EXPLORING THE FUNCTIONS OF EXPLANATIONS IN MATHEMATICAL ACTIVITIES FOR CHILDREN AGES 3-8 YEAR OLD: THE CASE OF THE ISRAELI CURRICULUM

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Explanations are an inherent part of mathematical activity. They serve various purposes and may take on different roles. This paper focuses on the functions explanations may serve in the preschool as well as in the beginning of elementary school. It investigates the use of explanation-related terms in the preschool and in the first and second grade mathematics curriculum in Israel.

Key words: Preschool; first and second grades; explanations; curriculum documents

INTRODUCTION

The mathematics education community has placed great emphasis on the giving and evaluating of explanations by students of all ages (Mueller, 2009; Yackel & Cobb, 1996). They are part of the reasoning processes we wish to encourage among students, “Students need to explain and justify their thinking and learn how to detect fallacies and critique others' thinking" (NCTM, 2000, p. 188). They are also part of communication processes which include “sharing thinking, asking questions, and explaining and justifying ideas" (NCTM, 2000, p. 194). From what age should we expect students to offer explanations when engaging in mathematical activity? Do the types of explanations and the purposes of giving explanations differ at various ages? This study examines the roles of explanations in official Israeli mathematics curriculum documents, focusing on the transition between preschool and primary school. We chose to examine official curriculum documents for several reasons. First, the national curriculum often sets the standards for what is learned and how that learning is assessed. Second, textbooks in Israel take their cue from national standards and must be approved by the Ministry of Education. In addition, prospective and practicing teachers are explicitly exposed to the curriculum during professional development and often use the guidelines to construct lesson plans. The curriculum is especially important for preschool teachers who often design their own activities based on curriculum suggestions as textbooks and other curricula materials are less available for this age group.

In Israel, there are separate curriculum documents for students of different ages. At the preschool level, the Israel Mathematics Preschool Curriculum (IMPC) (Ministry of Education, 2008) covers concepts and competencies that children should reach by the time they enter first grade. It also lists explicitly and separately which of those concepts may be promoted and which skills should be enhanced for children ages 3-4, 4-5, and 5-6 years old. For example, by the time children enter first grade they should be able to count backwards from ten but the curriculum suggests that this skill should be fostered from the age of 4 and not from the age of 3. The preschool curriculum also includes examples of activities that can be used to promote and assess the required skills. The Israel Mathematics Curriculum (IMC) for elementary school (Ministry of Education, 2006) covers concepts and competencies which should be fostered among students in grades one through six. Each grade is dealt with separately. In addition, an official supplementary document to the elementary mathematics curriculum was published by the Ministry of Education entitled "Milestones" (2009). This document includes the curriculum as well as specific standards for each content topic and explicit examples of activities that can be implemented in classes. Thus, in this study we examine the preschool curriculum document and the "Milestones" document. We focus on preschool children ages 3-6 years old and first and second grade students in elementary school (ages 6-8). The main questions of this study are: (1) Are young children (ages 4-8 years old) expected to give explanations when engaging in mathematical activities? (2) What are the functions of explanations at these ages?

THEORETICAL FRAMEWORK

How may explanations be characterized? Philosophers of science tend to view the concept of explanation as a logical relationship between questions and answers. According to van Fraassen (1980) an explanation must answer a why-question. However, Achinstein (1983) takes the broader view that many different kinds of questions may be asked when attempting to gain understanding and it follows that the act of answering any of these should be regarded as an act of explanation. Within the field of mathematics, the notion of mathematical explanation is closely related to other notions such as ‘generality’, ‘visualizability’, ‘mathematical understanding’, ‘purity of methods’, and ‘conceptual fruitfulness’ (Mancosu, 2008).

In mathematics education, explanations may be characterized by referring to their functions and forms. Yackel (2001) views explanations given in the classroom as a social construct whereby their functions and forms are interactively constituted by the teacher and students. Thus, an explanation is considered to be an aspect of discourse and its first function is communicative, “Students and the teacher give mathematical explanations to clarify aspects of their mathematical thinking that they think might not be readily apparent to others” (Yackel, 2001, p. 13). In a traditional mathematics classroom, an explanation may describe the steps of a procedure used. In the inquiry-based mathematics tradition, explanations communicate interpretations and mathematical activity to others in order to convince others that solutions are legitimate. Krummheuer (2000) found that when learning mathematics, students “’tell’ or ‘narrate’ how they came to their solution, or better put how one can come to a solution” (p. 24).

An additional function of explanations is to rationalize actions, both for the giver of the explanation as well as for the receiver. In that sense, explanations may also have the function of convincing oneself or another person of some assertion. In the field of argumentation (Krummheuer, 2000), an explanation may take on the role of data which supports some assertion, or a warrant which legitimizes a previous explanation, or a backing for the warrant. Thus, one may view explanations as the building blocks of argumentation. Convincing and explaining are also related to proving. When discussing proofs, de Villiers (1990) characterized an explanation as providing insight into why a statement is true as opposed to verifying the truth of the statement. An explanatory proof (Hanna, 2000) may help students see why a theorem is true; it is both convincing as well as illuminating. Nunokawa (2010) referred to proofs as full explanations which often contain critical ideas. He further claimed that explanations not only communicate the student's existing thoughts but may also generate new objects of thought by directing new explorations which may then deepen the student's understanding of the problem at hand. Thus, an underlying function of explanations is to expand students' mathematics learning.

METHOD

The first stage of this study was simply to gather each instance of when the curricula use explanation-related terms. In Hebrew, the term 'explanation' may be translated as 'hesber' ( ). This is the most common translation and when translating back the term 'hesber' to English one always gets 'explanation'. There is also a second word in Hebrew which is sometimes translated to explanation and that word is 'nimuk' ( ). However, 'nimuk' may also be translated as justification. On the other hand, a specific word for justification, other than 'nimuk', exists. This word is 'hatzdaka' ( ). Finally, as mentioned above, explanations are sometimes related to proofs; thus the term for proof, 'hochacha' ( ) was included in this study. To summarize, when examining the curriculum documents, four terms were taken into consideration: 'hesber', 'nimuk', 'hatzdaka', and 'hochacha'.

The second stage consisted of an inductive process, whereby each instance of the four explanation-related terms was analysed with the help of guiding questions. This led to the development of categories. A separate analysis was conducted for the introduction sections of the curricula and the other sections. This separation was due to the more general format of the introductions as opposed to the specific examples of activities given in the other sections. In the introduction sections, the following questions guided our analysis: How do explanations fit in with the general aims of teaching mathematics at each level? To what purpose do we use explanations in a mathematical activity? In the other sections of the curricula we asked ourselves: In the given context, would an explanation be an answer to a "how" question or to a "why" question? Would the explanation be used to evaluate procedural or conceptual knowledge? Does it seem that the explanation is the culmination of some mathematical activity or reasoning process or might the explanation be a stimulus for further mathematical activity? Each of the authors of this paper categorized the instances on their own and then compared the analyses. A third researcher validated the final categories.

RESULTS

As mentioned in the previous section, we began by counting the number of instances each term appeared in the different curriculum documents. As can be seen from Table 1, the term 'proof' does not appear in any of the surveyed documents and the term 'justification' appears once. Recall that the term 'nimuk' does not have a clear translation and seems to fall somewhere between explanation and justification. To sum up, there are 15 instances of explanation-related terms in the preschool curriculum and 11 instances in the elementary curriculum.

	Preschool (ages 3-6)	Elementary school (ages 6-8)
Explanation	13	8
'Nimuk'	1	3
Justification	1	-
Proof	-	-

Table 1: Frequency of explanation-related terms found in different curricula

We now focus on how the different terms were used, beginning with the introductions to the two curricula and moving on to specific examples of suggested activities.

'Explanations' in the curricula introductions

The introductions relay the general aims and reasons for teaching mathematics at these ages. In the preschool curriculum there is one instance of the term justification, one instance of the term 'nimuk' and three instances of the term 'explanation':

> Participating in mathematical activities develops mathematical skills such as the ability to count and enumerate, add different amounts together, and identify shapes and solids, as well as thinking skills such as such as the ability to make comparisons, the ability to sort, and the ability to **justify** oneself and it is important to develop both of these aspects when working on mathematical activities. In order to develop mathematical understanding as well as a child's (general) thinking skills, one should request the child to **explain** his actions. The **explanations** allow the child to justify (**nimuk**) his actions allowing the teacher to better understand what the child meant. (Emphasis not in the original, p. 12)

According to the introduction, there are two separate aims of engaging children in mathematical activities. The first is to promote their knowledge of mathematical concepts and skills and the second is to promote general thinking skills and abilities. How do explanations fit in with those aims? Regarding the first aim, having children explain their actions can promote their mathematical understanding. Thus, giving explanations is a means to achieving a goal. Regarding the second aim, giving explanations is the goal. We note that it does not mention that giving mathematical explanations is the goal or that explanations should be based on mathematical ideas and principles. Instead, we desire in general to promote children's ability to justify their actions and this ability can be promoted while engaging children in mathematical activities. Another general ability, being able to verbalize one's ideas, is mentioned later on in the introduction, "mathematical discourse … develops a child's verbal abilities, that is, his (or her) ability to formulate and **explain** in words what he (or she) is doing" (INMPC, 2008, p. 14). Finally, a child's explanations can be used by the teacher to assess mathematical understanding.

In the introduction of the elementary school curriculum the terms explanation, justification and proof do not appear but there are two instances of the term 'nimuk'. One of them is related to developing children's mathematical reasoning skills such as, "deductive reasoning, …raising conjectures, generalization and **justification** (nimuk), …etc." (p. 8). The second instance is related to evaluating students' thinking, "When the teacher is assessing the students, she should follow students' engagement with task… and listen to their **explanations** (nimuk) as they implement the mathematical activity" (p. 14). In the first instance, being able to offer justifications is seen as a mathematical skill which we wish to develop. The second instance reminds us of the introduction to the preschool curriculum which suggests listening to children's explanations in order to assess their mathematical knowledge.

To summarize, from the introductions we learn that explanations are both a means and an end – the means to achieve mathematical understanding but also a skill, onto itself, to be developed. In the preschool curriculum there is no direct mention that explanations should be mathematical in nature or based on mathematical principles. In the elementary curriculum they are connected to specific mathematical reasoning skills such as making generalizations. In the next section we gain additional insight into how explanations may be used in mathematical activities.

'Explanations' in suggested activities

Following the introductions, each curriculum suggests specific examples of activities which may promote different competencies. While in the introductions we find several reasons for promoting the use of explanations, we are left unsure of what is meant by the term 'explanation'. In addition, the introductions aim at general abilities and general mathematics reasoning skills, not only those related to explanations. In this section, we review the examples of activities that include explanations and analyze what the specific function of an explanation might be in a mathematical context. These functions were not explicitly written but arose from the contexts. Below, we list the functions we found along with examples of activities from the preschool and elementary school curricula which highlight these functions.

Function 1: Explanation as a description of one's thinking process or way of solving a problem (i.e. How did you solve the problem? Explain.)

<u>Preschool curriculum</u>	<u>Elementary school curriculum</u>
Mathematical topic: Comparing sets by counting	Mathematical topic: Operations with natural numbers
Suggested activity: In competitive games	Suggested activity: You can buy the

where the winner is the one with more or less cards, tokens, etc. the teacher can see the way children use counting in order to compare two sets and the way children **explain** how they acted.

following toys in the store. (A picture of several toys and their corresponding prices is given.) Tamar has 15 NIS. She wants to buy two toys. Which toys can Tamar buy? **Explain**.

In the preschool activity, the teacher is encouraged to request from the children an explanation of how they acted, what they did in order to compare the sets. In the elementary school activity, we infer from the problem situation that students are requested to explain how they arrived at an answer. The explanation can be in the form of an arithmetic sentence or a verbal description. In both the preschool and elementary cases, the explanation allows the teacher to evaluate procedural knowledge. In essence, the explanation tells us what the child did but not why he did what he did. This brings us to the second category.

Function 2: Explanation as justifying the reasonableness or plausibility of a strategy or solution (i.e. Why did I choose to solve the problem in this way?)

<u>Preschool curriculum</u>

Mathematical topic: Measurement

Suggested activity: When comparing the measurements of two items with the help of a mediating tool, does the child use a convenient and appropriate mediator? Can the child **explain** why he chose that specific mediating tool?

In the above activity, the child is asked to solve a problem, in this case a measurement problem. He is not asked to explain what he did but instead to explain why he chose to solve the problem in such a way. For example, when comparing the lengths of two tables, the child may use his foot to measure the length of each table. However, if asked to compare the lengths of two papers, he might use paper clips as a measurement tool. The child is then asked to explain why he chose to use his foot in the first case and paper clips in the second case. This type of explanation is thus different from the first type. Yet, in this second category, explanations do not necessarily draw on mathematical properties nor are they necessarily related to general properties. In the third category, explanations are also given as an answer to a "why" question, but tend toward more general mathematical properties. In the elementary school curriculum, no activity was found for this category.

Function 3: Explanation as an answer to a "why" question where the underlying assumption is that the explanation should rely on mathematical properties and generalizations (i.e. Why is this statement true/false? Explain.)

<u>Preschool curriculum</u>

Mathematical topic: Shapes

Suggested activity: The teacher will draw a picture for the child made up

<u>Elementary school curriculum</u>

Mathematical topic: Operations with natural numbers

Suggested activity: (a) For each of the

of different shapes in various sizes and orientation. The teacher then asks the child to color, for example, all of the triangles and asks the child to **explain** why he colored or not colored a certain shape.

numbers below, try to write an addition sentence using two consecutive numbers.

(b) Which (kinds of) numbers could be the sum of two consecutive numbers? **Explain**.

As opposed to the first category of explanations, this category focuses on conceptual, rather than procedural, knowledge. In the preschool activity, when asking the child to explain why he did or did not color a certain shape, we are essentially asking the child to explain why that shape is or is not a triangle. This type of explanation allows the teacher to evaluate the child's conceptual knowledge of triangles as well as their preconceptions of triangles (Vighi, 2003). The elementary school activity begins with specific arithmetic examples but then asks a general question. By requesting the child to explain her answer to this general question, we are encouraging her to think about the properties of natural numbers, more specifically, the properties of even and odd numbers. In this case, the explanation allows the teacher to evaluate children's conceptual knowledge of even and odd number and of consecutive natural numbers. In addition, the one instance when the term ‘nimuk’ was used in an elementary school activity, it was used in this sense.

Function 4: Explanations as a step in directing new explorations leading to generalizations (i.e. Find all possible solutions and explain)

<u>Elementary school curriculum</u>

Mathematical topic: Geometry

Suggested activity: Cut a rectangle along a straight line generating two polygons. Which (kinds of) polygons can be the result of this action? Can you get two squares? A triangle and a pentagon? **Explain**.

We found it quite difficult to interpret the term ‘explain’ in the above activity. It was clear that the underlying purpose of the request was to have the child verbalize his thoughts. This, of course, may be said of each of the examples given above. But, what might we learn from the child's explanation in this case? In order to gain a better understanding of the activity, we note that it was labelled by the curriculum as an inquiry-based activity. Children are requested to investigate what might result from cutting the rectangle along a straight line. While two possible results are suggested, the underlying aim is for children to try cutting the rectangle in different ways. The function of the explanation here could be viewed as a combination of functions mentioned previously. On the one hand, children can explain the situation by saying what they did – I cut the rectangle this way and got two triangles and then I cut the rectangle this way and got two rectangles. On the other hand, an explanation might rely on mathematical properties, such as explaining under what conditions two squares will result from the cutting. In our opinion, the ultimate purpose of the explanation in this activity is to encourage children to think of additional possibilities. If children explain what or why they did some action, it might lead them to think of other possible ways to cut the rectangle, which may possibly lead to a general conclusion covering all possibilities. In the preschool school curriculum, no activity was found for this category.

To summarize, following the introductions, there were 10 instances of explanation-related terms in the preschool examples of activities and 9 instances in the elementary curriculum. Table 2 summarizes the number of instances (%) related to each function.

Function	Preschool	1st and 2nd grade
1 – Explanation as a description of one's thinking process or way of solving a problem	5 (50)	3 (33)
2 – Explanation as justifying the reasonableness or plausibility of a strategy or solution	2 (20)	-
3 – Explanation as an answer to a "why" question relying on mathematical properties and generalizations	3 (30)	5 (56)
4 – Explanations as a step in directing new explorations leading to generalizations	-	1 (11)

Table 2: Frequency of explanation functions at different ages

Note that the most frequent function of explanations in the preschool curriculum was simply to have children describe what they did. These explanations are not necessarily mathematically-based. This seems to be in line with the preschool curriculum introduction which clearly stated that engaging children in mathematical activities can also promote general thinking skills. Moving on to elementary school, the functions of explanations become more mathematical in nature and less descriptive.

DISCUSSION

To begin with, we see that having children offer explanations while engaging in mathematical activities is encouraged from an early age. It is not something left to the later years. We also see that, as mentioned by the NCTM (2000), explanations are part of both communication processes and reasoning processes we wish to promote. We do see, however, a subtle shift from the preschool to the elementary school. In the preschool, it seems that more emphasis is placed on the communication aspect of giving explanations and less on the reasoning aspect while the opposite seems to be true in the first and second grade curriculum. During the early elementary years, it seems that explanations become more mathematical in nature, relying on mathematical properties and supporting mathematical explorations.

Looking back at the specific functions of explanations cited in this paper, we see much in common with the literature background. For example, both Yackel (2001) and Krummheuer (2000) claim that explanations are often given in a narrative format, conveying what was done in order to solve a problem. This is, in essence, the first function mentioned above. However, an explanation may also clarify the rationality of an action (Krummheuer, 2000), which is the basis for the second function mentioned above. The rationality of an action may or may not be based on mathematical properties. Thus, the third function of an explanation might be to specifically ground an activity in mathematics. Finally, in line with Nunokawa (2010), the fourth function of an explanation could be to lead students to new understandings and knowledge. However, despite the relationship between explanations and proofs found in the literature (e.g. Hanna, 2000), at this age there is no mention of proofs in either document and the term justification is hardly used. The absence of these terms is notable when considering studies which have shown that young children are capable of proving or refuting conjectures raised by themselves and others (e.g., Stylianides, & Ball, 2008).

We do not believe that our way of categorizing the above examples is the only way. In fact, we are in the middle of an international comparative study investigating the uses of explanation-related terms in mathematics curricula in four different countries. Initial results indicate that explanations may also serve other purposes such interpreting day-to-day occurrences in a mathematical way (e.g., explain what it means when the carton of milk says it contains 3% fat) and clarifying personal view points (e.g., explain why statistics is important). We also do not believe that each of the examples we presented in this paper necessarily falls into exactly one category and not another. Much is dependent on the classroom context. For example, the elementary activity presented under the first function could lead to explanations of the third type based on children's knowledge of numbers. This might be encouraged if the teacher were to ask for a general statement concerning the combination of prices leading to 15 NIS, perhaps an explanation that if one toy costs more than 7.5 NIS, then the second toy must cost less. Likewise, the preschool activity presented for the second function of an explanation may lead to additional exploration and comparison of lengths and measurement, thus qualifying it for the fourth function of an explanation.

Taking into consideration that our interpretations are just that – our interpretations – one might ask, why bother analysing how explanation-related terms are used in curriculum documents. To begin with, we wanted to raise the issue that the different explanation-related terms are open to interpretation and that even among mathematics education researchers, the same word may be used but with different meanings. In addition, as mentioned in the introduction, these documents are used by teachers and others when planning lessons and activities. Our analysis can be used as a preliminary investigation into how the curriculum may be interpreted by others. Our analysis may also serve as a guide to others in understanding how explanations can serve different purposes. Finally, we hope that our study will lead others to investigate the use of the term 'explanation' in additional contexts, such as text books and curricula materials and perhaps become more sensitive to the different purposes and functions that explanations, and perhaps justifications and proofs, may have when teaching mathematics at all ages.

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