WAYS OF ANALOGICAL REASONING – THOUGHT PROCESSES IN AN EXAMPLE BASED LEARNING ENVIRONMENT

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This study is about analogical reasoning in problem solving situations. As one part of the study, task sequences for an example-based learning environment were developed with the aim of initiating processes of analogical reasoning. While solving these tasks the subjects were asked to verbalize their thoughts. Their problem approaches were recorded by video camera and transcribed to Think Aloud Protocols. Referring to a two-dimensional process-model of analogy the protocols were used to trace, visualize and quantify ways of analogical reasoning in order to become accessible for classification. It appears that different ways of analogical reasoning occur and there seem to be suitable attributes to describe them as different classes of paths in the two-dimensional model.

INTRODUCTION

> "ALL [sic] our reasonings concerning matter of fact are founded on a species of Analogy, which leads us to expect from any cause the same events, which we have observed to result from similar causes." (Hume, 1748, Sec. IX, Par. 82)

David Hume refers to the possibility of drawing on experience to extend the (collective) knowledge as one of the main characteristics of analogical reasoning. In relation to the individual and in the context of learning mathematics, Pólya (1949; 1961) sees analogical reasoning in the frame of an heuristic approach. He outlines clearly defined procedures which should help the learner to fall back on his own experience. Yet, students' transfer performances are not – in general – very successful. In order to make this strategy available to students in the process of teaching mathematics, it is important to identify mathematical contents and learning situations in which analogical reasoning strategies can be used in a profitable way. Hence, the significance of processes of analogical reasoning within the scope of mathematical activities has to be clarified. However, this can only be achieved by taking knowledge of cognitive psychology into consideration.

Thus, the starting point of the below described empirical study is the following question: How do students use analogical reasoning as a possibility of drawing on mathematical experience? In the present study, we refer to the usage of solved sample tasks while working on a new problem that has a similar mathematical structure.

TWO DIMENSIONS OF ANALOGICAL REASONING

The principle aim of analogical reasoning is to make the structure of an untapped issue (*target*) available to learners by comparison to structures within the learner's field of experience (*source*) (English, 1997, p. 5). The most simple form of analogical reasoning, often used in intelligence tests measuring general intelligence *g* by analogy formation skills (for example Culture Fair Test CFT, CFT20 Catell & Cattell, 1963, Weiß, 2006), is seen as the establishment of a relational identity like 'A is to B, as C to D'. The concept of this identity is based upon the comparison of *object attributes* and especially upon the comparison of *relations* between objects involved (Alexander, White & Daugherty, 1997, p. 1997f).

Nevertheless, within the scope of learning mathematics not only the necessary skills to solve tasks, as mentioned above, determinate the value of analogical reasoning. In fact, students are to be enabled to analyze and compare a given situation to something that is already known so that they assess possibilities of mathematical action with regard to common structures. They finally transfer mathematical options to the unfamiliar situation ('Do you know a related problem?', Pólya, 1949). In the latest cognition research, this operation is described by the concept of *structure mapping* (Gentner, 1983). In addition to finding equivalents on the levels of objects and relations (Schumann, 2006), mathematical operations also have to be analogized from the *base* to the *target* so that the task can be achieved by analogical reasoning (Ruppert, 2010).

Thus, solving a task by analogical reasoning means finding equivalents on the levels of objects and relations on the one hand and analogization of mathematical operations on the other hand. This is the *First Dimension* of analogical reasoning.

Regarding analogical reasoning as a cognitive process, the chronological sequence is of particular interest. In a variety of experiments, Sternberg (1977) was able to identify several components of analogical reasoning. According to Sternberg, the study at hand takes the following four components as a basis:

(1) Structuring (Sternberg: Encoding, Inferring)

(2) Mapping

(3) Applying

(4) Verifying (Sternberg: Justification, Response)

These components constitute the *Second Dimension* of analogical reasoning.

So the statements mentioned above suggest studying processes of analogical reasoning in the field of the two dimensions: 'Level of Analogical Reasoning' and 'Component of Analogical Reasoning'.

Therefore, the underlying working hypothesis is that processes of analogical reasoning could be illustrated as 'paths' in a two-dimension model (Ruppert, 2012).

According to the collected data, the evaluation of analogy based lines of argumentation leads to diagrams of the following type:

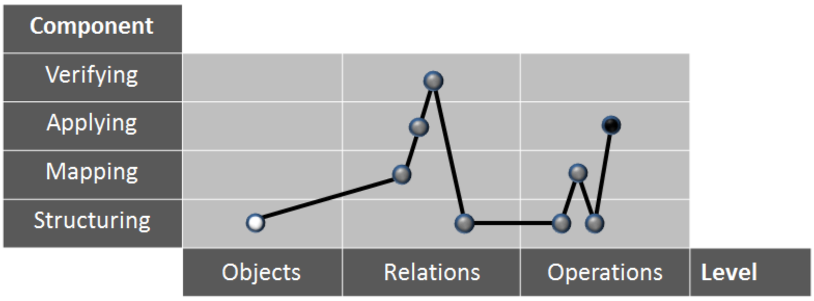

Figure. 1: Two Dimensions of Analogical Reasoning

Fig. 1 is read in the following way: The line of argumentation starts with structuring arguments on the level of objects and ends with the application of mathematical operations transferred from the base to the target domain. The outline itself passes argumentation in different phases on different levels. The extraction of diagrams of different analogy based argumentation processes defines the starting point of this present inquiry.

RESEARCH QUESTIONS

On this basis, the following questions should be particularly clarified:

- How do specific processes of analogical reasoning appear as 'paths' in the outlined two-dimensional model and how do they look?
- Is it possible to classify these 'paths' regarding both successful and failed processes of analogical reasoning?
- Which particular importance is attached to the transitions from the structural to the operational level?

RESEARCH-DESIGN: A FOUR-PHASE METHOD

If observing and describing a process in general and in particular a learning process like analogical reasoning, it has to be ensured in advance that this process actually occurs during the period of study. Therefore, it is necessary to create a learning situation which allows a transfer performance by analogical reasoning.

As a learning process represents an activity that widely defies direct observation, additional measures have to be taken so that this process is made observable.

Aiming at a systematical description of all observations referring to a present process model, the data basis has to be defined in order to find the most fitting one. Moreover, the criteria that allow the categorization of observed data into the present model need to be found.

To sum up, the following four questions have to be clarified:

1. How can processes of analogical reasoning be initiated?
2. How can processes of analogical reasoning be made visible and observable?
3. Which data can be used for the description of analogical reasoning?
4. Which criteria are suitable and necessary to systematize data on the basis of the two-dimension model?

Results of studies on *Example Based Learning* (Atkinson, Derry, Renkl & Wortham, 2000) lead to a two-stage study design in which the subject is given different exercises out of one sequence of tasks.

During a 'learning phase' the *source* of analogical reasoning is established. The subject is offered sample tasks with solutions and some instructions during this phase. Afterwards, the subjects complete further tasks of this sequence single-handed during a 'testing phase'. Now the intended transfer performance in form of analogical reasoning shall occur.

In a first step, various sequences of tasks from different domains have been developed in accordance with research results on *Example Based Learning*. As part of a preliminary study, these sequences of tasks were tested to ensure that the intended transfer effects are really initiated and to guarantee that the students are able to verbalize their underlying thoughts.

Yet, in this inquiry on thought processes, relying only on the originated results like for instance students' documents would fall considerably short in answering the research questions formulated above. That is why the main study resorted, in compliance with results of Schoenfeld (1985) and Haastrup (1987), to the process-related, introspective method of *Pair Thinking Aloud*. This method represents a variant of the *Think Aloud* method, which was mainly developed by Ericsson and Simon (1980, 1999), as one possible method for integrating verbal data into empirical studies. The creation of a natural situation and the necessity to keep the verbalization process of thoughts as complete as possible constituted, above all, the main reason for this kind of data collection. Haastrup (1991) writes concerning the advantages of Pair Thinking Aloud:

> „(…) by using pairs, one stimulates informants to verbalize all their conscious thought processes because they need to explain and justify their hypotheses (…) to their fellow informant. Furthermore, thinking aloud in pairs seems quite natural (…); It comes close to a real life situation." (p. 85).

So, in each case two students were asked to verbalize their thoughts loudly during the completion of tasks. Their statements were recorded by video.

In order to ensure a better reconstruction of individual processes of analogical reasoning, the data can be secured by a range of additionally measures (conf.

Haastrup, 1987; Borromeo Ferri, 2004). In the present study the following two dispositions were made:

On the one hand, the phase of task completion was divided into a phase of partner work and one of expert work. During the phase of partner work two students worked together on two tasks related to the domain they received instructions to. During the phase of expert work the processes of analogical reasoning were made more individual since two students who did instructions and partner work on different domains worked together. Now it was their job to complete tasks on each of both domains. It was expected that the student who was familiar with the current content would assume a leadership position. In fact, it appeared that in nearly any case, even during the phase of partner work, one of the students takes over a leading role. It is assumed, that in this case the process of analogical reasoning observed during the study is close to the thought process of the leading student.

On the other hand, after the completion of each task one of the students, if possible the "leading" student, was asked to track back the line of thought once again in a *Teach Back Phase* (Wallach/Wolf, 2001, p. 25; Vora/Helander, 2005, p. 375).

In conclusion, the study consisted of four phases (conf. Figure 2):

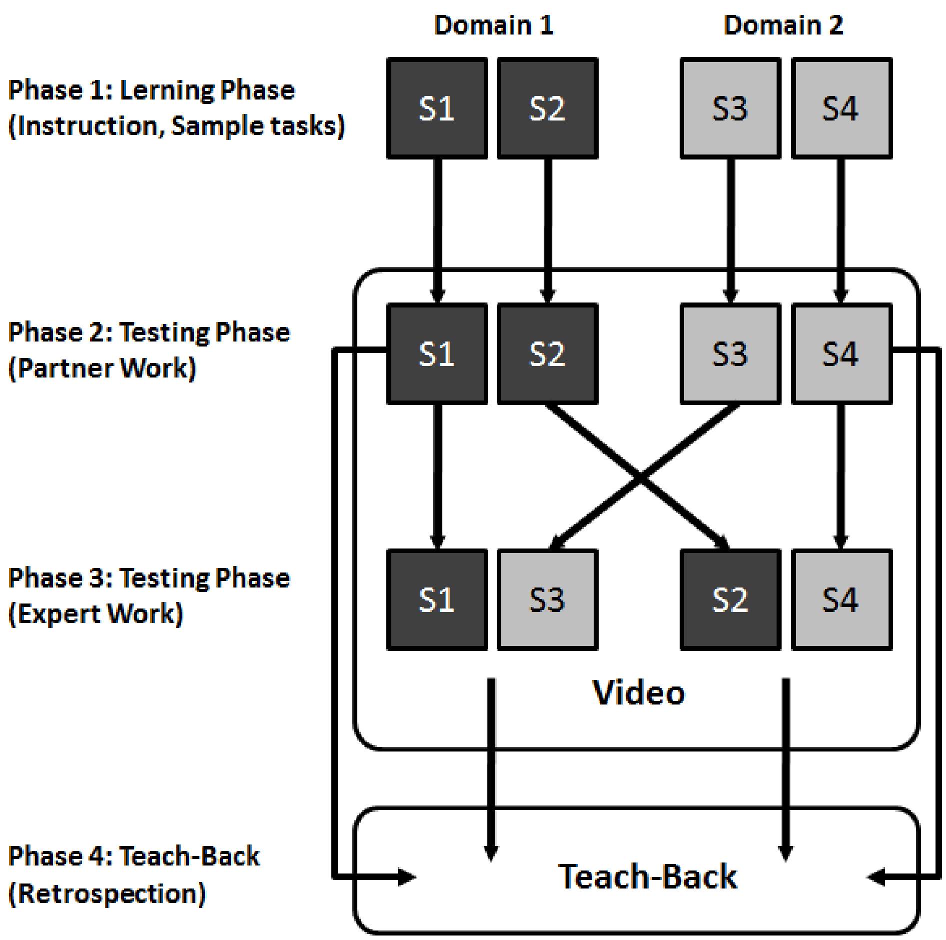

Figure 2: Four-Phase-Design of the Study

DATA ANALYSIS

With regard to analysis the following data are available:

- transcripts of students‘ dialogues and of the *Teach Back Protocols*
- video material of students‘ solutions (in particular: analysis of gestures)
- students‘ documents (solutions of tasks in written form)

On the basis of an encoding outline which was developed within the framework of another preliminary study, all verbal statements during the phases 2 and 3 of the inquiry were assigned to the different areas of the two-dimensional model (Fig.1). Therefore, different attributes of the students’ statements were outlined to characterize each area. Additionally, anchoring examples, which serve as typical examples of statements and help to locate statements to an area of the two-dimensional model, were identified. Teach Back Protocols, graphic material and students’ documents were used to bridge gaps in the verbalization processes as well as to substantiate the allocation to areas of the two-dimension model (for example by gestures like pointing at a distinctive part of an example task).

An example

The following dialogue is recorded during the *Testing phase*. Two students work on the problem of finding the number of different loop trails passing two villages A and B depending on the number of connecting trails between A and B. In the *Learning Phase* worked out examples of *complete graph problems* were presented.

1	S1:	A and B are the vertices.
2	S2:	Yes. And the trails are the edges.
3	[S2 draws a figure with two vertices and two edges]	
4	S2:	Loop trail means that you walk in a circle and that you don’t walk back on the same trail.
5	[S1 marks two trails between A and B to highlight a loop trail]	
6	S1:	Rather like this.
7	S2:	Exactly.
8	S1:	That means a loop trail needs two connecting trails.
9	S2:	Yes.
10	[S1 writes]	
11	S2:	Now we have to calculate the number of days depending on n, isn’t it?
12	S2:	‘Cause we don’t know how many trails, it could be infinitely many, so to speak.
13	S1:	Mhm.
14	S2:	And since you want to walk one loop trail every day …
15	S1:	… you need two different connecting edges a day.
16	S2:	Exactly.

17 [S1 writes, S2 takes the worked examples and points a finger on example 2]

18 S2: That means you have to, hm, …, here in the second (points on the worked example) … you mustn't divide by two.

19 S2: Because you have to count pairs, hm?

Both students begin their argumentation (l. 1-2) by applying an analogy on object level. Commonalities in the relations between the objects are used to visualize the relational situation (l. 3, mapping on relation level) and to argue about the mapped relations in the target domain (l. 4-16, applying on relation level). The argumentation by analogical reasoning in this time segment is confirmed by the use of the terms "connecting edges" instead of "trails". For the transition to the operation level the worked examples are explicitly used (l. 17, mapping on operation level; l. 18, applying on operation level). Finally, the assumption is evaluated (l. 19, verifying on operation level).

The diagram of this short dialogue in the two dimensional model is shown in Fig. 3:

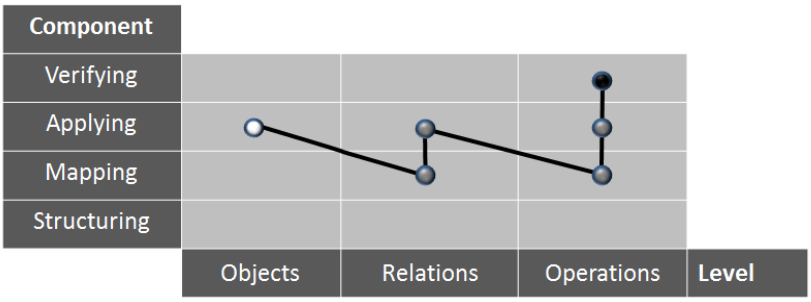

Fig. 3: Example of a "way of analogical reasoning" in the 2D-Model

For the coding of the data the software Videograph was used. The user interface of Videograph is partitioned into different fields which can be handled simultaneously: one window shows the video recording, a second window allows the transcription of the recorded dialogue and a third window shows a timeline on which time segments can be determined and assigned to predefined categories with respect to the coding scheme. One result of the work with Videograph was the graphic presentation of phases and levels of analogical reasoning on the timeline. In Fig. 4 the levels of analogical reasoning define the main categories and the phases appear as subcategories.

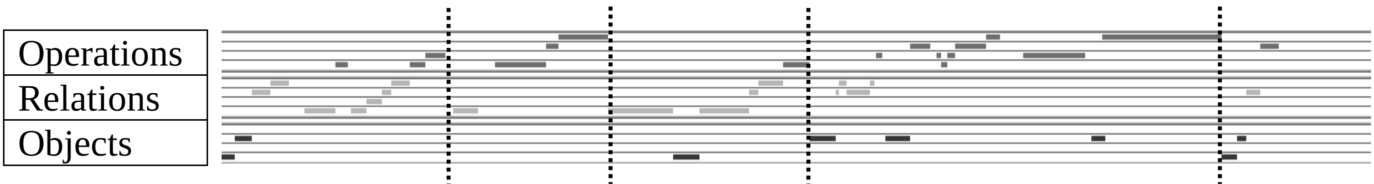

Figure 4: Ways of analogical Reasoning in form of a graphic representation created by the software Videograph

These diagrams could be already used for a first interpretation of analogical reasoning processes. For example, it could be noticed that there are several discontinuities in the graphic presentation. Drawing the attention back on the verbalizations it could be shown, that these discontinuities often coincided with an abandonment of one line of thought (dotted lines in Figure 4). With respect to this observation the processes of analogical reasoning could be split into sections by the use of the verbal data.

Moreover, the ‘paths’ in the two-dimension model represented (like in Fig. 1) a basis for further interpretation.

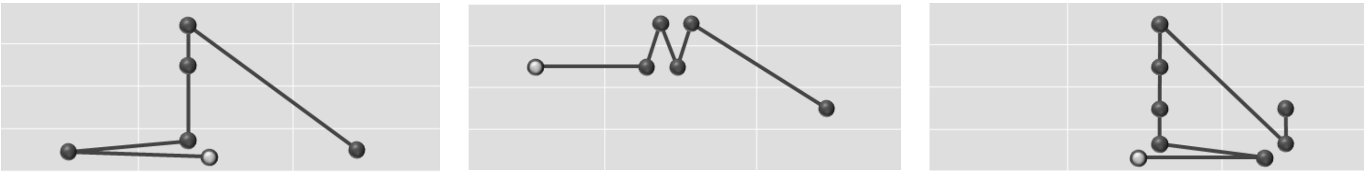

Figure 5: Paths of analogical Reasoning in the two-dimension Model

Based on these diagrams, one can try to find a classification of ‘similar paths’ during the investigation.

In order to have quantitatively substantiated statements, the ways of the two-dimension model are translated into a ‘Stopover Matrix’, too (conf. Figure 6). By means of a cluster analysis the data should help to establish a classification of paths.

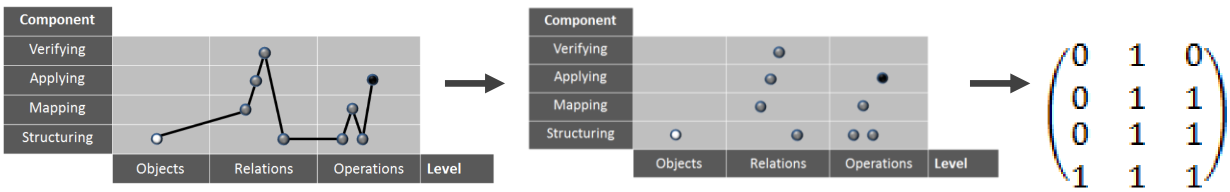

**Figure 6: Paths of analogical Reasoning (left)
and the corresponding ‘Stopover Matrix’ (right)**

FIRST RESULTS

The interpretation of the graphical representations taken from the software Videograph in combination with the students’ dialogues provides these findings:

- In a process of analogical reasoning which does not reach the target immediately, several sections (in thinking) can be identified.
- A new section in thinking always starts in the diagram at a lower level than the ending point of the previous section.
- Mainly, the sections in thinking show an upward trend when taking them separately.

Furthermore, the interpretation of the graphical representations on the basis of the two-dimension model shows the following:

- The recorded ‘paths’ run, generally speaking, from ‘left to right’ and from ‘top to bottom’.
- A new section in thinking basically starts at the object level and/or at the stage of mapping.

These findings enable to neglect the missing time information in the Stopover Matrices. They can be compared to each other with regard to an appropriate measure of distance. In this respect, the aim is to group ‘similar ways’ in a cluster analysis. Regarding first interpretations by the use of this method, the question arises whether ways which share sections at one level or within one stage of the process of analogical reasoning especially fall together in one category. This can be one starting point for the classification of ways of analogical reasoning. It appears that at least two path-types of analogical reasoning can be identified: One type of analogical reasoning prefers structuring on all levels before connecting the source and the basis domains. The other type prefers analogical reasoning on the object level before reasoning on relational or operational level.

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