HOW FOUR TO SIX YEAR OLD CHILDREN COMPARE LENGTHS INDIRECTLY

Johanna Zöllner, Christiane Benz

University of Education, Karlsruhe

Keywords: Kindergarten, preschool, measurement, unit

This article will introduce one part of a research studying how four to six year old children develop an idea about indirect comparison of length, with the aid of diverse tools. In a qualitative survey the solution processes of altogether 40 children have been observed. Hence, this paper will show the system of categories and will also present initial results.

INTRODUCTION

“We are convinced that dealing with measures is the most suitable way to introduce five-year-old children to the world of figures and mathematical terms” (Castagnetti & Vecchi, 2002, p. 14). In spite of this statement, Copley notices that “measurement is an often neglected mathematics topic for young children“ (2006, p. 17). Furthermore, Benz states that, during mathematical activities, German kindergarten educators mainly focus on arithmetic (2012, p. 22). Only 17% of the kindergarten educators who took part in that survey (n=589) answer that measurement is part of their mathematical activities in kindergarten. A fact which is “astonishing, because the reference to everyday activities is very obvious concerning measurement“ (ibid. 2012, p. 22). This is also true for the area of length “although of the […] physical quantities, length is undoubtedly the most primary“ (Buys & Moor, 2005, p. 18). It can be stated that in descriptions of the development of a concept of length, comparing objects indirectly constitutes an important aspect (e.g. Battista, 2006; Clements & Stephan, 2009; Piaget, Inhelder & Szeminska, 1975). Therefore, this article focuses on comparing items indirectly.

Looking on empirical studies it can be also stated, that there is a lack of studies investigating especially younger children`s ideas about length and also about children’s competencies of comparing indirectly (Gasteiger, 2010).

Nevertheless, there are some studies which investigated children`s concept of length, focusing especially on competencies of comparing indirectly. But these studies took place with children starting school or attending school (e.g. Hiebert, 1981; Nunes, Light & Mason, 2003; Schmidt & Weiser, 1986). The focus of this paper will be on competencies concerning comparing items indirectly of four to six year old children.

THEORETICAL BACKGROUND

The following section firstly deals with a normative aspect of what kind of competences are identified for comparing items indirectly. Empirical results are included as well while describing these competences. One must have in mind that all research results are referring to children in primary school. Only the study by Schmidt and Weiser (1986) children were asked who were just starting school.

Concepts of length and its linear character

In order to be able to compare lengths indirectly it is necessary to develop an idea about lengths and its understanding of the attribute (Nührenbörger 2002, p.100). This means that children have to learn to focus their attention only on the length of an object while ignoring its other attributes, for example like colour or taste.

Comparing objects directly

In order to compare objects directly understanding additivity (Clements & Sarama, 2009), decomposing (Battista, 2006) and part-whole relationships are essential. Referring to children just starting school, Hiebert (1984) showed with his survey that the idea that the length of an object consists of the sum of its different parts is not new to these children. Griesel (1996) and Nührenbörger (2002) are convinced that children who are capable of comparing objects directly have learned that it is possible to divide the longer object in two parts. Whereas one of these two parts is as long as the shorter unit that has to be compared and the second part is as long as the overhanging piece. Consequently, children who can compare things directly would also understand the different concepts of additivity, decomposing and part-whole relationships.

Conservation and transitivity

Regarding the importance of conservation there are different positions. Piaget et al. (1975) emphasize the importance of understanding conservation and transitivity. They are convinced that children are only able to measure when they have learned to understand transitivity and conservation. In contrast to that, Hiebert (1981, p. 208) observed that first-grade children who did not yet have developed an understanding of conservation and transitivity were able to implement different measurement strategies successfully (cf. also Schmidt & Weiser 1986, p.150f.).

Unit iteration

If a shorter medium is used to compare an object indirectly the comparison will only be successful if the children can control the unit iteration and if they are able to count these units. Together with the transitivity many authors highlight the key role of this expertise (Battista, 2006; Clements & Sarama, 2009; Piaget et al., 1975; Schmidt & Weiser, 1986). While repeatedly conducting the unit iteration, it is important to focus on the fact that there are no gaps in between the units. Schmidt and Weiser (1986) observed in their survey that even those children starting school who implemented the idea of the unit iteration did not attach value to an entire measurement without gaps.

Application of suitable adjectives

In order to be able to successfully compare objects indirectly children have to be able to describe the results with the aid of suitable adjectives. Relational terms such as *longer, wider, lower*... are prerequisites for being able to communicate about lengths (Gasteiger, 2006, p.11). Nührenbörger (2002) shows that faulty comparisons were not wrong because the comparison itself was wrong but because the comparison and its result were not described correctly with the second graders. Whereas Schmidt and Weiser (1986) could prove that children starting school use relational terms properly in connection with measures even if they do not have an idea about measurement. For example, when being asked if three meters or five meters are longer the children answered correctly. It seems as if children who have an understanding of figures a "transfer within" (Schmidt & Weiser, 1986) takes place. The relational term 'is longer than' can – at least verbally – be referred to relations between figures in a correct way ("is a bigger number than", "is bigger than", "is more than") (Schmidt & Weiser 1986, p. 145).

Regarding these aspects it becomes obvious that comparing length indirectly requires many different competences. Is dealing with measurement, especially length, (Castagnetti & Vecchi, 2002) to demanding for young children particularly an indirect comparison? In order to answer this question the following research question has been addressed.

RESEARCH QUESTIONS

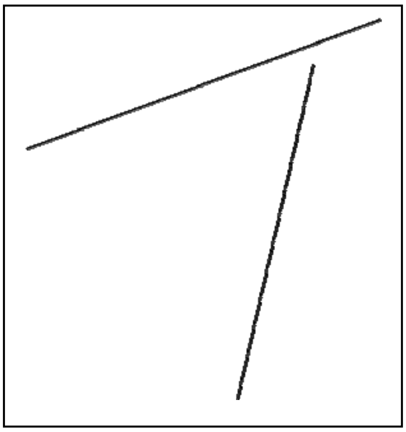
Which individual solution processes can be observed for the different aspects of comparing two different lines indirectly?

STUDY DESIGN

In order to get first insights into children's solution processes at the age of four to six for comparing indirectly a qualitative method was chosen: Qualitative interviews were conducted. All children had to solve the same task, however, the following strategy was oriented on the children's expressions and strategies. In this way, it was possible to capture individual processes. The qualitative interviews were videotaped. In the analysis of the interviews categories were generated to describe the children's processes.

The research was conducted with 40 children as a cross-sectional study with four to six year old children. The children were selected randomly.

On the basis of one task which was used in the interviews the generating of the categories will be described and first results will be illustrated. Following with Oswald there will be also some quantitative data given, due to the fact that quantitative data can be one aspect of qualitative reality (Oswald 2010, 186). Moreover, through quantitative data expressions like typically, generally, frequently, rare, can be avoided. These expressions are not clearly defined and might complicate the comprehension. Besides the interpretation of the qualitative data, there will be quantified details (e.g. number of children using the same arguments).



In the beginning, two stripes of sellotape (1,20m and 1,30m long) were taped on the floor (see figure). Initially, the children were asked to state which line is the longer one. After that, they not only had to give a reason for their idea but also had to give an idea about how to proof their answer. The second step included giving the children several tools which helped them to proof their first statement: different wooden sticks and cords, a 30cm long ruler, a set square, a measuring tape, a surveyors tape and a folding yardstick.

RESULTS AND INTERPRETATION

While the children were trying to solve that task the following steps could be identified: at first length comparison without any tools, selection of a tool, the application of the tool and the reasoning.

In the following, some parts of the classificatory scheme that was generated to analyse the children`s actions will be explained and interpreted.

First length comparison without any tools
All children followed the request to compare the two lines with each other. It was especially interesting how the children reasoned the results of their comparison. Thereupon, the following four categories could be generated:

- The endpoints were related to each other as if compared directly (n=10).
- “I have seen it” was explained as a substantiation (n=8).
- Imaginary units were counted (n=4).
- 18 children did not give any explanation.

Luka (female; 5;4 years old) can be included into the category *Imaginary units were counted*. She tried to implement the task in the following way:

Interviewer: What do you think, which of these stripes is longer?

Luka: (points her finger in the air) This one!

Interviewer: Why do you think so?

Luka: Because I have counted it.

Interviewer: Can you show me again how you did that exactly?

Luka: (taps her finger along the line and counts imaginary units) 1,2,3,4 .... 40 (doing this, she not only skips some numbers but also not every tap with her finger corresponds to one number).

Luka knows that if two lengths are measured and the figures are contrasted it is possible to compare them with each other. Thus, she equalizes the idea of measuring with the idea of counting. In the course of this, she does not consider equal units.

Selection of the tool
Referring to this solution step it becomes apparent that in this survey[1] fourteen children took a measuring tape, sixteen a folding yardstick, five a surveyors tape and five took a ruler. Solely four children used a stick or a lace as a non-standardized tool. Finally, one child did not use any tool[2] at all.
Nunes et al. (1993) could observe that most six to eight years old children – provided that there is one at hand – prefer a standardized tool to compare things indirectly. Consequently, the study underlying this article approves Nunes et al. observation for younger children. It can be assumed that children equate the process of comparing things indirectly with a measurement process and thus, while measuring, try to imitate adults. Likewise, it could be possible that the children taking part in the survey did not take sticks or laces because these are ordinary toys, whereas standardized measuring tools could be much more attractive in the children`s eyes.

Usage of the tool
While analyzing the usage of the tool the focus will be on how children apply it, whether they have taken into consideration to use the tool straight and on how the children deal with the endpoint. Here, not only the approaches of those children who used a standardized measuring tape have been regarded but also of those children who used a stick or a lace.

Applying the tool[3]: It is salient that all children applied the tool in a way that the stripe`s and the tool`s starting point meet each other (n=31). Except for the ruler, the starting point of each tool equals the scale`s zero point. Four children tried to apply the tool so that one of its endpoint coincides with the endpoint of the stripe and five children applied it without any visible relation to either the one or the other endpoint.

Process: To compare the lines successfully the children have to put the tool straight on the line. Due to the fact that when using an inflexible tool, such as a ruler, one automatically uses it in an even way. It is not possible to come up with a statement explaining whether the children have consciously tried to apply the tool in an even way and, thus, those cases will not be regarded. Six of those children who have used

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[1] In Zöllner (2012) there are slightly different numbers of children, because there vague situations were not included. In terms of intersubjectivity. These ambiguous cases were discussed later with a team of researchers and, consequently, they can be included here.
[2] Some children used different tools which have been counted.
[3] Regarding those children who used several tools, in the analysis the focus was on that solution process which the children favored most. Two children have processed several approaches with the same tool and, thus, both of these processes have been acquired.

a measuring tape tried to use it in an even way, meaning that they tried to use it straight.

Reading of measured values: Thirteen children either tried to read a number from the scale of the tool on their own or asked the interviewer to read it to them. In other words, those children paid attention to the scale`s figures and use them as an aid. Whereas others used a certain point as a marker (without paying attention to the figures) or they used their finger. On the other hand, eight children opened the tool so far that it equalled the length of the line. Eleven children completely ignored the endpoint of the line in relation to the endpoint of the tool.

Usage of the tool by the second line: Measuring the second line, ten children did not use any tool (neither the one used for measuring the first line nor another one). Therefore, one could conclude that they observed adults using the meter and, thus, try to imitate these movements without having developed an idea of comparing indirectly.

In observing of how the children applied the tool on the second line it was examined whether they did it in a similar way as they had tried to measure the first line or whether they chose another approach.

It could be observed that twenty-four children used the same tool in the same way to compare with the first as well as with the second line, whereas six children used a different approach for using the tool with the second line. . To determine the result twenty children used the same approach as before. Four children did not do that. Six children completely ignored the endpoint of the second line in relation to the endpoint of the tool.

Generally, it can be stated that those children who used a standardized tool were more successful in comparing the lines indirectly than those who used an arbitrary tool. Only one of these children that had used an arbitrary tool gave an explanation. Indeed, this explanation does not refer to the process of measuring itself but the child tried to correlate the endpoints as if it is a direct comparison.

Again, this result equals the ones that Nunes et al. (1993) observed in their survey with older children. This investigation showed that children are more successful with a standardized tool even though – accordingly to its construction – they did not use it correctly. This approach becomes visible when regarding Elsa`s (female; 6;3 years old) approach:

Elsa:	(opens the folding yardstick and tries to apply it in a way that its starting point coincides with the line`s starting point (130cm). Then she uses the edge of her hand to mark the line`s endpoint) So far, that much.
Elsa:	(still holding her hand on the folding yardstick, she walks towards the other line (120 cm) and applies it so that the “handmarked” point equals the

	starting point of the line. The yardstick`s endpoint is above the line`s endpoint) Well no, they are not of equal length!
Interviewer:	And which one is the longer one?
Elsa:	This one! (points to the line which is 130 cm long)

Elsa chose a folding yardstick to compare the stripes indirectly. She used it very proficient but, nevertheless, as if it was a non-standardized tool. This means that, considering the fact that she used her hand as a marker, the scale on the yardstick is not relevant to her at all.

Conclusion

Finally, after measuring the children have not only been asked again to compare the length relation of the two line, but they also have been asked to reason their thoughts. Some children (n=9) came up with their first explanation again – which shows that using a tool did not make them choose another reason to describe their result. Ten children did not reason their result at all. Twelve children reasoned their result with the aid of markers and a direct comparison. With the help of a marker, a figure or by adjusting the tool the children tried to depict the first line on the tool. After that, they compared their result with the second line and deduced their outcome. Two children gave up this task before they came to an end. Seven children reasoned because of the larger/ smaller relations between numbers, meaning that those children used a standardized tool and, if necessary, they asked someone to read the figure to them or they read it themselves. After that, they compared those figures to be able to conclude the stripes` lengths.

Luka can also be included into this category. Due to the fact that both numbers (130 and 120) exceed her active range of numbers, she always asked, after having measured both lines with the measuring tape, which of the two numbers is the bigger one. Nevertheless, she correctly correlates the bigger number with the longer line. As this example shows, it is noticeable that Luka accomplishes a “transfer within” (Schmidt & Weiser 1986) from the numbers to lengths.

Summary

Summarizing the results and referring them to the aspects that have been emphasized before the following results become apparent: The participating four to six years old children had no difficulties to clearly identify the different lengths and their linear character at this task. This is not surprising because concerning lines, the linear character and the usage of the concept “length” is clear. All children of that survey could easily compare objects directly. Some children had the idea: Measuring equals counting.

As it was described above, some researchers claim that developing an idea about transitivity is the most important aspect in order to be able to compare things indirectly (Piaget et al, 1975). Indeed, observing the children`s actions it becomes obvious that none of them employs transitivity in its mathematical sense (if A > B and B>C then A>C). Moreover, the tool has the function of representing the length of the first line. In the following, the children compare the second line directly with the aid of their chosen tool. In the case that the children consider the figures on their standardized tool, they use them as a marker. Then, they either follow the same approach as described above or, as Luka did, they compare the two figures and thus, identify the longer line. Consequently, in these situations those children do not think deductively and transitively. Concerning the conservation the following two aspects could be seen:

> Shifting a rigid tool (a stick, a ruler or a yardstick), none of the children wondered about the fact that its length always stayed the same.
>
> Nine children did not pay attention to the course (lace and measuring tape). Regarding those children it can be assumed that they either have no idea about conservation at all or that they cannot recognize the conservation in connection with this situation.

In order to compare an object indirectly, these young children prefer standardized measuring instruments and are more successful when using standardized measuring instruments. They use seldom shorter tools in order to compare indirectly, regardless of whether it is standardized or not. The unit iteration of a shorter tool was not observed.

In order to describe their result the children were able to choose correct adjectives. Due to the fact that the interviewer could ask the children in case of vagueness it can be assumed that the children understood the relational terms correctly.

DISCUSSION AND CONCLUSION

In conclusion it can be stated that many of the competences considering normative aspects for an indirect comparison can be observed by four to six year old children

All children had an idea on how to solve the task, and it became obvious that concerning comparing items indirectly children already have acquired some competences and thus do not enter school as tabula rasa. Many children already have experiences with comparing lengths. Dealing with length (Castagnetti & Vecchi, 2002), especially indirect comparison, can therefore constitute a challenging task for young children.

Looking on instruction about length this study reveals that emphasizing the unit iteration before using a standardized tool does not correspond the children`s natural approach. These insights can provide a basis for an instruction approach which is orientated on children’s own constructions.

REFERENCES

Battista, M. T. (2006). Understandig the development of students' thinking about length. *Teaching Children Mathematics 13*(3), 140–146.

Benz, C. (2012). Attitudes of Kindergarten Educators about Math. *Journal für Mathematik-Didaktik*; online first: http://www.springerlink.com/content/m30356400m472554/?MUD=MP.

Buys, K. & Moor, E. (2005). Domain Description Measurement. In M. van den Heuvel-Panhuizen & K. Buys (Eds.), *Young Children Learn Measurement and Geometry*; *A Learning-Teaching Trajectory with Intermediate Attainment Targets for The Lower Grades in Primary School* (15-36). Utrecht: Freudenthal Institute.

Castagnetti, M. & Vecchi, V. (2002). *Schuh und Meter*. [Shoe and Meter]. Weinheim: Beltz.

Clements, D. H. & Sarama, J. (2009). *Learning and Teaching Early Math. The Learning Trajectories Approach*. New York, London: Routledge.

Copley, H. (2006). *The Young Child and Mathematics*. Washington, D.C.: NAEYC; Reston, Virginia: NCTM.

Gasteiger, H. (2006). *Zwischenstandsbericht; Wissenschaftliche Begleitung der Implementierung der Lerndokumentation Mathematik im Rahmen des Projekts TransKiGs für das Land Berlin*. Retrieved from http://www.bildung-brandenburg.de/transkigs/fileadmin/user/redakteur/Berlin/Bericht_WissBegleitung TransKiGs_Dez06.pdf

Gasteiger, H. (2010). *Elementare mathematische Bildung im Alltag der Kindertagesstätte. Grundlegung und Evaluation eines kompetenzorientierten Förderansatzes.* [Elementary Mathematics Education in Everyday Life in Kindergarden]. Münster: Waxmann.

Griesel, H. (1996). Grundvorstellungen zu Größen. *Mathematik lehren, 78*, 15–19.

Hiebert, J. (1981). Cognitive Development and Learning Linear Measurement. *Journal for Research in Mathematics Education. 12* (3), 197–211.

Hiebert, J. (1984). Why Do Some Children Have Trouble Learning Measurement Concepts? *Arithmetic Teacher 31* (7), 19–24.

Nührenbörger, M. (2002). *Denk- und Lernwege von Kindern beim Messen von Längen: Theoretische Grundlegung und Fallstudien kindlicher Längenkonzepte im Laufe des 2. Schuljahres*. [Children’s Ways to Think and to Lern in Measurementsituations with Length]. Hildesheim: Franzbecker.

Nunes, T., Light, P., Mason, J. (1993). Tools for Thought: The Measurement of Length and Area. *Learning and Instruction, 3,* 39–54.

Oswald, H. (2010). Was heißt qualitativ forschen? In B. Frieberthäuser, A. Langer, A. Prengel (Eds.), *Handbuch Qualitative Forschungsmethoden in der Erziehungswissenschaft*. Weinheim, Münschen: Juventa.

Piaget, J., Inhelder, B., Szeminska, A. (1975). Die *natürliche Geometrie des Kindes*. [The Child’s Conception of Geometry]. Stuttgart: Klett.

Schmidt, S. & Weiser, W. (1986). Zum Maßzahlverständnis von Schulanfängern. *Journal für Mathematikdidaktik*, *7*(2/3), 121–154.

Zöllner, J. (2011). Längenverständnis bei 4- bis 6jährigen Kindern. In M. Ludwig & M. Kleine (Eds.), *Vorträge auf der 46. Tagung für Didaktik der Mathematik vom 05.03.2012 bis 09.03.2012 in Weingarten* 2 (969-972). Münster: WTM.