MODELLING IN FRENCH AND SPANISH SYLLABUS OF SECONDARY EDUCATION

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In this paper we present the exploratory phase of an ongoing comparative study on the teaching of modelling in France and Spain. The study aims to describe the place of modelling in the curricula. In this exploratory phase we will use some tools of the Anthropological Theory of the Didactic (ATD). We will discuss the rationale, theoretical framework and methods of this study. Then we will answer the question: is modelling designated in the curriculum as knowledge to be taught? We will try to give some conditions that could explain the place of modelling in syllabus. We will formulate other questions for the next step of this research.

RATIONALE, THEORETICAL FRAMEWORK AND METHOD

Blum and Ferri (2009) point out that "mathematical modelling (the process of translating between the real world and mathematics in both directions) is one of the topics in mathematics education that has been discussed and propagated most intensely during the last few decades. In classroom practice all over the world, however, modelling still has a far less prominent role than is desirable"(p.45). For us (as in (Cabassut, 2009) a modelling cycle is the process of solving a real world problem by translating it into a mathematical one (called model of the real world problem), then solving this mathematical problem before translating the mathematical solution back into solution of the real world problem, and validating this solution.

In the previous quotation of Blum and Ferri (and in the whole article) we note the gap between the importance of modelling in mathematics education research and the impact of modelling in classrooms. With this in mind, we compare here the role of modelling in French and Spanish curricula. To this end, the Theory of Didactic Transposition is extremely useful. In this theory "a distinction is established among: the 'original' or 'scholarly' mathematical knowledge as it is produced by mathematicians or other producers; the mathematical knowledge 'to be taught' as it is officially designed by curricula; the mathematical knowledge as it is actually taught by teachers in their classrooms and the mathematical knowledge as it is actually learnt by students" (Bosch & Gascón, 2006, p.55). In modelling, two kinds of knowledge are involved: extra-mathematical knowledge (for example, from everyday life or from sciences) from which the real world problem derives, and mathematical knowledge from which the mathematical problem derives.

We present the exploratory part of our on-going work. We concentrate initially on the question: is modelling designated in the curriculum as knowledge to be taught? This question will be answered in the second section. In further research we aim to see if modelling is taught knowledge, for this aim we will need to analyse textbooks, resources, interviews teachers, etc. Our study is limited to the secondary school

curriculum because a similar study about primary school was previously made between France and Germany by Cabassut and Wagner (2011).

In the third part of this paper we will try to justify the conclusion of the second part using the levels of determination proposed by ATD. Dorier (2010) gives a description of each level:

"In the description of these levels, a Subject is the lower level and is organised around one type of task and technique (like quadratic equations). A Theme is centred on one technology (like polynomial equations). A Sector is centred on a complex of praxeologies within a same theory (like polynomials). Of course different sectors are part of a same Domain, like algebra being a domain of which polynomial is a sector. The next level is the Discipline, here mathematics is the discipline of which the domain of algebra is part [...]. Therefore, after the discipline, Chevallard takes into account the level of the Pedagogy, i.e. the general teaching principles included in the description of the curriculum of an institution. Then the level of School, takes into account how the general curriculum is structured, the division into disciplines, the time allocated to each, the fact that teachers are monoor pluri-disciplinary, etc. The next level deals with Society, that is to say, the institutional organisation of the educational system in a country or a region, the most general level of the curriculum, etc. The highest level has to do with Civilisation, it takes into account variations between different cultures, like western versus eastern culture" (p.12).

Each level could help to explain the place of modelling in the syllabus.

To analyse the different curricula we refer to the official texts produced by the corresponding ministries of education of both countries and select those sentences where some words semantically related to modelling appear. From these collected citations we try to answer the question: is modelling designated as knowledge to be taught? In order to analyze levels of determination, we will use the same method but we will enlarge the reading to more general text, as European Parliament recommendations, PISA results, etc.

We have chosen a bi-national research team because "the comparative method seems to be a major tool in clinical questioning, making it possible to break with the apparent naturality of observations in each country, which encourages the constitution of multinational teams of research" (Cabassut, 2007 p.2431).

IS MODELLING DESIGNATED AS KNOWLEDGE TO BE TAUGHT?

The French secondary school

In France, secondary school lasts from grade 6 (11 to 12 years old) to grade 12 (17 to 18 years old) and, in contrast with primary school, mathematics teachers are subject specialists. Secondary school is organised in two parts. There is a common school from grade 6 to grade 9, *collège*, with the same curriculum throughout France. After this schools are differentiated (vocational, technical or general, *lycée*). We will consider the curriculum of both the *collège* and the *lycée*. The contents of the first year of *lycée* (corresponding to grade 10) are common for all students. In the final

two years (grades 11 and 12), students choose between literary, scientific or economic branches. Compulsory education ends at age 16, corresponding to end of grade 10 for pupils who have never repeated a school year. France is a centralised country and the same official texts edited by the Ministry of National Education (MEN) describe the curriculum and are applied everywhere. There are two kinds of official texts. The first describes the content to be taught for every year of the curriculum and is found in the newspaper of the Ministry of Education (Bulletin Officiel de l'Education Nationale [BOEN]). The second presents resources or advice produced by the MEN. For mathematics these texts are often produced under the responsibility of the body of General Inspectors of Mathematics (Inspection Générale de Mathématiques) that monitors mathematics teaching everywhere in France. The present curriculum was introduced in the *collège* from 2006 to 2009 and in the *lycée* from 2010 to 2012.

Curriculum of general education (from grade 6 to grade 10)

In France, the common base of knowledge and skills (BOEN, 2006) considers that

"the main elements of mathematics are acquired and exercised primarily by problem solving, especially from realistic situations. [...]. On leaving compulsory school, the student must be able to apply the principles and processes basic math in everyday life, in his private life as in his work. [...]. The student must be able [...] to model so elementary, to understand the link between natural phenomena and mathematical language which applies to it and helps to describe it." (p.6-9).

The introduction to the collège syllabus asserts that

"through problem solving, modelling of some situations and progressive learning of the demonstration, students learn little by little what a real mathematical activity is: to identify and to formulate a problem, to conjecture a result by experimenting on examples, to build an argumentation, to check the results by assessing their relevance for the studied problem, to communicate on a research, to give form to a solution" (BOEN, 2008, p.9).

For problem solving, the relations with everyday life or other subjects, and particularly sciences, are mentioned. After this introduction the mathematical content of the syllabus is described by mathematical domains: data organization and functions, numbers and computing, geometry, magnitudes and measures, and later analysis, statistic and probability, algorithmic, arithmetic...

For grade 10 the objective "is to train students in the scientific process in all its forms to enable them to model and to engage in research activities [...], to make a critical analysis of a result, of a process [...], to communicate in written and oral form" (BOEN, 2009, p.1). In the detail of the content of different mathematical domains we find references to modelling and models. For example, regarding problems related to first degree equations, the syllabus recommends that "each time the different stages of work have to be identified: setting equation, solving the equation and interpreting the results" (p.29), which is a reference to a kind of modelling cycle.

Scientific branch (grades 11 and 12)

This branch develops scientific education in grades 11 and 12. Moreover, in grade 12 pupils have to choose one speciality among earth and life sciences, physics and chemistry, or mathematics, with a supplementary syllabus. The introduction to the mathematics syllabus of the scientific branch specifies that "activities [...] should lead students to: search, experiment, model, [...] explain a process, communicate results in a write and oral form" (BOEN, 2010a, p.1). Different mathematical domains such as analysis, probability and geometry, refer to modelling and models (for example: "Diffusion model of Ehrenfest: N particles are distributed in two containers, and at each instant, a randomly selected particle exchange container" (BOEN, 2011a, p.18). We have also found nine explicit connections to the science syllabi. For example, in relation to sine and cosine was "progressive sinusoidal waves, mechanical oscillator" (p.6) or, in relation to probabilistic independence was "heredity, genetics, genetic risk" (p.12). We think that such relations with the science syllabi encourage students to understand how mathematics is used to model science. In the mathematics specialism syllabus of the final year, the study of the situations considered in the context of this course leads to a modelling work and places students in a position to undertake research.

Economic and literary branches (grades 11 and 12)

For students following the literary branch mathematics is optional, although, as with the scientific branch, those pupils who choose to apply mathematics have to be trained "to develop the following skills: to implement independent research; to conduct reasoning; to have a critical attitude towards their results; to communicate in writing and orally [...] to experiment and to model" (BOEN, 2010b, September 30, p.1). Different mathematical domains (algebra, analysis, statistics and probability, geometry) allude to modelling and models. In the economic branch curriculum, particularly in the last grade, we find, as expected, some explicit references to modelling. In this branch "teaching is based on problem solving. [...] The study of such situations leads to modelling work, and places students in a position to research". Examples of these problems are given, including "workflow, simple problems of graph partitioning under constraints: the traveling salesman problem management, road or air traffic, scheduling sports tournaments... modelling of inter-industry trade (Leontief matrices)" (BOEN, 2011b, p.10).

The Spanish secondary school

In order to study the teaching of mathematical modelling in secondary education in Spain, we focus on the official syllabus from the Ministry of Education. Spain is divided into 18 autonomous regions; each of these regions also has an official syllabus of secondary studies. Each regional syllabus must be framed within the national one, which is why, for this work, we centre our attention on the syllabus published by the Ministry of Education at Boletín Oficial del Estado (BOE).

The structure of secondary education in Spain is regulated by Education Law (BOE, 2006, p.17158-17207). It comprises two stages: compulsory secondary education

(from grade 7 to 10) and high school (grades 11 and 12) from which, after passing a test, students can access university. In contrast with primary education (grades 1 to 6), secondary teachers are subject specialists, with each subject taught by a different teacher. To analyse the content of every stage, we will focus on the study of the syllabi which regulate both the compulsory secondary (BOE, 2007, p.31680-31828) and the non-compulsory high school (BOE, 2008, p.27492-27608).

Curriculum of compulsory secondary education (grades 7 to 10)

All curricula refer to eight core competences or essential skills that students must reach across all subjects. These are communication skills, mathematical competence, competence in the knowledge and interaction with the physical world, information processing and digital competence, social and civic competence, cultural and artistic competence, learning to learn competence and autonomy and personal initiative).

In the introduction to the section on mathematics it is mentioned that, historically, mathematics "has been used by scientists of all times to build models of reality" (BOE, 2007, p. 31789). The objective is that students, at the end of compulsory secondary education, should "be able to use [mathematics] to think critically about the different realities and problems in today's world" (BOE, 2007, p. 31789). To achieve this objective, they recommend that the content is presented in a problemsolving context. Thus, problem solving stands as the cornerstone on which to work the mathematical content of the curriculum. In this introduction we find some explicit references to modelling, particularly in the domain of geometry "learning of geometry should provide continued opportunities for [...] modelling" (BOE, 2007, p. 31790), and also in the domain of relationships between variables from tables and graphs (analysis function) through which "students are intended to be able to distinguish the characteristics of certain types of functions in order to model real situations" (BOE, 2007, p. 31790). Finally, the syllabus offers methodological guidelines focused on mathematics as a discipline. For example, they recommend working on open situations as this allows students with higher levels of cognitive development to be able to "conceptualize progressively contents in order to ask questions about what is sought" (BOE, 2007, p. 31803), while these open situations can also serve to support and reinforce students with difficulties.

Spanish high school (grades 11 and 12, from 16 to 18 years old) comprises two years of study divided in three branches: literary, scientific and artistic. Mathematics is only compulsory for scientific students during their first year. Below we discuss aspects of the mathematics offered to students in the science and literary tracks; students in the artistic high school do not have to study mathematics.

In its introduction to the content for science-oriented students, the syllabus specifies that mathematics "gives rise to the necessity to solve practical problems [...] and [mathematics] are supported by their ability to [...] model real situations" (BOE, 2008, p.27574). Among the seven general objectives of mathematics for scientific students, no explicit reference to modelling appears. However, the need to "use scientific research strategies and skills specific to mathematics [...] for general research and explore new situations and phenomena" (BOE, 2008, p.27575) is stressed.

During the first year of high school, mathematics (Matemáticas I) is divided into four domains: algebra (and arithmetic), geometry, analysis and probability (and statistics). The only explicit reference to modelling appears in the domain of analysis, where pupils must be "able to model situations and phenomena with known graphics" (BOE 2008, p.27575). In its section on evaluation the syllabus asserts that pupils should be able to "solve problems drawn from social reality and nature involving the use of equations and inequalities, and must interpret the results" (BOE, 2008, p.27576).

During the second year of high school, we find no mention of inquiry based learning, problem solving or modelling in the mathematics content. However the evaluation criteria suggest that teachers should "intend that students manage information drawn from various sources and use available technologies [...] model situations, [...] extract information, make interpretations [...] and process mathematic data" (BOE, 2008, p.27577).

For students of the Literary branch mathematics is not compulsory, although in the syllabus for Mathematics Applied to Social Sciences it is emphasized that it should be worked from a practical point of view rather than from a mechanical point of view, going "beyond the mechanical resolution of exercises that requires only the immediate application of a formula" (BOE, 2008, p.27606). In order to understand the use of mathematics "activities arising should encourage the possibility of applying mathematical tools to analyse social phenomena particularly relevant, such as cultural diversity, health, consumption, coeducation, peaceful coexistence and respect for the environment" (BOE, 2008, p.27605). In the two years of high school, mathematics (as applied to social sciences) is subdivided into three domains: algebra, analysis and statistics (and probability). Regarding the domain of algebra, in the evaluation criteria we found that pupils must "use appropriate techniques to solve real problems giving an interpretation of the expected solutions" (BOE 2008, p.27606). In general, teachers of mathematics have to show students how to "deal with real life problems, organizing and codifying information, developing hypotheses, selecting strategies and using both the tools and modes of argumentation of mathematics to face new situations effectively" (BOE, 2008, p.27606).

Answer to the question and discussion

In the French secondary curriculum the reference to modelling is frequent throughout: it is always related to problem solving either inside pure mathematics or in relation to other subjects, particularly sciences and technology. This means that it is not clear if modelling is always referenced to real a world. For example, geometry, where optimisation problems are mentioned, can be considered as a pure mathematical world. That is, modelling in this context could be construed as operating within a mathematical world. We note clearly the reference to a kind of modelling cycle involving the three steps of setting an equation, solving the equation and interpreting the results (BOEN, 2008 p.14). All the mathematical domains are involved in modelling, and specially probability. The competences are mentioned often in a general context because most of these competences are related to general and transversal competences, like to be able to communicate, to be critical, to reason,

to argue... For all these reasons we can conclude that modelling is designated in the curriculum as knowledge to be taught, in all branches of general secondary education. The differences between branches are related to the mathematical level and the nature of the problems (scientific, economic or social) in which it occurs.

In the secondary Spanish curriculum, particularly in the mathematics syllabus of compulsory education we find some reference to modelling, most of which are in the domains of geometry and analysis. We think that the references to modelling that appear in the introduction to the scientific branch can increase inquiry based learning, however it is quite revealing that, in the final year before university we find no mention of modelling. We think the reason may be the following: at this level, the syllabus contains many new topics (matrix algebra, integration and limits) and preparation (often mechanical) for the final examinations for university applications prevents any kind of innovation in the classroom. In the syllabus of Mathematics applied to Social Sciences (literary branch) we find, as expected, several references to the relationship between mathematics and reality. So, we conclude that indeed modelling is knowledge to be taught. However, these official texts give no explicit guidance on how to work mathematics through modelling. They merely recommend that the subject should be worked through problem-based learning (problem must be taken from everyday life). Unlike France, in Spain any teaching resources are published by the Ministry of Education. Possibly further research may well lead us to resources published at the level of the regional government.

Discussion

In contrast with Germany, where modelling is one of the seven core competences of the secondary mathematics curriculum, in neither France nor Spain is modelling so explicitly defined. Official texts discuss modelling both explicitly and implicitly but it is not always clear if students are expected to apply a given model or construct a model in order to solve a problem. However in the French texts is mention of the part of the modelling cycle where the model is built. Indeed, there are several resources from the French Ministry in which can be found classroom tasks where models have to be built, like for example in probability (MEN, 2008). We have also remarked that it is difficult to compare the French and Spanish syllabi, since there are written in different contexts. For these reasons, we will study now why modelling is knowledge to be taught.

LEVELS OF DETERMINATION

At the level of civilisation and society, in France, the common base of knowledge and skills (BOEN, 2006) refers explicitly to PISA and to European parliament recommendations. In the text of the latest Spanish education law we find also explicit reference to both the Organization for Economic Co-operation and Development (OECD) and the European Union (BOE, 2006, p. 17160).

At the level of school we remark that in both France and Spain secondary mathematics teachers are subject specialists, which could make it more difficult for them to teach themes linked to other subjects.

In France, over the last few years, new curriculum structures have encouraged schools to integrate different subjects. For example, in grade 10, there is now an optional course on scientific methods and practices (BOEN, 2010c, p.1), which entails one and a half hour per week in students' time-tables. It allows them to explore different areas of mathematics, physics and chemistry, life sciences and earth and engineering sciences. Also, during grade 11, students of the scientific branch have to undertake a supervised project, called TPE (BOEN 2011c). Over eighteen weeks, small groups of students work collectively on a project, using various resources, on a subject chosen by them that connects two topics (as, for example: How can we use satellite images to refine forecasted monsoons? Modification of food is it progress?). TPE bring into play at least two disciplines, including one which is essential to the students' orientation. The realisation of the project is supervised by teachers of the relevant disciplines with two hours per week in the students' timetable. Assessment considers all aspects of the students' contributions, including written and oral presentations, and is part of the final mark for entering university. Clearly, modelling activities with an open building of the model are easier in this kind of structure (themes of convergence, exploration teaching, TPE) than in a one subject lesson. In the Spanish programme, such opportunities are not found. Some regional university institutes (called IREM) take charge of in-service training of mathematics teachers or offer resources, in relation to modelling. Starting in 2012, a network of science houses is developing in France in order to offer in-service training for the teachers and could offer training on modelling or on inquiry based approach. In Spain the problem is that the training of secondary school teachers in didactics is poor, as shown in the analyses of (Ferrando et al., 2012) and (García et al., 2007). In view of these observations, it is understandable that, still, the majority of Spanish secondary school teachers find working in accordance with the official syllabus guidelines difficult. To get a broader view of the resources available for teach modelling it would be desirable to extend our research to the study of textbooks. In any case it seems clear that, compared to France, Spain is far from incorporating modelling into its mathematic classrooms.

At the level of pedagogy, in France, the *college* syllabus proposes a common introduction for all scientific subjects and defines (BOEN, 2008, p.5) different *themes of convergence* to be worked together by different subjects and supports a common inquiry based approach that fits well with modelling activities. The Spanish education system insists in the connection between all subjects (all the competences have to be developed in all the disciplines). Obviously, modelling activities promote the acquisition of these eight competences. Moreover, the official text of the curriculum claims that "the teaching methodology must be communicative, active and participatory", fostering cooperative work and "highlighting the relationships between subjects and its relationship to reality" (BOE, 2007, p.31682). In the definition of the eight core competences, we realize that these make sense when teachers propose to work on real situations (near to daily reality of students). Indeed, we can conclude that both programmes are based in a pedagogy that could promote the use of modelling as a teaching tool.

At the level of the mathematical domains, in France, resources in numbers, geometry, magnitude, data-organisation and specially in statistics and probability, mention explicitly modelling and propose activities for the class. In Spain there are no national resources like in France. In particular, probability seems a domain underused for modelling in comparison with France.

At the level of mathematical themes and subjects, in France and also in Spain, it is difficult, from official texts, to find a clear link with modelling. We have to investigate resources like textbooks in a next study. Textbooks, in both France and Spain, are non-official resources designating the knowledge and skills to be taught and, when they are used by teachers and students, show what has been taught.

CONCLUSIONS AND OPEN QUESTIONS

The comparative method shows phenomena in one country (like for example official resources for the teachers in France) absent in the other. The method shows also that the same condition (for example modelling designated as knowledge to be taught) can produce different consequences because other levels of determination play different roles depending on the country. That is, the comparative method, in fact, helps us to understand more deeply the, not so obvious, conditions of one country in contrast with another. In a further study we will try to answer the following questions: Is modelling taught knowledge? Is it learned knowledge? What is the role of non-official resources - like textbooks - in these answers? What can explain the distance between knowledge to be taught, taught knowledge and learned knowledge for modelling?

NOTES

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