2005, chp 5.) refers to the mathematical technique of "pushing buttons"). And finally we take from Rabardel & Bourmaud (p. 669) a sensitivity towards the *orientation* of the mediation. Instrumented mediations can be directed towards (a combination of) the object of an activity (the solution of a task) other subjects (classmates, the teacher) and oneself (as a reflective or heuristic process). Hence my theoretical framework consists of the concepts: *instrumental genesis* as consisting of *instrumentation and instrumentalization,* the concepts *epistemic* and *pragmatic mediations* as well as a sensitivity towards the *orientation* of an instrumented mediation.

RESEARCH QUESTION

The research question that I address in this paper is:

How can the use of GeoGebra in an instrumented board game design activity, support pupils instrumental genesis with GeoGebra?

Furthermore I will investigate *the types of mediations* that GeoGebra serves to the pupils:

> To whom are these mediations directed? To fellow pupils? The teacher? or Towards fulfilling the task?
>
> Are GeoGebra used for epistemic mediations, and what knowledge is involved?
>
> Are GeoGebra used as a pragmatic mediation and towards what actions?

These questions are guided by two hypotheses. The first hypothesis that the nature of GeoGebra as a microwold will support that pupils uses it for epistemic mediations. The second hypothesis is that engaging in open ended design activities, within an epistemic frame allows that GeoGebra act as a mediating artefact towards other subjects and not only towards solving specific tasks.

METHODS AND PROCEDURES

The methodology can be described as design based, in the sense that we have been dedicated to an iterative approach and to the application of theoretically based analysis of learning goals and envisioned learning trajectory, as well as to the collection of empirical evidence (Cobb & Gravemeijer, 2008). The first game design scenario (the "fraction crusher") was developed with the mathematics supervisor from the school partner in the project, and used for a weekly ICT class that this supervisor taught together with a first language teacher. The intention of this intervention was to test the idea of board game design with GeoGebra as a mathematical activity among children in grade 5. The data from this intervention consisted of the students' productions as well as reflections from the two teachers and the researcher who participated in some of the lessons.

The second scenario, the "multiplication crusher", was developed together with the mathematics supervisor of the school and two teachers in grade three who tested the design in their classes. The intention of this intervention was to further understand the mathematical learning potentials in board game design with GeoGebra in grade 3 and to test if the idea of using board game design scenarios would work with different teachers. The data consisted of minutes from meetings between teachers, supervisor and researcher, as well as field notes and pictures from the researcher's participation in a total of 11 lessons in the two classes (six in one class and five in the other class). Both classes' spend 10 lessons working with the scenario. Furthermore a research student conducted four in depth interviews with pupils participating in the "fraction crusher" and four interviews with pupils participating in the "multiplication crusher" (Rosenkvist, 2012).

DATA

In both interventions the pupils worked in pairs. All the (pairs of) pupils developed a game. Most games had some mathematical theme, but for some of the pupils the mathematical theme was very weak. Designing board games was accepted as a meaningful activity by all children. The involved teachers found the pupils engagement and developed competence with GeoGebra to be positive and valuable aspects of the intervention.

The interviews (Rosenkvist, 2012, p. 142-143, here shown in translated form) revealed that the pupils considered their work as mathematical work. And furthermore the interviews revealed that the work they did in the interventions, were very different from the normal mathematics lessons. The pupils in general felt that the GeoGebra classes where much freer, building more on their own ideas than normal classroom activities.

The following quote from the interview protocol describes how one pupil experienced the relation between the activities in the intervention and mathematics:

Interviewer: What mathematics have you used when you made your game. Have you used math to do that?

Pupil: Yes, we have used mathematics. We have created the shapes of the game, we needed to make some shapes.

Interviewer: What kind of shapes?

Pupil: Mostly squares, we have also made some circles and some pentagons.

Interviewer: Have you used any other mathematics than shapes?

Pupil: Yes, we have also used calculations actually, when you landed on a field, then you might need to solve a task.

I have chosen to include this example because it shows the two types of reasons that the pupils gave for considering their game design work as mathematical work. Firstly, the use of GeoGebra for creating visual layout enforced the pupils to design through mathematical shapes, and hence – perhaps – to connect to mathematical concepts. Secondly the task of creating a mathematical game (perhaps together with the institutional context of having a math lessons, with your math teacher) did influence the students' design related discussions, as when the pupil in the transcript describes calculation tasks as a natural part of their gameplay. These two mathematical aspects of the pupils work where typical across the interviewed pupils.

In figure two, two examples of the pupils' game designs is provided. In both examples the students are working in pairs with designing a mathematical board game. In the first example the pupils are working with GeoGebra to design their game and in the second example the pupils are working with pen and paper to design their game, in the latter example the pupils implement their game in GeoGebra later.

The first example is two students that are making a game were you move on tiles (the small circles), around in different "worlds" (the larger circles and the corners). They point to a part of the board and say "this here is a multiplication world, here you have to solve three multiplication calculations and then you can fly on to the next world – which is an addition world".

The other example shows two students that are buiding their initial gamedesign with pencil and paper. They explain about their game that you have to throw a dice in order to get to a field; this could be the field that says 5 times 5. The other player then count to ten while you calculate, and if you get it right, and on time, you can go on, otherwise the turn is given to the next player.

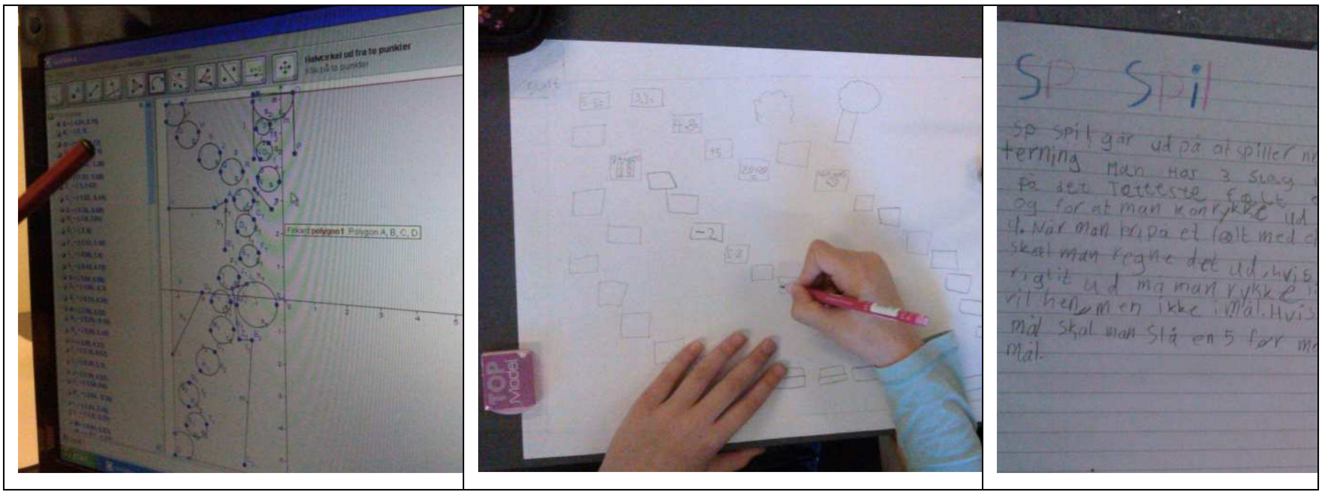

Figure 2 from left (1) Two pupils have made a game where you move between different mathematical worlds. (2) Pencil and paper activities also play a role for some pupils; here pupils are sketching a game board, later to be drawn in GeoGebra. (3) Writing rules for the game.

In this example you also see another aspect of the pupils' activities; they write rules. The rules written by the two pupils reads (translates from Danish): ”The purpose of SP Game is that player number one throws a dice three times, and if you throw a 4 you can move to the nearest next field. When get to a field with a multiplication calculation, you should do the calculation right and then you can move all over the game board but not into the target zone. If you are next to the target zone you have to throw a 5 with the dice in order to get into the target.”

ANALYSIS

In this section I will analyze the presented data in order to answer the research questions.

The main research question of whether or not the board game design activity, supported instrumental genesis with GeoGebra, can easily be answered with a yes. All pupils were somehow able to use GeoGebra for something after the intervention. This observation is not entirely trivial. It could have been the case that the software was too complicated or inappropriate to the age group or the task. However it is contestable if the mere application of GeoGebra to a visual layout task in any way can be viewed as an activity that relates to the teaching of mathematics. Two aspects do suggest that this could be the case: Firstly, the pupils were also doing a number of simple mathematics tasks with the software. Even though we do not have performance data from the pupils both observations and teachers' evaluation suggest that the pupils were able to use GeoGebra to visualise mathematical concepts and solve mathematical tasks. Hence it is reasonable to conclude that the combination of instrumented board game design activity and mathematical tasks allowed the pupils to develop instrumented techniques with GeoGebra, that relates to mathematical goals. Secondly, during the interviews the pupils described that the use of GeoGebra for developing the visual layout of the board game, did force them to reflect on aspects of mathematics. The piece of transcription provided in the data section is typical in the sense that the respondent point to the mathematical shapes as the way in which software made the pupils design work more mathematical. This can be viewed as a process of instrumentation and seen as a result of choosing to work with GeoGebra rather than any other visual layout tool.

The makers of GeoGebra most likely have not considered the type of visual layout activity that the pupils engaged in when designing games. Therefore the pupils often needed to find ways to make GeoGebra “do” various things such as change colour, fill figures completely, and remove points for aesthetic reasons, to mention a few typical activities. This can be viewed as a waste of time and as examples of bad choice of software for the task. However these activities also give the students an experience of appropriating a tool to their own need. Such experience with instrumentalization can be of potential value to the pupils later since it suggests that mathematical tools are open-ended and can be appropriated to different situations in school and life. As an example of this point, some of the fifth grade students did on their own initiative choose to use GeoGebra as a part of an assignment in an English class where an illustration was needed (an proudly announced it to the mathematics teacher afterwards). In that sense data suggest strong signs of the process of instrumental genesis with GeoGebra as a result of the intervention.

It is arguable to what extend we see GeoGebra used for epistemic mediations in the board game design activity. The observed dialogue among the pupils and the questions posed to the teachers, were mainly of a pragmatic nature. By wanting the software to support the development of specific visual layouts, the involved mathematical concepts were not object of investigation in their own right. They were used to get GeoGebra to do what the pupils wanted. However one aspect of the game design can be viewed as a mediation of a more epistemic nature. Many of the pupils included mathematical tasks in their game. The SP game in figure 2 shows a typical example. When developing these tasks some of the pupils were explicit in their discussion about what a difficult mathematics problem is and how such problems could make their game easy or hard. However GeoGebra was not used as a mediating artefact in these discussions.

GeoGebra was used for epistemic mediations by some pupils in some of the mathematical tasks that were done as a part of the intervention before and after the board game design. Especially the tasks that dealt with visualizing mathematical concepts, as for example the task “Make a drawing, which compares the 2/3 and 3/5: Which fraction is the largest? Draw ½, 5/6, 2/4, 6/8, 1/5, ¾, 4/6, 2/10” seemed to allow epistemic mediations.

When analysing the orientation of the use of GeoGebra as a mediating artefact, the situation of future use of the pupils’ game became apparent. While the pupils identification with professional designers was weaker than expected, their relation to the idea of their classmates playing their game was strong. In that sense the board game design did constitute mediation towards others, and this mediation often did have explicit mathematical aspects because it included posing mathematical challenges as part of the gameplay, and because the appropriation of GeoGebra to create a functioning and aesthetic layout did involve working with geometrical concepts.

CONCLUSION

In this paper I have presented an analysis of how an open ended design activity can support instrumental genesis with GeoGebra. The analysis suggest that board game design tasks support instrumental genesis, and allows GeoGebra to mediate to fellow students. Most use og GeoGebra for board game design can be characterized as pragmatic rather than epistemic mediations. we can conclude that board game design can be an interesting way to introduce strong tools into mathematics teaching and learning in primary school. Such tasks might lead to easy adoption of GeoGebra, familiarity with appropriating GeoGebra for different tasks, a positive attitude to mathematics among the pupils, and a re-scoping of primary level mathematics in direction where the discipline play a part in constructing cultural artefacts.

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