

NARRATIVE CONTEXT AND PARADIGMATIC TOOLS: A TALE FOR COUNTING

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We present a design study about the use of narration to frame a work on counting in a first grade class. We propose to use the ideas of ‘narrative’ and ‘paradigmatic’ ways of thinking (Bruner, 1986) in order to design, manage and analyse the development of children’s experience. In this specific experience the role of logical tools is played by the criteria used to carry on the count process, conceived as coordination of two different semiotic activities.

Keywords: storytelling, counting, primary school.

INTRODUCTION

Stories and storytelling continue to arise interest of practitioners in different research fields, from psychologists and anthropologists characterizing this kind of activities as a crucial one in human cognitive and cultural development, to educators recognizing their value in order to pursue educational goals for specific disciplines: “*The value of the story to teaching is precisely its power to engage the students’ emotions and also, connectedly, their imaginations in the material of the curriculum*” (Liljedhal and Zazkis, 2009, p. 3). Actually, stories can provide controlled contexts for experimenting parts of the complexity of life and for creating meaning in our lives and environments (Green, 2004). In this sense telling a story is a way of creating meaning also for mathematical structures: in a story context we can offer pupils an authentic experience of modelling by leading and supporting them in recognizing the mathematical structures as embedded in facts and embodied in human ways of managing them. Our research group has been working on this idea for several years, with the aim of recognizing some features of narratives – and of the way of using storytelling – which can enable mathematical meanings to emerge among the several ones intertwined in a narrative context. In this paper we present a particular design research (Cobb et al., 2003) built upon the theoretical perspective of the paradigmatic and narrative ways of thinking pointed out by Bruner (1986). We used this frame in order to actually design a teaching-learning trajectory using storytelling with first grade children, with the aim to explore children’s counting strategies and suitable didactical mediations to make their mathematical skills develop. In particular we aim at exploring opportunities (in terms of capacities of designing settings and interpreting the correlated dynamics) provided by a driving-conjecture about counting as *focusing of attention*.

THEORETICAL FRAMEWORK

As many scholars have been pointing out in the last years, a mathematics classroom practice exclusively built on standardized word problems and routine exercises does not educate students to a genuine mathematical modelling disposition (see for

example Verschaffel et al., 2000). One of the hypotheses put forward claims that the verisimilitude and likelihood that something told could be true and real in some coherent “world” (even though it could be not true and real at all) have been very often neglected in the stories used to do mathematics at school (see for example the narrative ruptures in the word problem texts detected by Zan (2011)). On the contrary storytelling can – by means of its verisimilitude – represent a special experience of teaching and learning also in a mathematics classroom (see for example Lilledhal and Zazkis (2009)), precisely for its power to evoke situations (even though completely fantastic) in which children can project themselves according to their previous experience and own sensitiveness. Indeed telling a story, as well as proposing a “story problem”, entails a triggering process of the children’s natural interest for human actions and interactions (Donaldson, 1978).

According to Bruner what makes a narrated tale really interesting for people is its “violation of canonicity”, that is a situation that a listener feels as unusual with respect to the expectations determined by the contextual background (Bruner, 1990). The problem of restoration of canonicity is crucial in the development of the tale and we think that it can be used in an education perspective as an occasion to introduce or to build new cultural tools. In particular, Bruner (1986) points out two complementary ways of thinking: the narrative one arranged as an open space of possibilities that frames the temporal sequence of events and the choices taken time after time, and the paradigmatic one linked with the formal constraints (logical, algebraic, physical and so on), which rules the space of possibilities opened by the tale. On the mathematics education level we assume that within a story the violation of canonicity can be used to introduce or build paradigmatic mathematical tools in order to establish again the scripts of the narrative space. For this goal we also recognize the need to let children directly experience the actions described in the narration. In this view dramatization represents a natural step to be explored, along with the development of the story (for another example at kindergarten level see Mellone et al., 2012). Outside of the dramatization – and while it develops – the children, supported by adults, can explore how to overcome the breaches in canonicity building the paradigmatic tools that are useful to solve the tricky situations.

The design study we are going to outline here is placed in the trend focused in the use of stories in order to build new mathematical tools and meanings: in particular, it concerns a work around the counting process. Regarding the paradigmatic aspects of counting we refer first to Gelman & Gallistel (1978) who describe the counting process through five principles: the One-one Principle, the Stable-order Principle, the Cardinal Principle, the Abstraction Principle, the Order-irrelevance Principle. Such principles have the merit of carefully describing the counting process giving, at the same time, instruments to detect the difficulties that children meet during this activity. But, nowadays the new insights coming from neurosciences (see for example Piazza (2010)) show that the counting process is, beside being an external

phenomenon, an active mental process of either recognizing a naturally discretized aspect of reality, or imposing a more or less rhythmical discretization of continuous magnitudes. In both cases the core of such a process is a (perceptual and) internal gesture of *focusing of attention*, in the sense that what we count is always a “closed” internal action-of-individuation, variously correlated to others (from utterance to external doing). This conjecture becomes useful mainly when we want to study the counting of events developing along time. In this situation, one of the problems that needs to be firstly faced is that of deciding what are the rules to proceed into the count, that means understanding “what we count when we count”. This metacognitive activity can be supported by a semiotic activity (in a vygotskian sense) by expressing, and in this way stabilizing, the external events in correspondence of which we can add one to the count: in other words, using paradigmatic tools to lead the focusing of attention.

Our hypothesis is that this aspect of the semiotic activity linked with counting is different from – and needs to be coordinated with – the ones described by Gelman and Gallistel, which mainly deal with the use of labels (“numeros” for them). In other words, in counting events children need at first to recognize the discrete events corresponding to “add one to the count” and then to coordinate this recognition with the use of labels (as described in Gelman and Gallistel’s principles).

THE DESIGN STUDY

The research has been carried out in two primary school classes. The research team is composed by the class teachers and by some researchers. Two researchers play an active role in the ordinary educational work, spending about two hours a week in each class throughout the whole school year. The research is still going on, following the class through years. In this work we only account a part of the path developed in one of the involved classes, when it was at first grade. This specific part lasted two months at the beginning of the school year (that is seven working sessions of two hours each). Our research work can be defined as a design experiment (Cobb et al., 2003) for its ecologic and engineering features in exploring the opportunities that the conjecture of counting as focusing of attention can offer in planning and managing long term educational paths framed in a narration context. The classes also dealt with number issues in several other ways, working in the daily activities with the teachers. Nevertheless the teachers’ work and our design research were closely intertwined and reciprocally supported.

We believe the goals of mathematics education should include letting children experience rich activities that, in the case of counting, mean also involving them in purposeful activities of counting events. Having this in mind we built an ad hoc narrative frame, encompassing an element bringing about a breach in canonicity and letting children give themselves up the story development and directly act (as characters) in dramatized forms of the tale. We aim at leading children in building their own (geometrical and numerical) semiotic tools and use them in order to enable a restoration of canonicity and a consequent development of the story. In particular, in order to support the children into the coordination of the two semiotic activities

linked with counting, we propose to lead them to move to a representation level, where *both* the events and the objects to be counted are represented with certain mobile marker objects (in the case of counting objects the markers should be different from the objects). In the analysis we will develop in the following paragraph, we underline the moment when we “went out” of the tale and the way we worked on the coordination of the two semiotic activities described in the previous paragraph. Our data consist in researchers’ and teachers’ observations, in excerpts from classroom discussions and children’s representations.

Lastly, we would like to highlight the choice we made to put problems that were difficult enough in order to be hardly handled without the use of suitable tools (as marks or signs) and, at the same time, accessible and comprehensible enough in order to let the children recognize them as manageable. In our case, a difficulty was related to the presence of several rhythmical processes to be simultaneously controlled; another difficulty concerned the fact that the number of times (or the number of things each time) to be counted, though small enough, exceeded the limits related to the *subitizing* (for a review on this issues, see (Dehaene, 1997)).

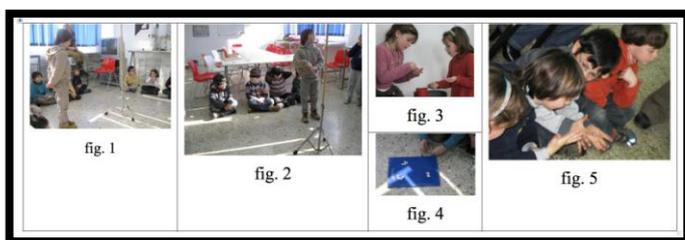
THE DEVELOPMENT OF THE TALE

Temujin has been taken captive and is compelled to serve a sentence: he has been tied to a grindstone and forced to go round and round, in order to hull rice. Every morning Temujin is woken up by two beats of a gong, struck by a guard, and begins its job. He has to stop at the evening, when the guard beats the gong one time. Each time Temujin completes a turn around the grindstone, four rice grains are hulled and cumulatively collected in a holder. Several ‘accountants’ have the task of counting the number of turns accomplished by Temujin and the number of days of work, in order to report them to an inspector.

In this phase, as in the following ones, the action presented in a narrative form is also contextually dramatized by the children, so that they have to deal with the necessity of orienting their actions, leading to a certain development of the story. A child played the role of Temujin (fig.1); two children played the role of the grindstone, having autonomously to decide when the rice grains should be dropped in the holder (fig.3 and fig.4); a researcher played the role of the guard (actually signalling the end of a day every time Temujin completes two turns, but this was not made explicit with the

children); some children had to count the number of turns completed by Temujin (fig.5); other blindfolded children had to count the number of the days spent (fig.2). The dramatization has been repeated many times and each child has played several roles in order to let

children became familiar with the canonicity of the situation. Each dramatized sequence lasted about six “story days” (twelve child’s turns) and, at the end of each sequence, a sharing was held in order to check if the counting worked.



The design of the story has been driven by the conjecture according to which counting is basically an oscillation between recognizing and imposing a rhythm, meant as complementary aspects of an intentional and aware addressing of attention and action. The considered situation is characterized by different rhythms: Temujin has to comply with the rhythm marked by the gong; the rice dispensers have to decide the right moment when they have to drop the rice grains and – at the same time – they have to warrant the right number (four) of dropped grains at each turn; the accountants have to establish how to record number of turns or days they counted. We intended to support children to try and orient themselves among these different rhythmical structures, by mean of discussions devoted to collectively treat each issue.

Initially we observed the counting strategies children adopted. In this phase we distinguished three kinds of attitudes:

1) Some children count properly. They recognize the occurrence of a repeated structure, distinguishing the base elements: two beats plus one beat of gong; four rice grains. These children are able to express the operative procedure they follow in order to rhythmically modulate the action

Elena: Maya shouldn't add four grains if Alyce has already dropped them [Maya and Alyce played the role of the grindstone], because Temujin didn't do two turns! [...]

Ilario [a counter of the days]: I add one after two beats, he after one beat [turned to another counter of the days]. It's the same! We always count the days, only I count the mornings and he counts the evenings.

Here Ilario evidently recognizes there are alternative criteria for deciding when one has to proceed in the counting.

2) Other children tendentially singsong, in the sense that they count throughout the whole development of an action (i.e. during a whole day or a whole turn), even if they understand there are moments when the counting has to be stopped, for example during the night, when one sleeps. The conduct of these children is coherently commented by the first ones:

Stefano: Gianluca [a counter of the days who singsongs] is counting the minutes or the seconds, not the days.

Alyce: The seconds, not the days! [...]

Camilla: Ornella [a counter of the turns who singsongs] isn't counting the turns! She's counting the steps!

3) In an intermediate position there are children that struggle to keep counting properly. These children understand the constraints to be complied with in order to proceed with counting, but they apparently make a great effort to avoid the singsong, or to avoid counting two times the same base elements. It is interesting to highlight Lucio's conduct. As a blindfolded counter of days, he counted "one" in

correspondence of morning beats and “two” in the correspondence of the evening one, while he tried to avoid this mistake:

Lucio: One... two... oh no, wait!

With the exclamation “wait!” aimed at himself, Lucio brings out the fact that counting is actually an oriented activity, which implies effort. An analogous situation concerns those children who play the role of parts of the grindstone. Maya’s case seems to be emblematic. At the beginning Maya had difficulties in playing alone the role of grain dispenser; then she completely relied on Alyce, who helped Maya and suggested her when dropping the grains. In the meantime Maya tried to autonomously move, oscillating among three facts: the peremptory indications by Alyce, the continuous wheeling around of Antonio (who played the role of Temujin) and her attempt to coordinate herself with them and to autonomously decide when dropping the rice grains.

In a second phase, we provided the counters with some objects, as bottle tops and seashells, to be used as markers during the counting. Nevertheless, the introduction of these supports did not change the scenario. In fact, the use of markers seems to be not particularly useful – at least for little quantities – for those that are already able to count properly, as highlighted by these comments

Eleonora: I count in my head!

Lars: I make it like this [counting on his fingers] and I make it well!

At the same time those who had difficulties in controlling the several rhythmical scans appear to be even more confused by the invitation to use the markers. At this point we considered opportune for the group to move outside from the temporally ordered narration and from its heuristic dramatization, in order to build the atemporal paradigmatic prostheses, the explicative tools aimed at establishing rules and controlling procedures in counting. This is a crucial point in the development of our intervention. Our tale comprises several activities, which can be effectively managed through some kind of (mathematical) formalisation. Now, the activity aimed at formalising does not belong to the narrative. It rather needs for another setting, other actions and other discourses, in order to build the tools used for explaining and controlling what happens at the narrative level. Therefore children are continuously invited to go back and forth between the dramatized narration and the paradigmatic level, in alternating and complementary phases. So we transferred into the gymnasium to work on structuring the space, in order to be able to determine when a child completes a turn. The children formed a circle and were invited to discuss about the movement of some classmates, who had the task of completing a turn around that same circle. The discussion was aimed at settling criteria for understanding when a child completes a turn. At the end, the group agreed on the advisability to mark out a line on the floor, in order to recognize the end of a complete turn. We made the same for what concerned the counting of the days spent. In this case the group should have dealt with a structuring of the temporal events: the discussion focussed on the

distribution of beats of gong in a day and on the different ways to recognize the passing of the time, following some suggestions given by those who count properly.

After this activity, we came back to the narrative context, providing children with markers that were used by most of them. Therefore objects were used in order to record the number of turns and days, as well as signs were used to decide 'what' had to be counted: the crossing of a line, the sound of two close beats, the sound of a separate beat, a special sequence of sounds (two close beats plus a separate one). It seems interesting to recognize the usefulness of support given from the practice of proper counting together: being supported by others, children that have difficulties can find time and ways of coordinating words and gestures with what they perceive, distinguishing the events to be counted and being able to record all of them, without getting confused in the complexity of those operations.

Subsequently, a second part of the story was told.

Suddenly an inspector arrives, with the task of verifying whereas Temujin has served the sentence. The inspector declares Temujin will be released when he will have worked for no less than 7 days (and no one more) and will have completed 16 turns. The accountants are so consulted in order to decide about the Temujin's destiny. But different groups of accountants express contrasting opinions about the possibility of setting Temujin free. Therefore the inspector gets initially angry. At the end he will accept other conditions, but he will ask for a written document, which certifies the actual Temujin's work.

The new situation is clearly designed so that the condition put by the inspector is incoherent with the constraints of Temujin's activity: for those who count days, he may be free; for those who count turns, he may not. This is the point where we introduce in the tale a studied violation of canonicity. A character with a normative role (the inspector) provides a criterion to decide if the prisoner can be set free or not. According to canonical expectations, the given condition should be granted or not, so that one should be able to decide about the release of Temujin. In our case, the contradictory judgements of the accountants, and the consequent impossibility of deciding, put the group at an impasse. This doubtful situation needs explanations and strategies to allow a plausible development of the tale. At this point the paradigmatic mode, the arithmetic in our case, is committed to explain the rifts in the narration (Bruner, 1990). Through the necessity of settling this matter, we aimed at making explicit (at least at a certain level) the multiplicative structure, which is always intertwined to the counting operation (characterized by the ratio of two turns per day, which was left hidden during the whole development of the activity). At the same time, we aimed at raising the necessity of producing a representation of what had happened in order to encompass the several rhythms that had characterized the considered processes.

Meanwhile, one looks for plausible solutions to the missing satisfaction of the conditions necessary to set the prisoner free. Anyway, no child observed that the inspector's conditions couldn't be fulfilled together: this suggests that the ratio,

which correlates number of turns and number of days, is not well understood by pupils even if it is clearly “smelled”: here we move in a zone of proximal development (Vygotsky, 1934). In fact, when we ask to dramatize again the situation, children begin to identify the correlations among the several rhythms as a crux, even if they do not have a symbolic system which enables them to highlight the inner contradiction of the inspector’s request

Elena: You finish the day too soon! You should let him do more turns! [talking with the researcher that beats the gong]

Researcher: I only beat when the night comes.

Elena: We can change both the things. Day and night longer and Temujin faster.

Lucio: Or we could say to the inspector Temujin worked for seven days and did sixteen turns!

Elena: We can’t say he did sixteen turns! We can’t cheat! The inspector will realize it!

Lucio: How does he realize it?

Elena: If he counts the grains, he realizes it!

We are at a moment when the children try to identify the crucial ratio between turns and days, but they are not able to make it explicit. We decide to intervene, simplifying the situation and addressing the work toward a clarifying semiotic activity: the inspector negotiates his request, asking Temujin to work for 8 days doing 16 turns. In return for the prisoner’s freedom, the inspector asks for a written document, which certifies the exact number of days of work, completed turns and hulled grains. Children work in little groups on drafting several documents using mobile markers. This activity is carried out under adult’s supervision, gradually helping to organize the groups work. The adults support and partially address the activities, fostering the circulation of ideas among groups, asking for explanations, giving suggestions. At the first, we attempted to share ways of representing days, turns and grains:

Eleonora: We can make two circles, which are the two beats.

Stefano: But I counted a day each three beats.

Eleonora: But I don’t find it good, so.

Researcher: What do you mean about making a white circle, as a sun, in order to represent a day?

All children agree.

This short excerpt represents an attempt of coordinating the two semiotic activities of focussing of attention and labelling. In fact during the first phase of the path the adult mediation led the semiotic activity related to identifying events that corresponds to add one to the counting. Now one has to coordinate this semiotic activity with that

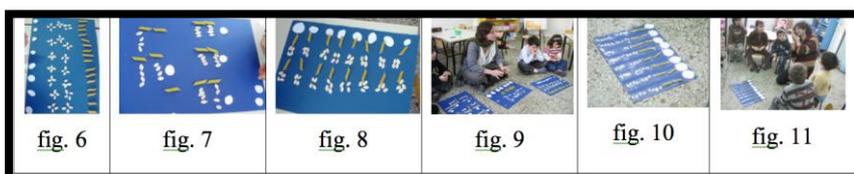
described by Gelman and Gallistel (1978). By means of the use of shared markers (white circle for the days, macaroni for the turns, pumpkin seeds for the rice grains) the groups build several documents, actually several ways of giving form to the rhythmical structures and to their mutual correlations (see for example fig. 6, fig. 7, fig. 8, fig. 9). The produced representations are shared and read together (fig.9), in order to test their comprehensibility and coherence (fig.11). In some cases the particular rhythmical disposition of the markers on the sheet, as well as their spatial relationships, enable to highlight the more “difficult” relations which were not yet made explicit, as the number of hulled grains per day, or the number of turns per day. A final discussion (fig.9) aims at selecting the “best” document for the inspector (or at producing a shared new one).

Camilla: According to me, one better understands our one [fig.8], because it really says what happened. [She points out the first column of the array] There is the day, then he has made a turn, so the grindstone has dropped four grains, then another turn and other grains.

Elena: Oh yes, but our one [fig.10] is more orderly. The sun, then two turns, then the grains.

We would like to emphasize the different features of the strategies adopted in these two representations, according to their authors’ comments that had the opportunity to emerge due to the use of mobile markers. On the one hand, Camilla is interested in producing a representation faithfully keeping the temporal development of the narrated and dramatized action. On the other hand, the introduction of markers as meaningful signs led Elena’s group to reconsider and reorganise signs according to their meaning, in a semiotic activity that actually presents a fruitful ground for the

further dealing, in a significant way, with the roots of the multiplicative structure and its dimensional issues.



CONCLUSION

With regards of managing a storytelling for the purpose of carrying out mathematics education activities, our design study showed the relevance of encompassing the possibility of getting out from the tale, in order to build mathematical tools. For example, children that presented difficulties in counting the events, such as turns and days, didn’t seem to take advantage of using mobile markers during the dramatization. Rather children benefited by discussions aimed at establishing semiotic rules, which helped them to focus their attention, such as marking out a line on the floor in order to recognize the end of a complete turn, or focusing on the sequence of beats in order to recognize the end of a day. After the second part of the story, the use of mobile objects and the work with their arrangements supported the children in the difficult task of catching and precisely expressing the several

simultaneous rhythmical processes in the story (see the last remarks by Camilla and Elena), enabling children to progressively appropriate of role and meaning of conventional representations. In fact the following step was dealing –within the frame of the same story – with the base-ten representation (Guidoni et al., 2011).

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