THE SECOND DISCERNMENT INTO THE INTERACTIONAL NICHE IN THE DEVELOPMENT OF MATHEMATICAL THINKING (NMT) IN THE FAMILIAL CONTEXT

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As a reflection of mathematics education on developmental psychology and cognitive development, Götz Krummheuer created the concept of the "interactional niche in the development of mathematical thinking (NMT)" as a new theoretical framework in the mathematics education. This theoretical framework has been adapted as a developmental niche in the familial context (NMT-Family). Through an empirical study in familial play situations more details of the NMT are investigated. In this paper it will be stepped into the second discernment into the NMT-Family through the study erStMaL-FaSt.

Keywords: play, family, support job, Mathematics Acquisition Support System (MASS), geometric shapes.

1. INTRODUCTION

The IDeA (Center for Research on Individual Development and Adaptive Education of Children at Risk) is a research centre, which investigates extensively the development of children at risk and the processes of individual learning. This research centre is constituted by the German Institute for International Educational Research (DIPF) and Goethe Universität Frankfurt.The financial support provided by the Ministry of Higher Education, Research and the Arts from the state of Hessen. [1]

One of research project of IDeA Center is a Project erStMaL (early Steps in Mathematics Learning), which investigates the mathematical development of children with regard to their migration background. It is designed as a longitudinal study to follow children from the age of three, until the third year of primary school from a socio-constructivist perspective. While the first survey period contains only kindergarten children, the second survey period contains the same children in primary school ages (see also Acar Bayraktar et al. 2011).

In the scope of the project erStMaL, a family study is performed, which is designed as a longitudinal study and named as erStMaL- FaSt (early Steps in Mathematics Learning-Family Study). The study deals with the impact of the familial socialization on the mathematics learning and due to the following three criteria, 8 participants are chosen from the project erStMaL. The criteria are the ethnic background (German or German/Turkish), the duration of the formal education of the parents and the sibling situation within the families (see Acar Bayraktar and Krummheuer 2011, Acar Bayraktar 2012). Data collection comprises of recorded videos and their transcripts. Once in a year, an appointment is arranged with each family. This leads step by step

to a collection of data from each child. In each appointment the erStMaL child is video-recorded together with members of the family while they are playing in different settings.

For the family study two mathematical domains are chosen: Geometry and Measurement. Four play situations are conceived, due to these two mathematical domains. The members of the family are supposed to choose at least 2 games out of 4 and to perform them. For participation of all families, instruction manuals of each play are made both in German and Turkish, which can be spoken freely by families during play situations. The game materials are provided and put at the disposal of the family in the recording room. Currently, the new play situations are set up for the third observation phase in September.

In this paper it will be stepped into the second discernment into the NMT-Family.

2. THEORETICAL BASIS

> "The play, for the child and for the adult alike, is a way of using mind, or better yet, an attitude toward the use of mind"(Bruner 1983, p. 69).

erStMaL-FaSt enables families freely to play with their children. During each play situation with maintenance of father/mother/sibling the child explores something about the issued mathematical domain. This accompaniment of family provides to the child some "learning offerings" and interactive negotiation about the mathematical play. During the interaction of such various mathematical learning situations, there occur different emerging forms of participation and support.

For the comparison among the various mathematical learning situations and for the longitudinal analyses, the concept of the "interactional niche in the development of mathematical thinking" (NMT) will be used, which has been constituted by Krummheuer (2011a, 2011b). He explains NMT as in follow:

> The concept of the "interactional niche in the development of mathematical thinking" (NMT), consists of the provided "learning offerings" of a group or society, which are specific to their culture and will be categorized as aspects of "allocation", and of the situationally emerging performance occurring in the process of meaning negotiation, which will be subsumed under the aspect of the "situation" (Krummheuer 2012).

NMT- Family is a *subconcept* of NMT and offers the advantage of more close analyzes and comparisons between familial mathematical learning occasions in early childhood and primary school ages.

In view of the design of FaSt, three components of NMT-Family are shown and then their details given below:

NMT-Family	**component: content**	**component: cooperation**	**component: pedagogy and education**

aspect of allocation	mathematical domains: Geometry and Measurement	Play as a familial arrangements for cooperation	developmental theories od mathematics education and proposals of activeness for parents on this theoretical basis
aspect of situation	interactive negotiation of the rules of play and the content	leeway of participation	folk theories of mathematics education, everyday routines in mathematics education; MLSS

Content: In the practice of erStMaL-FaSt, children and their families are confronted with mathematical play situations, which are – as mentioned - either in mathematical domain "Geometry" or in mathematical domain "Measurement". The play situations in erStMaL-FaSt are designed to offer the families opportunities for interactive negotiations. From the situational perspective, in these play situations, processes of negotiation emerge, in which the rules of play and/or mathematical topics might be chosen as themes.

Cooperation: The process of cooperation between the adult and child provides the opportunity to refine their thinking and to make their performance more effective. Depending on this cooperation, a different leeway of participation comes forward. Krummheuer meant Leeway as a colloquial meaning of "room for freedom of action" (Krummheuer 2012).

"Leeway of participation" („Partizipationsspielraum", Brandt 2004) is one of the interactionistic approaches, by which a child explores his/her cultural environment while co-constructing it. "Leeway" is taken here in the colloquial meaning of "room for freedom of action". So, this is a concept belonging to the situational aspect. Brandt (2004) explains that the participants interactively accomplish different margins of leeways of participation that are conducive or restrictive to the mathematical development of a child. (see also Krummheuer 2011c; 2012). Alongside of contents, the children are involved in the social settings in the play situations, which are variously structured as in child-parents interaction and/or child-sibling interaction. These social settings need to be accomplished in the process of interaction.

Pedagogy and Education: Developmental theories and theories of mathematics education describe and delineate learning paths for the children's mathematical growth from which point of view. With the respect to the folk pedagogy, the participating adults and children become situationally active and operant in the concrete interaction. The cognitive development of each individual is constitutively bound to the participation of these individuals in a variety of social interactions. During these interactions and participations in the mathematical discourses, there occurs a "support system", which is proposed as a concept for the learning of mathematics and called "Mathematics Learning Support System" (MLSS) , with

respect of Bruner's concept of a Language Acquisition Support System (LASS) (Bruner 1986, p. 77; see also Acar Bayraktar and Krummheuer 2011, Krummheuer 2011b, Tiedemann 2010).

In the patterns and routines of interactions between child and families, MLSS occurs in different ways. In the mathematical competitions of play situations in FaSt, adults maintain the play, in which, possibly, emerges a support system. During these play situations they impart *their knowledge* by giving for example explanation to the statements during the negotiation of meaning. Not only through "the right given instructions" but also through "the wrong given instructions" by families, there occur some different types of support. Through the negotiation of the given instructions, children and parents lay out new interpretations, which support the development of the child either in a positive or a negative way (see Acar Bayraktar and Krummheuer 2011, Acar Bayraktar 2012).

With the respect to all these three components, it will be introduced one chosen scene as an example to show how the spatial abilities (spatial thinking) in the interactional niche in a familial context emerge.

3. A PLAY: BUILDING 02

The mathematical play "Building 02" refers to geometry and spatial thinking. The family is supposed to build three-dimensional version of the picture with wooden bricks, which all are in the same size and weight. Supposedly, they perform the relations between two- and three-dimensional representations. The player chooses one card from the deck and builds a wooden corpus from the image on the card. In the play, cards are placed on the table face down. Each card has a difficulty level ranging from 1 to 4. The cards with the number 1 are the easiest. The cards with the number 4 are the hardest, whose transitions between the various blocks are fluid and are purposed more complicated.

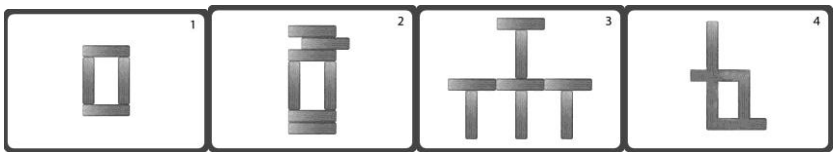


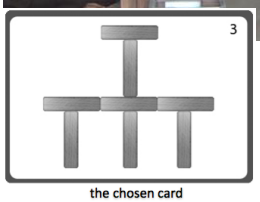
Fig 1. The game cards in different levels

An Example: Family Ak

The required information about Family Ak is in the following table:

Aleyna	6;9 years old	Single Child	German, Turkish rudimental
Father	Studied 15 years	Higher Education	German, Turkish rudimental
Mother	Studied 12 years	Higher Education	German, Turkish rudimental

Fig 2. Information about Family Ak



the chosen card

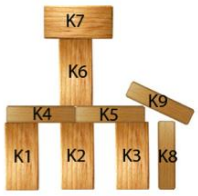
In the chosen game, Aleynas' game partner is only her father. Her mother is bystander and accompanies them behind the cameras by watching and making interpretations during their negotiations. In total, they play 5 rounds by turns. The chosen and transcribed scene is the first round, which begins with Aleyna. She chooses a card, which is shown on the left side. She builds the figure up while her father reads the instruction manual of play. Until she builds the figure up, in each step she asks her father and mother, if she does it right. They both don't give explanatory answers but the mother motivates her by saying "slowly" and "be concentrated". She accomplishes the corpus vertically on the table as shown on the right side. After she finished this construction, a conversation between father and daughter emerges, whose transcript is shown in the following.

the built figure

After her corpus is done, her father poses a question:

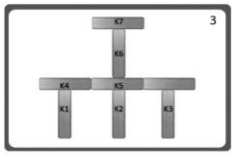
Transcript [2]

09	Father:	is it correct? No.
10	Mother:	just look accurately at it, Aleyna. there are two blocks, on it- or? there comes one more block up on it.
12	Father:	O.K. be quiet. don't interfere. *has a look at mother*
13	Aleyna:	*grimaces* ..nooooooooo! it is correct.
15	Mother:	just look
16 #	Father:	*shows with his right indexfinger on the card* just look. there are three parts. one, two, three.
18 #	Mother:	it is not true like that. honey?
19 <	Aleyna:	I've- *opens her mouth, looks grimly, handles K8 with her right hand*
21 <	Father:	yes. you've lost. *takes the card away* now it is father's turn-
23 >	Aleyna:	*takes the card furiously with her left hand from her father* nooo! *sets K8 on the Z Side near K3*
26 >	Father:	but it can't be played like that
27	Aleyna:	*puts K9 diagonally on K8 and K5*
28	Father:	no not like that.. not like that *smiles*
29	Aleyna:	*lays K9 under* ııııhhhhhh!
30	Father:	O.K. now it is dady's turn. *removes K6 and K7, puts them into the box*



the built figure

The father asks if aleyna built up the figure correct. He rephrases her question and answers, that the figure is not built correct. Hereby he expresses no further reason. By this he obviously deprives Aleyna of becoming informed about "the right figure" and also he limits her leeway of participation. Beside Aleyna's father, her mother tells her that she should look at the built corpus carefully. This

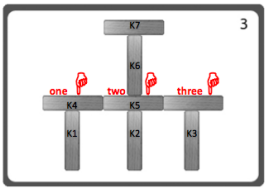


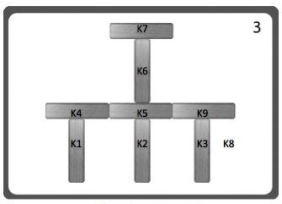
the chosen card

the built figure

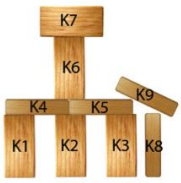
reaction can be interpreted, that she calls her attention to the block, which actually has to be laid next to the K5 on the right side on the card, in the built figure and hereby she gives her a chance to think about her oversight. Then she tells her daughter, that there are only two blocks on "it". Probably the pronoun "it" refers to the structure, which consists of K1, K2, K3. Hereby she might try to open a discussion that could foster her daughter to think about her oversight. Likely to make easier Aleyna's task, her mother gives her a clue, that how many blocks on which structure should be put. But then her father punctuates and warns her mother about, to be quiet and not to interfere herself to their play. After the both commentaries made by her parents, Aleyna gives a response, which is not clear enough to determine if she gives it to her mother's commentary or her father's. But as a consequence, she does not accept the critics and tells that the built corpus is correct.

After Aleyna's denial, her mother proposes her to look the built figure carefully. Then her father tries to call her attention to her oversight (?) on the card. By showing with his index finger, he counts the interjacent blocks on the card. This reaction of him could be interpreted as he is influenced by his wife's struggle to show Aleyna her oversight. Hence, he may try to show and make it clear how the structure actually has to be built. As a confirmation of the father's explanation, the mother tells her that the built figure is not correct without the third block, which has to be laid next to K5. The mother calls her, probably to get a response from Aleyna. Aleyna tends to respond her mother and probably tries to rectify her own oversight. But the father cut her off and says that she lost the turn. Hence, he takes the card away and tells that his turn begins. Presumably, he does not have enough perseverance to explain to Aleyna what her oversight is and how she can rectify it. Against her father's reaction, she takes the card furiously from him and puts K8 vertically near K3. Possibly she does not loose the game. It also could be that she very much involved in the correction of her initial conctruction and would like to continue regardless what game thy are playing. Then the father reacts as if she is in the contravention of the play rules by working over the built figure. Without saying anything, Aleyna goes on constructing the figure and puts K9 diagonally on K8 and K5 (see picture in below).





the chosen card



the built figure

As response, the father gainsays her action and tells her smilingly that the built corpus is not the same with the figure on the card. This could be as a interpreted as the smirk of the winner of the game or as a cordial attempt to mitigate the tension in the dispute. Aleyna lays K9 back on the table as if she abandons insisting, that the built figure is correct and she did not loose her turn. At the end of Aleyna's turn, the father says that his turn is up and removes blocks of the built corpus in the box. Obviously this is the end of Aleyna's turn.

4. CONCLUSION

As a summary, , a developmental niche for Aleyna emerges slightly in the chosen play situation, although there are an oversight of Aleyna and deficient information by father. While her father cuts out that she did wrong and lose the turn, her mother tries to give some hints and to call her attention to her oversight. With both emotional motivations, encouragement by her mother and declining reaction by the father, Aleyna realizes her oversight and tries to work on it again. In spite of her endeavour her father insists, that she doesn't build the corpus correctly and thus loses her turn. However this reaction of her father works on Aleyna and urges her to continue improving her construction. Actually and finally she does not come up with the correct solution but her activities give reason to assume that her competencies in spatial structuring and visual discrimination are enhanced.

She sets the different faces of blocks in the figure, and also she cannot coconstruct the built figure. According to the National Research Council Committee on Early Childhood (National Research Council 2009, pp. 186), the children at age 4 can identify the faces of 3-D shapes to 2-D shapes, can match faces of congruent 2-D shapes and can represent 2-D and 3-D relationships with objects. Furthermore at 5 years old they can build complex structures from pictured models. Considering these statements, it could be assumed that Aleyna's spatial thinking is not developed enough to think about parts and to relate them to the whole. But after constructing the figure with oversight, realizing and trying it to coconstruct, are the indicators to perceive Aleyna's developmental niche. Both together, encouragement by her mother and disapprovement by herfather, directs and provides her to see "mistakes" and to struggle getting at the "truth".

According to this analysis the three components of an interactional developmental niche in familial context can be structured as in follow:

Component "Content": Block Building provides a view of children's initial abilities to compose 3-D objects. In the chosen play, four goals are pursued: Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3-D shapes to 2-D shapes. By National Research Council is also reported that 5-years-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the relating parts and wholes level (National Research Council 2009, p.191). Aleyna realizes the spatial relations between 2-D and 3-D objects. She can relate some parts with the whole. So, as an allocation aspect occurs the spatial structuring and operating with shapes during the play. As a situational aspect, Aleyna negotiates with her father and mother. In this trial structure, there emerges a negotiation about the built figure. But neither her mother or her father assists her to explore how the figure actually has to be and what she overlooks on the built figure, there occurs a *consensus*, that Aleyna built the figure incorrectly.

Component "cooperation": The play situation is changeable, but directed by father. While he plays with his daughter, the mother does not refrain herself from interfering in their play and giving help to her daughter. Thus she plays such a role as a contributor in their play.

While her mother encourages Aleyna seeing her oversight, she also gives her a chance to an exploration and discussion on it. This means that she opens up a leeway of participation for Aleyna. While her father disapproves her, he also motivates her to struggle getting her due. So, the parents together in their seemingly uncoordinated moves open up a leeway of participation for Aleyna. On the other side, because the father always tries to end up her turn and brings no more discussion about the built figure, he limits the leeway of participation for Aleyna. Thus in all, father and mother together co-construct a leeway of participation in this play situation.

Component "pedagogy and education": The chosen play situation refers to the spatial structuring in geometry. In the chosen scene, by the organizing and setting objects, the graphical- and spatial-development of Aleyna are slightly assisted. While she is called attention to her oversight and is motivated co-constructing the built figure, she is not directly assisted, how this figure actually has to be. One can conjecture that the parents follow a rather constructivistic idea of helping and educating.

NMT **Family Ak** **Building 02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3-D shapes to 2-D shapes.	Playing with father, and Mother as a contributor	Theory of the development of spatial skills and spatial structuring
aspect of situation	negotiation between father, mother and Aleyna about the built figure; *consensus*	Different leeways of participation offered by the parents	Emotional motivation from parents provides Aleyna to explore her oversights and to endeavour coconstructing the built figure.

"The play under the control of the player gives to the child his first and most crucial opportunity to have the courage to think, to talk, and perhaps even to be himself " (Bruner 1983, p. 69).

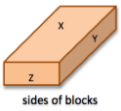
In the chosen example, with mother's encouragement and father's disapprovement, Aleyna realize her oversight and tries to resolve it. Although there occurs an "antagonisms" between Aleyna and her father, she can interpret her deficiency. This is an over careful learning progress, in which the interactional developmental niche slightly occurs for Aleyna. Clements and Samara reports that spatial processing in young children is not qualitatively different from that of older children or adults, but children with the age produce progressively more elaborate constructions (2007, p. 512). Hence, It will be really exciting to go on augmenting the examples of Aleyna, and to find out how NMT-Family functions work on Aleyna's spatial development in her familial context.

5. NOTES

1. http://www.idea-frankfurt.eu/homepage/about-idea
2. Rules of Transcription

Column 1	Serially numbered lines.
Column 2	Speech timing
Column 3	Abbreviations for the names of the interacting people.
Column 4	Verbal (regular font) and non-verbal (italic font) actions.
underlined	Speech is in Turkish
bold	Accentuated word.
<	Indicates where people are talking at the same time.
>	The next block of simultaneous speech is indicated by a change in arrow direction.
#	There is no break, the second speaker follows immediately from the first.

The sides of block are defined as X Side, Y Side, Z Side in transcript.



sides of blocks

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