TASKS IN TECHNOLOGY ENVIRONMENTS: THEORY OF DIDACTICAL SITUATIONS AND INSTRUMENTAL GENESIS IN A CABRI ELEM ENVIRONMENT

Kate Mackrell

Institute of Education, University of London, UK

Michela Maschietto

Università degli Studi di Modena e Reggio Emilia, Italy

Sophie Soury-Lavergne

Institut Français de l'Education, ENS Lyon, France

The contributions of two theoretical frameworks (Theory of Didactic Situations and Instrumental Genesis) to the design of a sequence of tasks in the Cabri Elem environment, where task and technology design are closely linked, are shown. Considering the potential for instrumental genesis as a theory of technology design reveals a fundamental difficulty in dealing with representations. It is hence suggested that the role of the artefact be broadened to include environments, tools, and entities.

INTRODUCTION

The first part of this paper consists of a summary of some aspects of our ICMI-22 submission on task design, in which we analyzed a particular sequence of tasks created using Cabri Elem in order to illustrate the interconnections between the affordances of the technology and the ability to implement particular didactic principles. Our choice of theoretical frameworks to use for this analysis was based on an analysis of tasks conducted as part of the Intergeo project (Trgalova, Soury-Lavergne & Jahn, 2011) which used Brousseau's theory of didactical situations (TDS) (Brousseau, 1998) together with instrumental genesis (IG) (Trouche, 2005).

In part 2 we begin a critique of instrumental genesis as a theory of task/technology design with the aim of both creating greater links between TDS and IG and enabling IG to become a more effective framework for design.

PART 1: THE TECHNOLOGY

Cabri Elem technology was created to serve the needs of primary students and also to enable the creation of "applets" in order to enable teachers to engage more confidently with technology (Laborde and Laborde, 2011). It consists of a task design environment in which "activity books" consisting of a succession of pages incorporating a sequence of tasks may be created, and a more restrictive task performance environment in which activity books may be used by teachers and students. Cabri Elem has the affordances of earlier Cabri technology for direct manipulation of geometrical objects and numbers, together with additional features such as 3D models and tools. A major difference between Cabri Elem technology and other dynamic geometry software, and also other generic technology, such as graphical calculators, CAS or spreadsheets is that the user interface of the task performance environment is under the control of the activity book designer, who must decide which objects (tool icons, images, text, geometric figures, etc.) to arrange on initially empty pages, and who may program control actions on these objects. Creating an activity book hence involves issues of both task and technology design.

THE THEORY OF DIDACTIC SITUATIONS

In this theory (Brousseau, 1998), knowledge is a property of a system constituted by a subject and a "milieu" in interaction. Learning occurs through this interaction: the subject acts within and receives feedback from the milieu. Technology, or the part of technology relevant to the mathematics concerned, may form part of the milieu, and the milieu related to a student changes as student knowledge, both technical and mathematical, develops. With a learning task in a technology environment, the author determines the possible milieu and hence the potential for learning by creating all the elements the student will deal with: the objects the student will manipulate, the possibilities of actions on these objects and the feedback provided by the environment.

Key aspects of a didactical situation are the mathematical problem and the task, where the task involves learning objectives and the mathematical problem. The teacher assumes that achieving the task will cause the student to learn. The goal of a task, whether teacher or student determined, should be clear, together with criteria for success or failure. A task is performed by concrete and conceptual student actions, with the existence of a space of uncertainty and freedom for the subject about appropriate action and strategy. This contrasts with the common dynamic geometry tasks such as "drag this point and observe" where the student has no choice of action and is uncertain about what is relevant to observe. The task corresponds to phases of the didactical situation and is related to different values of a set of didactical variables. Didactical variables are parameters of the situation, with values that affect solution strategies. The effects can be of three kinds: (i) a change in the validity of a strategy, where a strategy that produces a correct answer with a certain value of a didactical variable will produce an incorrect answer with another value, (ii) a change in the cost of the strategy (for example counting elements one by one is efficient for a small number but much more costly for a larger number) (iii) the impossibility of using the strategy. A combination of the different didactical variable values contributes to the task definition. The learning situation is a choice of different tasks that lead the students to construct the appropriate strategy. Thus task design will consist, for a part, in identifying the didactical variables of the situation and then choosing the succession of appropriate combinations of didactical variable values.

INSTRUMENTAL GENESIS

Instrumental genesis involves the processes of instrumentation, whereby a person builds personal utilization schemes for an artefact, and instrumentalization, whereby a person adapts an artefact to their own purposes, with the result that an artefact becomes an instrument to be used in the pursuit of a goal (Rabardel and Bourmaud, 2003).

IG originated in ergonomics (Rabardel, 2002) and is also used in computer-supported collaborative learning (e.g. Cerrato Pargman, 2003). It is well established as part of the instrumental approach (Artigue, 2002) dealing with the integration of technology in the mathematics classroom, but also, being derived from the work both of Vygotsky and of Piaget (Rabardel, 2002), has links to socio-cultural approaches and, less explicitly, to constructivist approaches and constructionism.

THE CABRI TASK

We will now look at an activity book and discuss the links between this sequence of tasks and the theoretical frameworks from which the tasks were generated. The “Target” activity book addresses the French primary school level CE1 (7 year old students) and deals with the representation of numbers using place value notation. The idea arose from comparing counters on a scoreboard, where the value of the counter depends on its position on the board, with the way that the value of a digit depends on its position in a written number. It was designed by a team of ten researchers (including two of the authors of this paper), teacher educators and teachers involved in a French national project [1] whose purpose is to create resources for the teaching of mathematics in kindergarten and primary school.

The process of creating the activity book involved elaborating the milieu by choosing appropriate objects, possible actions and resultant feedback. In our example, the objects are essentially the scoreboard with three different regions, the counters, the target number and the score, as shown below.

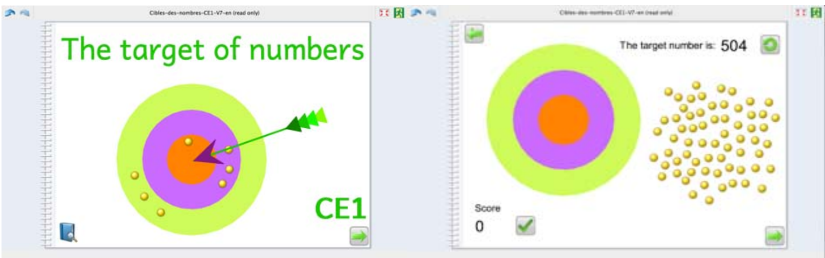

Figure 1. Title page and a task page from the “Target” activity book

The actions on the objects are simple: dragging the counters, clicking on a button to get a new target number and reset the counters and clicking to get an evaluation.

Didactical variables played an important role in the task design process. Some were identified a priori, while others emerged during the design process as the authors became more aware of what aspects of the situation could be changed. Once a potential variable was identified, an analysis of the ways in which this variable could be changed produced a better understanding of the possible tasks and their consequences. It also enabled the creation of strategy feedback.

Three kinds of feedback were essential to the activity book design. Evaluation feedback is related to the achievement of the task or part of the task. Strategy feedback aims to support the student in the course of task resolution, like scaffolding (Wood et. al., 1976). It is a response to the strategy used by the student. The authors needed to identify (i) configurations that were typical of a strategy and hence enabled a diagnosis and (ii) new objects or actions that could be provided to help the student without changing the nature of the task. Such feedback could consist of help messages, or a graphic enlightening of contradictory elements. Another possibility is to modify the values of didactical variables in order to make the student aware of the current strategy limitations. Direct manipulation feedback is the response of the environment to student action, and may serve the function of either of the previous types of feedback.

The first page of the resultant activity book, shown in Figure 1, is a title page. In page 2, the main objects are presented. The student may interact with these objects, by dragging counters to different positions on the scoreboard and noticing how this affects the score. This is dynamically calculated: one, ten and one hundred for each counter in the green outside region, the purple intermediate region and the orange central region respectively. The aim of the page is to give time for instrumentation to both teachers and students. They can explore interactions with the elements that will constitute the milieu without the constraints of a particular task. It also contains a reset button which, when clicked, replaces counters in their initial positions, and a button which allows students to move on to the next page.

The changing score is direct manipulation feedback that shows students not only the effect of their action, but also that action on one object (moving a counter to a different region) will affect another object (the score). The score is always displayed in some pages, but displayed only after a specific sequence of actions in other pages.

On page 3 the student first receives evaluation feedback. A specific task is given: to reach a score equal to a target number, randomly generated between 1 and 999 (see Figure 1). Clicking on the reset button now in addition generates a new target number. Another new action is that the student may, in addition to comparing whether the score matches the scoreboard, click on a new button for evaluation feedback: a red frowning face if the answer is wrong, and a yellow smiling face if the answer is correct. In case of failure, the student can continue to drag counters and ask for a new evaluation: a new smiley will appear to the right of the previous one. It is important that new feedback is only generated at the student's request: otherwise a trial and error strategy not stemming from mathematical considerations could lead to success.

From page 4 to 7 students are no longer given the direct manipulation feedback of seeing the score. They hence need to take into account the value of the counters in the different regions of the scoreboard to determine the score. "Score" was identified a priori as a possible didactical variable, with two values: visible or hidden.

In page 5, the number of counters is reduced so that, if the target number is over 27, a strategy that consists in placing counters only in the green units region will fail. A strategy which takes into account that a single counter can have another value than 1, i.e. using the inside regions of the scoreboard, is necessary. Therefore, another potential didactical variable is identified: the number of available counters, with two values, 3x9=27 and >27. In page 6, the target number is a multiple of ten, between 10 and 990. As there are enough counters to either leave the green region empty or to fill it with multiples of ten counters, a change of strategy is not necessary. In page 7, however, a single counter is fixed in the green region. Therefore, new strategies are required, involving the placement of a multiple of ten counters into the units region of the scoreboard. The "fixed counter" didactical variable is identified, with four values: no fixed counters, or fixed counters in the units, tens, or hundreds region.

Page 8 contains input boxes for the student to enter the values of a counter in each region of the scoreboard. The aim of this task is to summarize the key idea of the activity book, i.e. that the value of a counter depends on the scoreboard region.

Other pages of the activity book are not devoted to student tasks. The first page is designed with the aim of attracting teachers and students to the activity book with an iconic representation of some of the main objects. Pages 9 and 10 contain commentaries for teachers, reporting the main aspect of the task, the evolution from one page to another, possible student strategies (correct or not) and also the solution. The structure of the pages of the activity book was used to organise these notes and the didactical variable analysis helped to determine what information was useful.

TRIALING THE ACTIVITY BOOK

This occurred in spring 2012 in two primary school classes: CE1 with the version presented here and CP (six year old students) with a version where the target number size was limited to 99. Teachers used the activity book as one resource for learning about place value and instrumentalized the book by printing pages to construct related paper and pencil tasks. They were enthusiastic about student engagement, mathematical reasoning and the evolution of strategies, but raised a number of issues.

It was expected that the strong metaphor between the task situation and real scoreboard situations would both provide a meaningful context and minimize the need for instrumentation. Students expected, however, that moving a counter would require tossing it in some way and were initially uncertain about how to do this using the software. Teachers also proposed that instrumentation would be enhanced by modifying page 2 to include a target number chosen either by the teacher according to the constraints of the class, or chosen by students in order to challenge each other.

Some students used the target number update not only to get a new number after finding a previous target but also, unexpectedly, to get a number they knew they were able to deal with, showing the ability to diagnose their level of expertise. It is planned to modify pages to provoke problem resolution, but also to locally enable this usage. This example of students' instrumentalization of a functionality to adapt it to their level of expertise is a new, generalizable element in activity book design.

The number of available counters was not a didactical variable for most CE1 students, who used each region of the scoreboard and limited the number of counters they needed to drag. Many of them did not notice the reduced number of counters on page 5 and were surprised to apparently have to solve the same task again. However, for many of the younger CP students who used only the units region of the scoreboard the number of available counters was indeed a didactical variable. The status of page 5 will hence be changed in further developments of the book. Instead of being automatically displayed to CE1 students, it will only be displayed as necessary, i.e. if the unit region is repeatedly filled with many more than 10 counters. The strategy feedback, resulting from our analysis in terms of didactic variables, will consist in reducing the number of counters to better fit the sum of digits of the target number and choosing a target number over 50.

DISCUSSION

Both TDS and IG provide a useful lens to explore aspects of the design of the task. However, there is almost no integration between the two theories in our above analysis, and such an integration would be useful: for example student instrumentalization (described by IG) will affect solving strategies, and hence the milieu and the learning, as described by TDS. Identification of possible instrumental geneses should hence contribute to an a priori analysis in the framework of the TDS. A further issue is that technology design issues, crucial in the Cabri Elem environment, are not readily addressed within TDS.

In part 2, we will first address the potential for IG as a theory of technology design, and then show how an extension to the role of the artifact may both resolve some of the issues in its use in technology design and enable further integration with TDS.

PART 2: A CRITIQUE OF INSTRUMENTAL GENESIS

In the field of human-computer interaction, IG is already recognised as a theory for the design of technology: in fact design is the most common issue addressed with the approach (Kaptelinin & Nardi, 2006, p. 110). General design principles are that artefacts should be designed for efficient transformation into instruments through enabling flexible user modification and through taking into account the real needs of users while appropriating the artefact. It also explores user contribution to design, particularly through instrumentalization (Kaptelinin & Nardi, 2002, p. 111).

Further design principles may be derived from Rabardel's (2002) analysis. One example is the idea of the transparency of an artefact, which has to do with the visibility of the artefact's technical system. Subjects using a "black box", such as a carpenter hammering a nail, feel that they are acting directly on the environment without being impeded by the artefact. The artefact only becomes visible when problems arise, such as when the hammer behaves unexpectedly. In contrast, an artifact must be a "glass box" when subjects need to understand what the artefact is doing in order to act. Rabardel introduces the term "operative transparency" to refer to the properties of the instrument pertinent to the subject's action and the manner in which these are communicated to the subject. This varies with the subject's goals and constraints: in a professional situation the aim is to make the action easier, safer and more reliable.

However, in a learning situation, the aim may be to construct constraints that lead the subject to use and elaborate cognitive constructions. This has relevance in analyzing the types of action and feedback that should be enabled.

Another aspect of IG is that there is now fifteen years of research focusing on the analysis of constraints and possibilities with particular technologies in the classroom which could provide a base for a constructive critique of the technology being used.

However, although used in the design of tasks, such as the consideration of instrumentation issues in the tasks reported in this paper, or more interestingly by using the constraints of technology to promote student learning (e.g. Fuglestad, 2007, using spreadsheets), IG has not been much used as a theory of technology design in mathematics education. An interesting exception is the current constructionist exploration of the significance of instrumentalization in design (Healy & Kynigos, 2010).

Kynigos, Bardini, Barzel and Maschietto (2007) give a list of artefact affordances that are perceived to enhance instrumentation (to constitute exploration spaces, mediate between formal and informal, provide executable representations, offer dynamic manipulation, evoke interplay between private and public expression and generate interdependent representations). These might provide a focus for IG and design: however, we have been unable to find any research which elaborates or deals specifically with the effect of these affordances on instrumentation. It is also noticeable that reports on the design and development of new technologies in CERME 7 do not mention IG. Lagrange (2011), for example, discusses the design of Casyopée with no connection to a framework which he was one of the first to use (Lagrange, 1999).

Another issue is that IG in mathematics education was first considered in an environment (CAS on the TI-92) (Lagrange, 1999) where researchers had no control over the technology, and instrumentalization (such as creating programs) was difficult. Despite Trouche's (2005) acknowledgement that the complexity of the IG of an artefact is related to the complexity of the artefact, the perception seems to be that IG is inevitably complex. Is it possible that the idea of designing technology environments for effective instrumentation has come to seem pointless?

However, there is a more fundamental problem with IG that may preclude its use in design without significant modification. We are currently reviewing the reports and papers of past CERME technology working groups – and aspects of representation are, by far, the most commonly discussed. A problem is that IG, with its focus on artefacts as potential instruments, has language to talk about instrument-mediated actions upon objects (such as finding the tangent line at a point on a graph), but currently not about objects which are given existence by the artifact (such as the graph, or the tangent line) – and such objects are critical in any discussion of representation. This is not an intrinsic limitation: Rabardel (2002) makes it clear that an artefact may serve as object as well as instrument.

A second problem is that the role of the artefact in providing an environment within which actions take place is also not considered, although the need to consider the environment has been raised both by Hegedus et al. (2007), and Trouche (2005, p. 139), This is also not an intrinsic limitation: artefacts in technology may act as environments rather than tools (Kaptelinin & Nardi, p. 255)

When analysis is performed at the classroom level, these limitations are less important. For the teacher, instrumental genesis involves developing a system of documents from a variety of artefacts: an artefact is a potential instrument for facilitating mathematical learning, and a particular representation is one such instrument.

However, for the student, the artefact may provide not only instruments for action but also the objects upon which they act and the environment within which action takes place. We will hence consider a technological artefact to consist of an environment (within which action takes place), tools (potentially the instruments mediating the action) and entities (which are acted upon). An entity is defined to be anything given existence by an environment which is perceived as a whole and which may be an object of action. Entities range from simple images through representations of complex mathematical ideas (such as the graph of a function) A tool is a means of performing an action in the environment, which is very often the creation or manipulation of an entity. Tools may be represented by entities such as icons to click, or help text to read, but also require action from the subject, such as moving the hand to drag, typing, etc. An environment is that which gives existence to entities, provides tools which enable particular actions, and gives feedback as to the way entities change through interaction. This enables a link to the TDS concept of milieu, but, as stated earlier, an environment and a milieu are not identical.

We will now use the Cabri Elem activity book analysed in part 1 to exemplify these concepts and also to consider whether the idea of “genesis” of environment or entity might have any useful significance, where “genesis” is loosely considered as developing cognitive schemes in order to more effectively meet the goals of the activity. The environment for the student using the activity book is the activity book itself, composed of pages which the student must learn to move between. This illustrates that the ability to navigate to different parts of the environment might be one aspect of “environmental genesis”. The first part of the environment encountered, the title page, contains text and images which have the aim of enabling the student to make the connection to a familiar activity in a real-world environment, in which certain actions are relevant to meeting the goal of the activity. This creates expectations as to the actions possible and relevant, and not possible or irrelevant in this environment. The student would expect the action of placing counters on the target: the student would not expect, for example, to use the target to create music. A second aspect of “environmental genesis” might hence be about learning about the activity and the types of actions that are possible and relevant to the goal of the activity without necessarily knowing how to perform these actions. In page 2 the counters are presented as entities which may be dragged. A number of different types of schemes may result: standard “instrumental genesis” in which students must discover that dragging is the appropriate instrument for placing counters on the target, and a more subtle development of meaning for the counters and target. In pages 3 to 6 the instrument of “using counters and target together to change the score” is progressively generated, through constraints which require the student to use the tool in increasingly sophisticated ways with increasing understanding. However, counters and target also constitute a mathematical representation: an equivalence is created between a configuration of counters and a set of numerals. In contrast the score is an entity which, while it serves to give feedback on the results of an action (an important aspect of TDS), is not a means of action in itself but does undergo a genesis of meaning through its connection with the counters and the target. It is hence possible that the genesis of an entity may involve instrumental genesis only (as when the student learns that clicking on a particular entity causes a particular action to take place), meaning genesis only, as with the score, or a combination of the two, where, as with the counters and the target, the entities both provide the means to perform an action and a representation of the result of that action.

It would hence seem that the concept of “genesis” of an environment or an entity might be useful and worthy of further elaboration and definition.

CONCLUSION

In part 1 of this paper we have shown that two theoretical frameworks, TDS and IG, contributed effectively to the design of a sequence of tasks in the Cabri Elem environment where task and technology design are closely related.

In part 2, in exploring further questions of technology design and integration between the two theories, two limitations of instrumental genesis, in looking at representations and at the environment were discussed. A suggestion has been made to broaden the role of the artefact to include environments, tools and entities. A consideration of the artifact-as-environment would in particular enable connections to the TDS concept of milieu. Consideration of the geneses of environment and entities has been shown to be useful in analyzing the original task.

We stress that the additions to IG and its integration with TDS will require further elaboration: in particular, although the door is now open to a consideration of representation within IG, drawing on the rich literature on representation within mathematics education, such a consideration has been beyond the scope of this paper.

NOTES

[1] The « Mallette » project is supported by the French Ministry of Education and conducted in collaboration between the IFE Institut Français de l’Education and the COPIRELEM Commission of IREM http://educmath.ens-lyon.fr/Educmath/recherche/equipes-associees/mallette/

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