"I DON'T WANT TO GET BURNED OUT": DESCRIBING ONE TEACHER'S FIRST EXPERIENCE WITH INCLUDING HISTORY

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In the following narrative we share the perspectives from our two different roles in thinking about history of mathematics in mathematics education. The first author's perspective provides insight into how a university professor thinks about her graduate students' learning with regard to incorporating history of mathematics in teaching. The second author's perspective presents the reality of the classroom teacher making a first attempt at both studying history of mathematics and incorporating history in teaching mathematics.

INTRODUCTION

In this paper we present simultaneous accounts from two different perspectives about promoting the inclusion of history of mathematics in teaching secondary school mathematics in the United States. The first author's account is from the perspective of university professor, teaching a graduate-level history of mathematics course for teachers and prospective teachers. The second author's account is from the perspective of a secondary mathematics teacher, who took the graduate-level history of mathematics course in summer 2012 and subsequently pursued ways to incorporate history of mathematics in her teaching.

Thus, this paper is less about presenting a formal research study (i.e., with the expected literature review, research question(s), methodology, data collection and analysis, and assertions and conclusions) and more about describing the experience of a classroom teacher's efforts to include history of mathematics in her teaching. In the field of history of mathematics in mathematics education there are several examples of theoretical and empirical contributions, discussing a variety of issues ranging from ideal ways to implement a history of mathematics course for prospective teachers (Clark, 2008; Charalambous, Panaoura, & Phillippou, 2010) to studies on students' experiences with learning mathematics using original source material (Jankvist, 2010). However, there remains a gap in the available literature that provides insight into how and why secondary mathematics teachers include history of mathematics in teaching (or, elementary teachers teaching mathematics, for that matter) [1].

Avital (1997) stated that "[m]athematics is by nature a cumulative subject; most of what was created millennia ago – both content and processes – is still valid today. Exposing students to some of this development has the potential to enliven the subject and to humanize it for them" (p. 3). Moreover, Avital claimed that, "[i]f we want to change the present situation we have to do it through the teachers" (p. 3). Consequently, when working with mathematics teachers with the goal of determining when, how, and why to incorporate history of mathematics in teaching, it is a worthwhile task to examine the different claims for each.

RELEVANT BACKGROUND

In summer 2012 five prospective mathematics teachers [2] and one classroom teacher completed a course on using history in teaching mathematics (or, "Using History" course). The course was taught as a hybrid, in which the course met one evening per week for six weeks and as an asynchronous online course for the remaining six weeks of a 12-week semester. The course was also designed to contain two parts. Each of the six graduate students possessed a different mathematics background and knowing this, the course was designed so that the first eight weeks focused on studying several mathematical topics from an historical perspective. Then, the remainder of the course was devoted to pedagogical concerns and the ways in which history of mathematics can or should be incorporated in teaching mathematics. Table 1 displays the schedule of topics of the course.

Week Number	Topics
1	Overview of research skills and the foundations of early mathematics
2	Content Focus: Cultural and historical development of mathematics; number systems; development of the complex number system
3	Content Focus: Solving polynomial equations
4	Content Focus: Solving polynomial equations, continued
5	Content Focus: Geometry
6	Content Focus: Geometry, continued
7	Content Focus: Advanced mathematics topics
8	Content Focus: Calculus
9	Pedagogical Focus: Using original sources
10	Pedagogical Focus: Using original sources, continued
11	Pedagogical Focus: Emphasizing historical problems
12	Pedagogical Focus: Cultural- and ethnomathematics

Table 1: "Using History in Teaching Mathematics" course schedule of topics.

The primary intent of the course was to provide future and current teachers with resources, strategies (e.g., aspects of the "hows" per Jankvist, 2009), and reasons for including history of mathematics as part of their practice in teaching mathematics (i.e., the "whys" per Jankvist, 2009). In particular, there were two key course

products that prompted students in the course to plan for the role of history in their future teaching. The first, weekly contributions to asynchronous discussion board prompts, allowed the students enrolled in the course to share ideas prompted by weekly assignments, which included mathematical tasks, original source readings, and articles from the field of history in mathematics education.

The second product was the capstone assignment for the course, for which students were required to create a historical lesson for a mathematical concept or topic that they would potentially teach in their future teaching assignment. For the historical lesson, students were directed to include historical information and historical problems or processes. The reason for the requirement to include both anecdotal and mathematical aspects in the lesson was to eliminate the possibility of creating a purely "history as interesting story" (Clark, 2011) lesson, that would most likely not have the potential to engage pupils in learning mathematical content.

THE NARRATIVE

Kathleen's Perspective.

During the course it was clear to me that Lisa was thinking about how history of mathematics could enhance her instruction in ways that the other students could not. This was, of course, primarily due to the fact that Lisa was the only classroom teacher taking the course. In addition to having taught for many years, Lisa was experiencing what many teachers often describe as "hitting a wall" – or becoming somewhat disenchanted with the way they have always taught particular topics or courses. As Lisa explained, she just "didn't want to get burned out".

In this section, I share two important aspects of my interactions with the students in the course. First, I present examples of Lisa's contributions to the online course discussion board. The sample posts are helpful in revealing Lisa's disposition towards to role of history in teaching. Secondly, I briefly describe the historical lessons that the students produced for their capstone assignment, with particular attention to Lisa's historical lesson.

Discussion Board Contributions.

During the seventh week of the course, students were asked to select one of the topics of focus [3] and to share on the discussion board why they would be most excited to teach the topic using history. Furthermore, students were asked to consider the explicit use of historical problems (or, applications). That is, the students were challenged to consider using history for more than just the exciting stories, anecdotes, or biographies. In response to the discussion board assignment, Lisa stated:

> Trigonometry is usually introduced after the lessons on right triangles. After working on the historical aspect of trigonometry, I think it would have more meaning if the lesson could be taught after the circle concepts. With the circle concepts (especially chord relations) fresh in the students' minds and providing real-world problems from the ancient world, I think trigonometry will make more sense to the students. They may

> actually have fun with the concept. I do need to do more investigating as to the types of astronomy-based problems to give the class to solve. I am looking forward to building on this lesson and to compare the type of learning from previous years of introducing trigonometry as just ratios in a right triangle. (Discussion Board, Week 7)

Later in the course, Lisa responded to a discussion board prompt that asked for students to express how some of the course readings helped them to think about the possibilities, benefits, and obstacles to using history in teaching:

> Avital's article contained many wonderful ideas that I have been looking for; that these ideas will help me use history in the classroom is a bonus. These days the focus [in teaching] is on reading and rightly so. We also need the same push to work on students' ability to think. If the answer does not come quickly, so many students shut down. I have even heard of instances where teachers become frustrated and make the work easier, but making it harder for the student to develop their thinking skills. In the article, Avital mentioned that we should educate students to conjecture "It cannot be done" and then try to prove this conjecture. The problems he presented as examples make this "doable". An example was asking eleven-year-olds to obtain the sum 45 by adding 8 numbers taken from the set 1, 3, 4, 7, and 11. (This is very doable and the students would love the challenge.) (Discussion Board, Week 9)

Finally, Lisa contemplated a specific way to incorporate history of mathematics in teaching (e.g., Jankvist's "hows") toward the end of the course. In her reflection about Jankvist and Kjeldsen's article in *Science & Education* (2011), she observed:

> What drew me to the article was [Jankvist's] teaching module. We have been talking about bringing history to the classroom as more than an additive. We need to give it substance and have meaning. Many times it has been suggested that if the students 'witnessed' the struggles of the men and women as they toiled through the beginnings of mathematical concepts, then the students would find more meaning in their learning of the same concepts. I have been wrestling with the idea of how to incorporate history of mathematics in the classroom in order to share these struggles with the students. I feel Jankvist's module is a good start. I like the questionnaire that was given to the students before the study. I always wonder what students really thought of math and their ideas of where the subject came from. How did math come about or did it just appear? Many of the historical developments seem to have had some controversy associated with the people involved. The discussions seem to bring the students into the controversy with the debates with one another. All students have an opinion and I feel even the quiet, struggling student will gain insight to the human side of the mathematics. (Discussion Board, Week 11)

Lisa shared a valuable perspective for research in the field of history of mathematics in mathematics education. In particular, Lisa sought out the "Using History" course because she was interested in finding and implementing new strategies to meet the needs of her students, as well as to prevent her own burn-out from teaching

mathematics as she always had. Secondly, she was able to read, analyze, and apply ideas that she read about during the course to her particular experience as a veteran classroom teacher. Finally, Lisa used her new learning about history of mathematics, different approaches for incorporating history in teaching mathematics, and reasons why history may contribute to student learning to create her historical lesson in the course.

Historical Lessons.

The six "Using History" students produced an extensive historical lesson as the capstone assignment for the course. A brief description of the lessons is provided in Table 2.

Student	Lesson Topic	Grade Level	Details
Lisa	Pythagorean Theorem	High School	4 activities; different cultures included; base-60 calculations; proofs
Student 2	Probability	Middle School	1 one-hour lesson; original readings; questions on the readings; mathematical problems
Student 3	Multiplication Strategies	Elementary	3 multiplication methods from three cultures; open-ended questions; mathematical problems
Student 4	Volume of a Sphere	High School	3 methods from three cultures
Student 5	Method of False Position	Middle School	Primarily a description of Fibonacci's application of the method; only two historical problems (out of seven questions)
Student 6	Conic Sections	High School	No history included

Table 2: Brief description of historical lessons.

Four of the six "Using History" students designed lessons that met the required elements of the capstone assignment (Lisa and Students 2, 3, and 4). It was apparent from Lisa's historical lesson that she was an experienced teacher; her lesson was organized with four delineated activities. Each activity represented part of an overall progression to engage pupils with the mathematics of several ancient cultures, ranging from different numeration systems to an example of measurement (e.g., the Pythagorean Theorem). The other students' lessons differed from Lisa's in this key regard. In particular, the other historical lessons contained one main activity, with

several aspects (cultural background, historical readings, mathematical tasks) included in the activity.

Lisa's Perspective.

In this section Lisa shares her perspective, both as a classroom teacher and as a graduate student enrolled in the course on using history in teaching mathematics. In the following, then, the first person "I" is used by Lisa.

I have been teaching mathematics at the secondary level for the past 18 years. I enjoy mathematics and would always look for real world applications in the form of activities to make mathematics exciting for the students. In my reflections over the past few years, it seems as though the students are becoming more disconnected from mathematics. In this "quick fix" world, students are looking for formulas to solve all the problems. Consequently, I wanted to find something that would broaden the students' understanding of mathematics as well as give them a whole new perspective of the subject.

As part of my search to find content and strategies to include in my teaching, I met with Kathleen to discuss course offerings at the Florida State University. She told me about her "Using History" class and I immediately thought this could be the answer to my dilemma. I always believed my understanding of mathematical concepts would be enhanced if I knew about the origins of the concepts. And, more importantly, I hoped that it would do the same for my students.

The course proved to be fascinating. The courses I teach range from middle school mathematics to honors-level geometry. As I studied the required course readings, my thoughts were always focused on how I could use the material in the classroom. I found many ideas in the course text (*Math Through the Ages: A Gentle History for Teachers and Others*), as well as the assigned journal articles. For example, Avital (1997) suggested introducing open problems to develop critical thinking and problem solving. Meavilla and Flores (2007) presented an interesting activity of analyzing problems as they were given in their original language. One of my concerns has been students' poor problem-solving skills. Thus, the ideas suggested by Avital and Meavilla and Flores caught my attention as a potential strategy to help students develop this important life skill.

I also found the lives of mathematicians fascinating and feel students would enjoy learning the stories behind the individuals responsible for mathematical ideas. Additionally, I feel that my students could possibly relate to the struggles these great thinkers faced. Many of these mathematicians' lives contained conflict. It would be interesting to hear my students' opinions of what particular mathematicians dealt with in their lives. My thoughts during the "Using History" course were to make plans for when to introduce these interesting activities throughout the year. During the summer, I had other (though unexpected) encounters with the history of mathematics. At one conference I attended, one of the lesson plans presented involved Eratosthenes' measurement of the Earth. A brief history was given and an outdoor

activity for participants followed. This lesson provided proof that introducing history into lesson plans does not take the huge commitment of time that many teachers fear.

Since completing the "Using History" course, I must admit that I have been slow to introduce history into the classroom. My lessons for the academic year began with integers and measurement and this presented the perfect opportunity to introduce the many number systems of the ancient world. I began my 7^{th} grade classes with the different numeration systems of the ancient Egyptians, Babylonians, and Romans. I wanted to give students a brief history before expanding their knowledge of different number sets, including integers and rational numbers. I expanded on the Roman Numerals by mentioning that it is believed that it was motivated by the human hand. The numbers 1 through 4 were the fingers and the numeral for 5, "V", was the hand, which showed how the thumb formed a "V". In class, we discussed what it would have been like working with other groups from different parts of the world, with each having their own number system. The students were very interested and asked many questions about the origins of number systems and made observations about the different systems.

The greatest challenge in planning lessons informed by history of mathematics, however, deals with the resources available. Thus far, as I develop lessons and activities, I started small and as I continue to work on the plans, the ideas continue to evolve into many sub-phases. My slow start with introducing history as part of my lessons is due in large part to my lack of confidence with the many sources that available. For example, I read several interesting articles and books to learn about a number of mathematicians and found enough discrepancies to make me hesitant about the reliability of the information that I would use with students.

This, however, will not deter me from developing more lesson plans that draw upon the history of mathematics. For example, as a result of the class discussion on ancient number systems, I believe it will be fun for them to perform operations within each of the different number systems and I will design a lesson on this topic. In the lesson, I want students to compare the different number systems to each other, as well as to the way in which we perform operations in our base-ten numeration system. Although my efforts to include history in my lessons may appear to be a slow start, I do not plan on letting history go by the wayside.

Kathleen's Additional Response.

As a final orientation for the reader, the narrative returns to Kathleen's first person perspective.

Lisa's reflections brought three important issues to my attention. When Lisa first shared with me the lesson on Eratosthenes' measurement of the Earth that was presented at a professional in-service meeting for classroom teachers, I missed a valuable opportunity to share published examples of this activity with Lisa. Indeed, seeing a lesson plan that used history of mathematics – in the context of the in-service meeting – was an important cue for Lisa regarding the viability of incorporating

history in her mathematics teaching. However, taking the time to share further examples with Lisa, particularly from resources that she would have valued (especially given her desire for more scholarly resources resulting from her experience during the "Using History" course), would have contributed to the ongoing discussion in which we were (are) engaged. For example, sharing lessons on the topic found in the *Historical Modules for the Teaching and Learning of Mathematics* (Katz & Michalowicz, 2005) or the excerpt on the story of the ancient history of Earth measurement (Führer, 1991) would have, taken together, emphasized Lisa's idea that there are important stories to be shared with students and the mathematical content.

Reflecting on the story presented by Führer (1991) prompts another reaction to Lisa's perspective. As Lisa shared in her narrative – as well as many times during our meetings together – she felt overwhelmed by the number of resources that she perceived as providing different historical information. Führer stated the exact situation that Lisa found herself in: "[Most mathematics teachers] would be quite ready to include historical aspects in their lessons, provided someone would help them to overcome their feelings of precariousness concerning the uncertainty of history and the methodological problems of teaching it seriously" (p. 24). Though I tried to alleviate Lisa's "feelings of precariousness", her perceptions remained. Quite possibly, however, her strong feelings were tied to the need to please me, as her university course instructor. As a result, in my future work with Lisa I will be sure to emphasize Führer's exact sentiment that, "It cannot be the job of the mathematics teacher to idolize the standards of historical science. The teacher must not lie, but should free herself from the heavy burden of exactness" (p. 24).

Finally, Lisa's revelation that she discussed the origin of the Roman numeral "V" as it related to the angle formed by the thumb and index finger of a human hand reminded me that the work to incorporate history in teaching mathematics must – at least from its initial stages – be a true collaboration among classroom teachers who want to provide an historical perspective for their students and mentors (e.g., math historians, mathematics educators, history of mathematics course instructors). With respect to the example about the origin of the Roman numeral "V", Lisa and I could have worked together while she planned for including this story (and subsequent discussion with her students). Then, when confronted with a possibly too convenient explanation about the possible origin of "V", we could have searched sources together to investigate alternative explanations. For example, Menninger (1969) discussed the development of the Roman numerals I, V, and X from notches cut into tally marks: "At the very beginning, the Roman numerals observed the basic and simple laws of ordering and grouping, like the notches in tally sticks" (p. 241). Furthermore, V may have resulted from a half-symbol (the "V" formed from half of the "X" symbol). [4]

NEXT STEPS

I argue that the inclusion of history of mathematics in meaningful ways (i.e., addresses both content and affective dimensions for learning mathematics) may most successfully come to fruition in classrooms when sought after by teachers who are "looking for another approach" or are fearful of becoming "burned out". In our joint narrative Lisa and I described first, her initial journey to investigate how to incorporate history in her teaching – and her first steps to do so. Lisa's desire to seek formal instruction on how to do so marked the beginning of her journey. From my perspective – that of a mentor of sorts and fellow traveller – introducing teachers to the tools that they can use to plan for incorporating history in teaching mathematics is essential. In the case of Lisa, studying and working with a variety of resources presented tangible ways for her to begin to change her practice, but this also presented a challenge for her. Our continued collaboration will include discussing Lisa's lessons as she develops them, with attention to appropriate resources.

Our initial collaboration prompts important considerations for future research. Indeed, I have not had access to classroom teachers interested in using history of mathematics in their teaching in a long while. I have renewed interest to investigate the following with Lisa – and hopefully others:

(1) How do mathematics teachers select historical stories to use with their students?

(2) In what ways do teachers envision that historical stories will promote or support the learning of mathematics content?

(3) What are the most effective resources available for teachers to use to incorporate history of mathematics in teaching, in their current classroom context?

(4) What resources are needed to promote or support teachers' further use of history of mathematics in teaching, and how can academic historians of mathematics contribute to the construction of these resources?

NOTES

1. In this paper I use "elementary" to refer to pupils aged 5 to 11 years and "secondary" to refer to pupils aged 12 to 18 years.

2. These five students were enrolled in a master's (graduate) program designed to gain certification to teach secondary mathematics.

3. Topics for week 7 included early number theory, trigonometry, probability, the concept of infinity, or set theory.

4. The first author is grateful to Jan van Maanen for his suggestions on how to improve the original draft of this paper.

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