THE EFFECTS OF DYNAMIC GEOMETRY SOFTWARE ON LEARNING GEOMETRY

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The aim of this paper is to present the findings of a pilot study which was designed to collect data about the effects of using dynamic geometry software in the tenth grade geometry lessons on the students' geometric thinking, geometry achievement and ability of doing proofs in geometry. It was a quasi- experimental study in which treatment groups were given dynamic geometry activities during the geometry lessons. The findings revealed that the students' scores from geometric thinking test and proof test increased significantly with respect to the students' who did not experience those activities.

Keywords: dynamic geometry, geometric thinking, problem solving, proof skills

INTRODUCTION

Geometry is "a complex interconnected network of concepts, ways of reasoning, and representation systems that is used to conceptualize and analyze physical and imaged spatial environments" (Battista, 2007, p.843). Thus, learning geometry entails visualization and construction of images of geometric concepts and making appropriate relationships between the concepts. Van Hiele also identified visualization as an indicator of one's geometric thinking level. He noted that visualization is the first level of geometric thinking such that everybody should possess accurate concept images to attain higher levels of geometric thinking (Battista, 2007). Research findings show that students fail in geometry because they have difficulty in visualization of geometric concepts such that they are unable to analyze geometry problems and draw appropriate figures for the problem (Clements & Battista, 1992; Healy & Hoyles, 1999; Presmeg, 1997; Yerushalmy & Chazan, 1993). The conceptualization of visual objects, in other words having a valid matching of concept image and concept definition in mind (Vinner & Dreyfus, 1989), is vital to understand geometry (Battista, 2007). Therefore, teacher-oriented teaching strategies are not effective enough to help students understand geometry and improve their geometric thinking and proof skills (Reiss, Heinze, Renkl, & Gross, 2008).

Improvement in information technologies arise questions about how to use those technologies for educational purposes and how they affect students' understanding and learning. It is noted that using technology for educational purposes increases students' motivation for learning, helps for understanding and gives opportunity for repetition of the subject matter (Dörfler, 1993; Healy & Hoyles, 1999; Knuth & Hartmann, 2005). The studies indicated that although students can solve algebraic problems by following some algorithms they have difficulties solving problems including images and shapes (Healy & Hoyles, 1999). Because dynamic software enables students to observe the properties and the relationships by drawing figures and manipulating them easily, it has potential to decrease such problems (Healy & Hoyles, 1999). Indeed, the studies investigating the effective ways of teaching geometry suggest that dynamic geometry software (DGS) helps students visualize geometric concepts and understand geometric rules, generalizations and relationships between the concepts (Healy & Hoyles, 1999; Jones, 2000; Marrades & Gutierrez, 2000). Hence, using DGS in geometry lessons may be effective in terms of increasing students' geometry achievement and developing their geometric thinking and problem solving skills.

DGS not only contribute to the development of geometric thinking and problem solving skills but also facilitates understanding and proving hypotheses and conjectures (Jones, 2000). Doing proofs entails making a good plan for proof and following it by justifying each step (Heinze, Cheng, Ufer, Lin, & Reiss, 2008) and similar pattern is required when doing constructions with DGS (Scher, 2005). For instance, to construct a square with DGS students cannot just draw equal segments and connect them together but they need to use the idea of perpendicular lines, right angles and transformations. As students engage in fundamental constructions with DGS, the better they appreciate the fact that they have to follow an order to make constructions correctly. However, the studies about the effects of DGS on students' proof skills are limited, more research are needed to support that relationship (Hollebrands, Laborde, & Straesser, 2008).

Although the effectiveness of using DGS on learning geometry was investigated by many scholars, most of them were limited in terms of sample size, context, the variables were investigated, the way of integrating DGS into geometry lessons or data collection tools. Still, there is a need for large-scale experimental studies supported by both quantitative and qualitative data (Battista, 2007). Therefore, the aim of this study was to investigate the effects of using DGS on students' geometric thinking, geometry achievement and proof skills. However, the results presented in this paper emerged from the pilot study of that large-scale investigation.

METHODOLOGY

In this pilot study, quasi-experimental research design was used to investigate the effects of using DGS on the tenth grade students' geometric thinking, problem solving and proving skills. It was conducted in the second term of 2011-2012 academic year in Turkish high schools.

Sample

A total of 227 students from 6 different schools participated in this study. The schools were assigned into groups in terms of their preference. Three schools preferred to be in the experiment group and two schools preferred to be in the control group. In one school, one of the tenth grade classes was assigned to the experiment group and the other class was assigned to the control group. Then 145 students were in experiment group and 82 of them in the control group. However, because not all students were available during the testing days some of data was lost. Furthermore, in two of the experiment schools all DGS activities were completed but in others they were not. Therefore, the data analyzed for this paper is based on 12 students from the experiment group who experienced all DGS activities and took all the pretests and posttests and 37 students who took all the tests from the control group.

Data collection

Five DGS activities were determined in line with geometry curriculum and they were applied during the geometry lessons. The teachers used the computer laboratory only for the activities and used their regular classrooms otherwise. The activities were about triangle inequality, angle bisectors, medians and perpendiculars, transformations and Euclid, Menelaus, Ceva and Carnot theorems. Geometers' Sketchpad Program (GSP) was used as dynamic software. The students were given three types of tests namely, Geometric Thinking Test (GTT), Geometry Achievement Test (GAT) and Geometry Proof Test (GPT) prior to and at the end of the study. The tests consisted of questions related to triangles and transformations which were covered during the second term in the curriculum. The items in all tests were in supply type form such that students were expected to solve the questions and explain and/or justify their answers. GTT test consisted of seven items such that the items were written in the line of first four levels of van Hiele's geometric thinking levels (recognition, analysis, order, deduction and rigor) and Driscoll's (2007) geometric habits of mind (reasoning with relationships, generalizing geometric ideas, investigating invariants and balancing exploration and reflection). By GTT, it was aimed to measure students' knowledge of geometric concepts (how they describe given concepts), ability to interpret the relationships in geometric constructions and ability to transfer their knowledge into application and make inferences. GAT consisted of ten items such that they were chosen from the 10^{th} grade geometry textbooks and it was aimed to measure students' geometric knowledge. GPT consisted of six items such that they were the proofs recommended to be covered in the curriculum. It was aimed to measure students' proof skills (i.e., choose an appropriate type of proof, to make a plan to use it, write conjectures and justify them) in GPT. Although during the pilot study qualitative data was not collected, for the main study qualitative data in terms of interviews and videotapes will be collected.

Data analysis

The test results were analyzed by using software for statistical analysis. For the content validity of the tests table of specifications were prepared. In addition, for the GTT items van Hiele's and Driscoll's (2007) ideas for geometric thinking were taken into consideration. The content validity of the tests was agreed by one academician and three experienced geometry teachers. The concurrent validity of the tests was also checked. For the reliability test-retest reliability analysis was held. Furthermore, rubrics for each test were prepared for scoring and the interrater reliability was checked.

RESULTS

The content validity of the tests was agreed by one academician and three experienced geometry teachers. For the concurrent validity the correlation between each test was found. Then, for the pretest the correlation between GTT and GAT was .63 (p=.000), GTT and GPT was .38 (p=.001) and GAT and GPT was .51 (p=.000). For the post test the correlations were as follows: .50 (p=.000), .38 (p=.000), and .47 (p=.000), respectively. Test-retest reliability was sought and Pearson's r for GTT, GAT and GPT tests calculated as .56 (p=.000), .67 (p=.000) and .68 (p=.000), respectively. The tests were out of 100 points. Two academicians rated the tests. The interrater reliability for GTT was .96, for GAT was .98 and for GPT was 1.00.

The descriptive statistics about the tests are given in Table 1.

	Experiment Group					Control Group				
	N	Min.	Max.	Mean	SD	N	Min.	Max.	Mean	SD
GTTPre	12	7	55	30.08	14.34	37	7	67	39.73	14.50
GTTPost	12	22	63	43.42	14.59	37	7	81	44.70	19.89
GATPre	12	0	33	9.25	10.50	37	0	47	16.68	13.14
GATPost	12	0	48	21.17	11.51	37	0	82	31.27	23.51
GPTPre	12	0	13	3.17	4.99	37	0	60	15.73	14.97
GPTPost	12	0	20	7.92	5.42	37	0	45	12.11	12.20

Table 1: The descriptive statistics about GTT, GAT and GPT

The pretest results for each group were compared. There were no statistical difference between GTT ($t_{47,\ .05}$=2.007, p=.050) and GAT ($t_{47,\ .05}$=1.778, p=.082) but there was significant difference between GPT ($t_{47,\ .05}$=4.405, p=.000). The posttest comparisons showed that there were no significant difference between the groups in terms of GTT, GAT and GPT results ($t_{47,\ .05}$=.206, p=.838, $t_{47,\ .05}$=1.982, p=.055, and $t_{47,\ .05}$=1.639, p=.109 respectively). The pretest and posttest results for each group were compared. For experiment group significant differences were obtained in terms of all types of tests but for control group there was significant difference in terms of geometry achievement. For the experiment group the following t-scores obtained for each test: For GTT, $t_{11,\ .05}$=2.945, p=.013, for GAT, $t_{11,\ .05}$=3.856, p=.003, and for GPT, $t_{11,\ .05}$=2.844, p=.016. Because the number of participants taken into account was small, Wilcoxon Signed Ranked Test was also applied for the experiment group. The findings were aligned with t-test results such that for GTT, z=-2.432, p=.015, for GAT, z=-2.701, p=.007, and for GPT, z=-2.174, p=.030. For the control group the following t-scores obtained for each test: For GTT, $t_{36,\ .05}$=2.025, p=.050, for GAT,

$t_{36, .05}$=5.562, p=.000, and for GPT, $t_{36, .05}$=1.980, p=.055. The following results were obtained from Wilcoxon Signed Ranked Test: GTT, z=-1.753, p=.080, for GAT, z=-4.275, p=.000, and for GPT, z=-2.021, p=.043. Except GPT , t-test and Wilcoxon test results were compatible in the control group.

The items for each test were analyzed. The maximum points for each item in GTT were 30, 9, 9, 8, 12, 12 and 20, respectively. In the first item of GTT the students were asked to write the definitions of fifteen geometric concepts including line, angle, median, incenter of a triangle, transition and reflection axis. In the second item, the students were asked to explain the relationships between the sides of a triangle (triangle inequality). In the third and the fourth items they were asked whether given information is enough to construct a triangle and find the measures of its sides and angles. The fifth question was about transformations and in the sixth question they were asked to distinguish congruent triangles among the given set of triangles. Finally, in the seventh item the students were given four different descriptions of geometric figures and they were asked to draw them by using appropriate labels and symbols. For instance, students were asked to draw a triangle ABC such that $|AB| = |AC|$ and the intersection of angle bisector of the angle A and the median of the line segment AC is P. In the figure 1, two examples from the students work are given. For the group taken into consideration there was nobody deserved full credit for the first, the second, and the seventh items in GTT. Furthermore, although the credits for each question differed in GTT, the results showed that the weighted mean for the second item was the lowest (2.80 out of 6 points) while the weighted mean for the last item was the highest (11.41 out of 20 points).

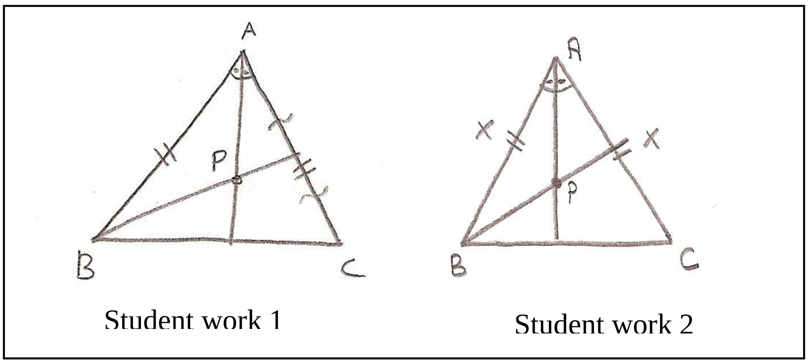

Figure 1: Typical student answers to the construction item of GTT.

In GAT, all items were worth 10 points. The test items were about triangle inequality, angle bisector, median, Pythagorean, Euclid and Menalaus theorems, congruency, similarity, area, and transformations. In GAT, there were students who received full credit from the items. However the mean of the tenth item (transformations) was the lowest (1.73 out of 10 points) and the mean of the sixth item (similarity) was the highest (5.61 out of 10 points). In the sixth item the students were given the following problem: *As shown in the picture, a bridge will be built up* *over a river.* $\overline{BD}$ *segment represents the bridge and* $\overline{AE} \perp \overline{AB}$ *and* $\overline{CE} \perp \overline{CD}$. *Use the given information in the figure and find the length of the bridge*. In the figure 2, a typical student answer for the sixth item is given.

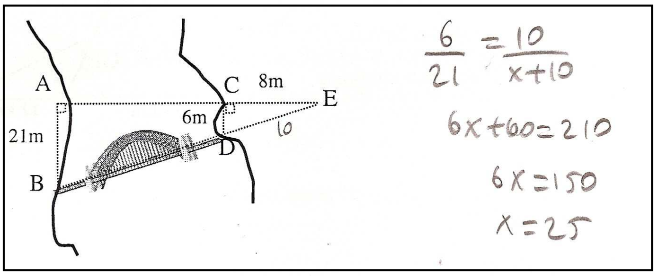

Figure 2: Typical student answer to the sixth item of GAT

In GPT, the third and the sixth questions were out of 20 points while others were out of 15 points. In the first item of GPT, the students were asked to prove that an exterior angle of a triangle equals to the sum of nonadjacent interior angles. The other items were about proving angle-side relationship in a triangle, area of a triangle, Sinus theorem, Carnot's theorem and Euclidean theorems, respectively. In GPT test the mean of the items were generally low but the first item had the highest mean (6.53 out of 15 points) and there were students who got full credit from the first and the sixth items.

DISCUSSION

The aim of this pilot study was to determine the reliability and validity of data collection tools and effectiveness of GSP activities. The data analyzed in this paper was based on 49 students who took all pretests and posttests. The mean scores of the tests and test items provide some information about the students' geometric thinking, geometry achievement and proof skills. It was apparent that students' geometric thinking levels, geometry achievement and proof skills were low.

Although comparing the means of the test items may provide information about the students' weaknesses and strengths in geometry, such inferences should be supported by some qualitative data in terms of structured interviews or other tests. In this pilot study, qualitative data was not collected but a detailed analysis of students' responses for each item is likely to provide some concrete data about students' geometric knowledge, geometric thinking and proof skills. Although such detailed analysis is not the scope of this paper, an instance from GTT is noteworthy to be discussed. In GTT, the seventh item had the highest weighted mean such that students were asked to draw geometric figures by using information given about them. In the first item, the students were asked to write the definitions of some geometric concepts. Although some of the students failed to provide complete and valid definitions for the geometric concepts in the first item, they were able to use those concepts to make some of the constructions in the seventh item. For instance, when they were asked to define angle bisector some of them wrote “it divides an angle in two” or “it is a segment divides an angle in two equal parts”. In the seventh item there were descriptions in which angle bisector could be considered as a line segment and those students used appropriate symbols to indicate that the line segment was the angle bisector. Hence they received full credit from that construction if other parts of it were valid. However, they failed to draw another figure in the same item because they failed to extend the angle bisector, that is, because they did not know that angle bisector is a ray not a line segment. Therefore, having higher mean score with respect to other items may not be indicator of students’ ability to analyze geometric figures and definitions. Furthermore, this instance supported the importance of the consistency between concept image and concept definition (Vinner & Dreyfus, 1989). In this case, the students probably wrote the definitions of the concepts in terms of their images in their mind. However, DGS provides opportunities to eliminate such misconception of students’ because the images and tools in DGS are compatible with their mathematical definitions such that an angle bisector is drawn as a ray or line. Then, experiencing constructions with DGS may enable students to define geometric concepts correctly.

Furthermore, the mean scores of some of the items in GTT and GPT scores indicated that students’ proof skills were low. In the most cases, the students who attempted to prove the given statements gave examples to show that it worked rather than proving them deductively. One of the reasons for poor skills in proving is probably the fact that teachers did not spend time for proof activities even though they were in the curriculum. Therefore, students did not know how to prove theorems and they skipped most of the items in GPT. Because deduction is one of the levels in geometric thinking, for the main study, the teachers will be encouraged to cover some of the proofs written in the curriculum. GAT scores revealed that students do not know geometric concepts and relationships and they have difficulties to understand transformations. However, because DGS enables students to manipulate geometric figures and do transformations, more practice with DGS may help students understand and visualize transformations. Thus, higher scores from GAT may be obtained during the main study.

Because only post test comparison was not enough, the pretests and posttests within the group were compared. The results revealed that there was significant increase in terms of students’ geometric thinking, geometry achievement and proof skills. These effects of DGS also supported in the literature (e.g., Jones, 2000; Marrades & Gutierrez, 2000). This result was noteworthy because although improvement in skills requires more practice and longer time, even five GSP activities created such a difference for the experiment group. Therefore, ten GSP activities which were planned for the main study will be likely to create such difference between the groups. However, because not all students took all types of tests during the administration of the posttests, the majority of data is lost. Furthermore, some teachers in the experiment group could not do all GSP applications so the data from those schools was excluded for this paper. Therefore, the number of students who experienced all GSP activities and who took all the tests was low. This was the limitation of the study. For the main study, there will be ten GSP activities and they will be done throughout the academic year. The exam weeks will be determined for each group and students will be encouraged to participate in the tests and do their best. Therefore, the completion of all activities and minor loss in data is aimed to achieve.

The values obtained for the reliability and validity of the tests were acceptable for supply-type teacher made tests (Miller, Linn, & Gronlund, 2009). It indicates that these tests can be used for the main study for the next academic year. However, the test may be revised for the main study for some reasons: First, the mean scores for GPT were quite low with respect to other tests. This was compatible with findings of many studies about students' proof skills (Clements & Battista, 1992; Harel & Sowder, 1998; Hershkowitz et al., 2002; Reiss, Klieme, & Heinz, 2001; Stylianides, 2008). Although there was statistically significant increase in the GPT scores in the experiment group, the GPT may not be considered as a single test for the main study such that some of the items may be replaced with the items in GTT and proofs may be asked during the interviews. Second, there were items in each test were aiming to measure the same or similar learning outcomes. Those items may be excluded or replaced with other items. For instance, the second, the third and the fourth items in GTT required students to explain an identity or rule and justify their reasoning those were aimed to measure the fourth geometric thinking level of van Hiele. Therefore, integrating some of the items GPT into GTT may not cause any loss in terms of content validity of the tests. Even, those items might be replaced the ones having the lowest mean in GTT such as the second and the fourth items. Similarly, the items with the lowest mean scores will be changed in GAT. Briefly, the tests will be revised for the main study although the values for reliability and validity were at acceptable levels.

In this study the homogeneity of the groups was not satisfied because the assignment of the schools into the experiment and control groups were based on their choice because of some administrative reasons and the pretests were administered after forming the groups. In fact, students' mathematics achievement in two of the control schools was higher than the students' in two of the experiment schools. Therefore, comparing each group's posttests did not provide valid information about the differences between groups. For the main study, some demographic information about the students and the schools will be collected prior to study and the pretests will be administered before assigning groups as experimental or control. If possible, the classes will be assigned into groups rather than the schools to achieve homogeneity between the groups.

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