REASONING IN PRIMARY MATHEMATICS – AN ICT-SUPPORTED ENVIRONMENT

Angela Bezold & Silke Ladel

University of Wuerzburg & University of Saarland

The public view on mathematics is often reduced to and seen as 'only calculating'. General mathematical competencies like exploration or discovery are hardly considered. One reason therefore is a lack of didactical concepts of how to promote reasoning skills together with an instrument to assess these competencies. Another reason is, that some children rarely get the chance to act in an explorative way because they have difficulties in calculating. The aim of the project KLIC *(Kinder lernen in computergestützten Lernumgebungen) is to support the development of general mathematical competencies, in particular exploration and discovery, of all children. We currently develop and analyze self-differentiating tasks regarding their suitability to support argumentation and we analyze in what way and with which specific aims ICT can support the mathematical learning processes.*

Keywords: digital tools, primary school, relational thinking

INTRODUCTION

We understand mathematics as a *"vital science of dynamic patterns which can be developed globally in the curriculum as well as explored, continued, re-shaped, and invented locally by the learners themselves"* (Wittmann, 2011, p. 1). Mathematical competencies do not only include mathematical contents like subtraction or addition but also general mathematical competencies such as exploration or reasoning. In this regard children should be enabled to explore patterns and structures, to discover mathematical relations or to scrutinize mathematical phenomena. In an empirical study about the development of reasoning skills in primary school (cf. Bezold, 2009) we were able to determine a didactical concept to promote these skills and support the teacher in their daily work. In spite of the positive outcome the concept needs to be modified with regard to low achieving students. In the project KLIC we want to extend this concept and to explore in what ways ICT can support the development of childrens reasoning skills. First experiments (cf. Ladel, 2012) showed the high potential of an ICT-supported environment that has to be explored and examined more deeply.

Our research questions are:

- What kind of tasks are suitable to support exploration and discovery of *all* children?
- How can ICT support the children to explore and to discover mathematical relations?

REASONING IN PRIMARY SCHOOL

„From children´s earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons." (NCTM, 2000, p. 56) According to the NCTM (The National Council of Teachers of Mathematics) standards, reasoning and proof is distinguished as an essential part of teaching mathematics. Furthermore, even the current research reflects a strong interest in reasoning (cf. Schwarzkopf, 2000; Reiss, 2002; Krummheuer, 2003 and 2006; Meyer, 2007; Bezold, 2009; Fetzer, 2011). In the German educational standards, which show noticeable parallels to the NCTM standards, reasoning means questioning mathematical statements and proofing their correctness, recognizing mathematical relations and developing assumptions, and searching and understanding reasoning (KMK, 2005). Following the educational standards, we already consider the expression of presumptions of (relevant) mathematical features as an argumentative activity. These presumptions are for examples based on discoveries or individual examples that have to be questioned afterwards. Questioning implies on the one hand that the necessity of reasoning is conceived ("Why is it true?") and on the other hand that mathematical statements are deliberately considered as a presumption ("Is it really true?"). Processes of reasoning finally lead to finding a mathematical truth, i.e. to accept or reject the presumption. These considerations clarify that reasoning can only be seen as an argumentative activity (cf. KMK, 2005). Therefore, mathematical reasoning in primary school is particularly characterized by three components (Fig. 1). KLIC concentrates on the improvement as well as the analysis of the first and the third component.

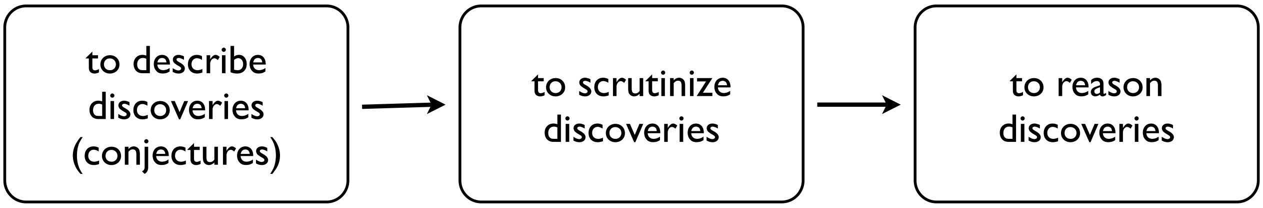

Figure 1: Three components of mathematical reasoning

'EXPLORATIVE TASKS' IN SELF-DIFFERENTIATING LEARNING ENVIRONMENTS

The question arises which types of exercises support all children with regard to exploration and discovery. Explorative tasks that are arranged close to heterogeneous or substantial learning environments (cf. Hengartner, 2006; Wittmann, 2002) are not only designed to provide all children access to the topic, but also to offer a challenge for highly skilled pupils (cf. Hengartner, 2006). We define explorative tasks as follows:

Explorative tasks …

- … give various opportunities to discover mathematical phenomena.
- … set requirements for different levels (self-differentiating).
- … manifest potential for reasoning and proof.

(cf. Bezold, 2009; Verboom & Nührenbörger, 2005)

Studies on primary math education that focused on written reasoning proved that self-differentiating learning environments initiated argumentative processes with different levels of performance to the vast majority of children (cf. Bezold, 2009). About one third of the children were able to reason their discovery. The proportion of reasoned findings increased with the complexity of the described numerical relations. However there were few children who couldn't develop their reasoning skills resulting from a lack of calculating abilities that prohibited the discovery of mathematical relations. Giving these children the opportunity to improve their reasoning skills with the help of ICT is one of the aims of KLIC.

THE ROLE OF ICT

In her article about "A Secondary Analysis from a Cognitive Load Perspective to Understand Why an ICT-based Assessment Environment Helps Special Education Students to Solve Mathematical Problems", Van den Heuvel-Panhuizen and Peltenburg (2011) point out three important aspects regarding the role of ICT to exploit the dynamic approach to assessment. In the project KLIC we refer to the aspect that ICT can reduce cognitive demand and hence enable low achieving students to *"more fully demonstrate their understanding of the mathematical concepts they have learned"* (Van den Heuvel-Panhuizen and Peltenburg, 2011, p. 24; cf. Bottge, Rueda, Kwon, Grand, and LaRoque, 2009; Elbaum, 2007).

The exploration and discovery of mathematical relations is often combined with calculation abilities. Hence, many tasks are worded like *„Calculate and …"*. This combination leads to the fact, that general mathematical competencies can only be acquired in the sense of a temporal differentiation. In this way calculation abilities state an essential requirement to get able to reason (cf. Grassmann et al., 2010). Krauthausen and Lorenz (2011, p. 171) calls this a time-killing and exhausting matter. Low achieving students who didn't yet automatize those abilities cannot develop and demonstrate their abilities to explore and to discover. In consequence, the view on mathematics for those children will persist in 'only calculating'. If the aim of a task isn't primarily to enhance calculation abilities but to develop general mathematical competencies, the calculation states only a secondary kind of activity. To delegate the calculation to ICT is one way to help low achieving students and to give them a chance to develop reasoning skills[1] (cf. Krauthausen and Lorenz, 2011; Ladel, 2012). In the following we focus this delegation in consideration of *computational offloading* which Rogers (2004) defines as *"the extent to which external representations can reduce the amount of cognitive effort required to solve a problem."* The calculation skills are offloaded and hence the children are enabled to focus on the mathematical relations and thus can develop their reasoning skills. Actually, this use of ICT does not only give access to what the children are actual able to do, but also to what they are able to do with some help in the sense of Vygotskys *zone of proximal development* (cf. Vygotsky, 1978).

THE THREE-STAGE MODEL: TO EXPLORE – TO DISCOVER – TO INVENT

In our considerations we focus the question how to support the three components of argumentation - to describe, to scrutinize, to reason discoveries (Fig. 1). Therefor we designed the three-stage model (Fig. 2). Firstly the pupils are engaged with the task on their own and have the possibility to explore and express first assumptions. Subsequently they exchange ideas on their discoveries and deal with further explorative tasks in a so-called group phase. The three components of argumentation are located in this stage. During the stage of invention the children produce variations of the task. This three-stage model serves as a teaching model for the explorative tasks. It enables the children to deal with the tasks in a self-acting way on their individual level but also allows cooperative learning. The pupils work on the explorative tasks in three stages that pass into one another and that don't necessarily have to be linear.

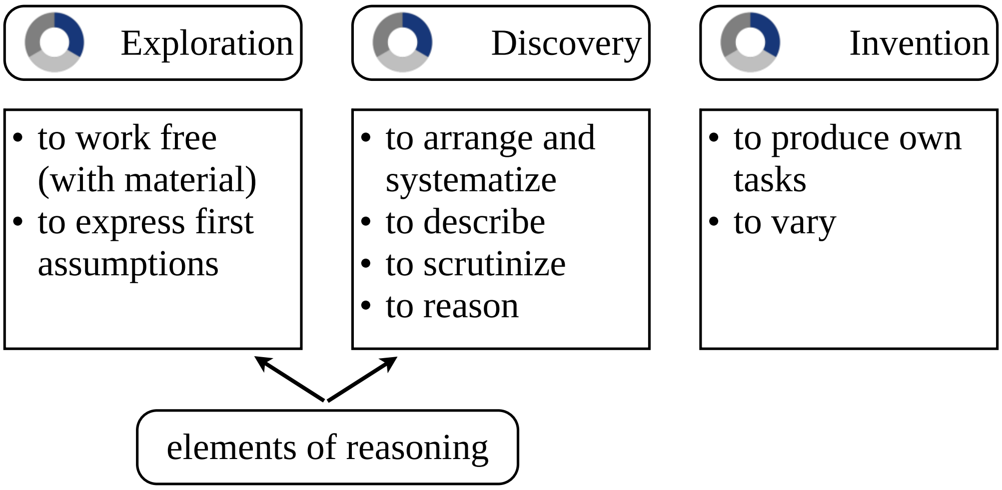

Figure 2: Elements of reasoning

Observations showed that the first assumptions of pupils are mostly based on intuition and creativity and that they are related only on few examples. During the stage of discovery the children deepen their assumptions. The children describe, scrutinize and reason their discoveries; the arrangement and systematizing support these activities. In the last stage the children apply their findings and transfer them to other tasks.

EXAMPLE: THE NUMBER ANGLE

In order to exemplify the activities of the three stages, we take a look at the explorative task *number angle* (Fig. 3)**.**

> *A task to solve* ***alone****: Fill in the numbers from 1 to 9 so that each arm add up to the same sum. Do you find different ways to do so? Did you find a strategy how to fill in the numbers?*

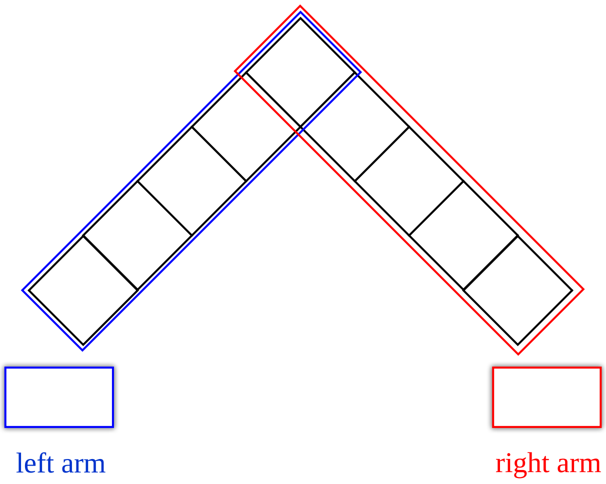

Figure 3: The number angle

During the stage of exploration the child has to conjecture (spontaneously) about strategies, possible solutions or special terms of the number in the top. The stage of discovery requires activities like describing, questioning and reasoning. These activities are initiated through the following explorative task that has to be worked on in cooperation (Fig. 4):

> *A task to solve* ***together****: Sort your number angles! Compare the sum of each arm and the numbers in the top. What do you notice?*

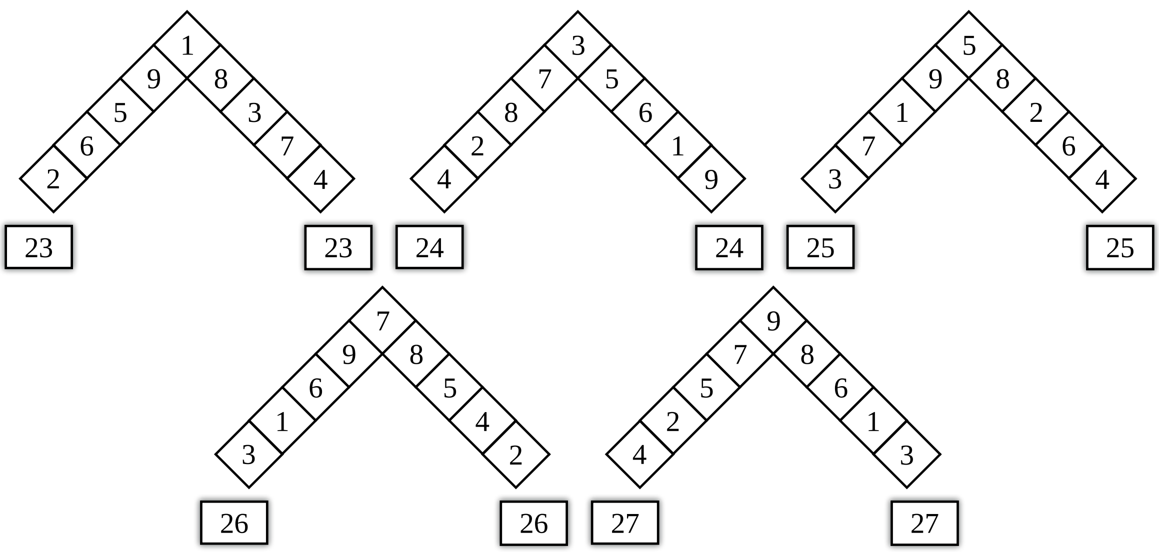

Figure 4: number angles to explore together

In case of children not discovering any mathematical relation explorer hints should be applied. Explorer hints are either orally or written given hints which are offered individually by the teacher if necessary. The following explorer hint draws the childrens attention to the fact that the top of the number angle only contains odd numbers and provides ideas for the reasoning of this phenomenon.

> *Explorer hint: Sum up the numbers from 1 to 9: 1+2+3+4+5+6+7+8+9=*
>
> *Distribute this sum on both arms of the number angle equally. Chose a number for the top and subtract this from the sum! Can you distribute the remaining amount equally on both arms?*

Finally, during the so-called stage of invention the creative side of math should be emphasized. Obtained results should be deepened and developed. Therefore, the children are either encouraged to invent their own number angle with altered numbers or to invent new rules for other children.

The ICT-supported environment is realized with the CABRI ELEM CREATOR. We created an activity book for the number angle in which children can work in different number ranges so that it can be used already from pupils at the age of 7. In the following example ICT is supposed to support the childrens activities in two ways:

- The calculating is delegated to ICT. In this way all children should be enabled to explore and to discover mathematical relations even if their calculating abilities aren't well developed.
- Children with difficulties in reading have the opportunity to get the task read. This also reduces their amount of cognitive load and enables them to concentrate on exploration and discovery.

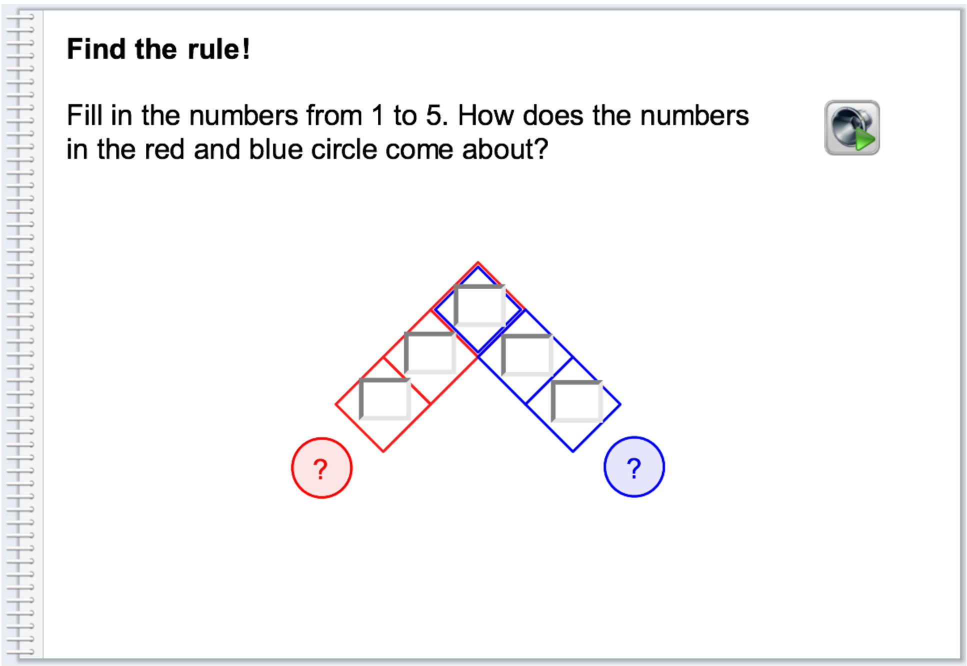

Figure 5: Screenshot

Another aspect we have to consider is that ICT-supported environments open up new kinds of tasks. This is because the development of the used tools (must) also leads to a development of the conceptual formulation. One example therefor is *“Find the rule!”* (Fig. 5). The children may fill the numbers in the empty fields in different ways and explore how the numbers in the circles change. Thus they can come from one special example to the general and are thus enabled to explore the rule.

FORECAST

We are currently working out explorative tasks and design the paper-and-pencil environment as well as the ICT-supported environment. There already exist positive results from each kind of environment itself. We want to bring both concepts together and explore the combination of both, especially the use of the ICT-supported environment more deeply. Finally, we aim to answer the research questions about the impact of the explorative tasks and ICT for children with different levels of performance in small groups.

NOTES

1. E.g. another way would be to give the children a row of already calculated tasks and ask them to explore mathematical relations. But this way constitutes only a static view and doesn’t support a dynamic approach (cf. Ladel, 2012).

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