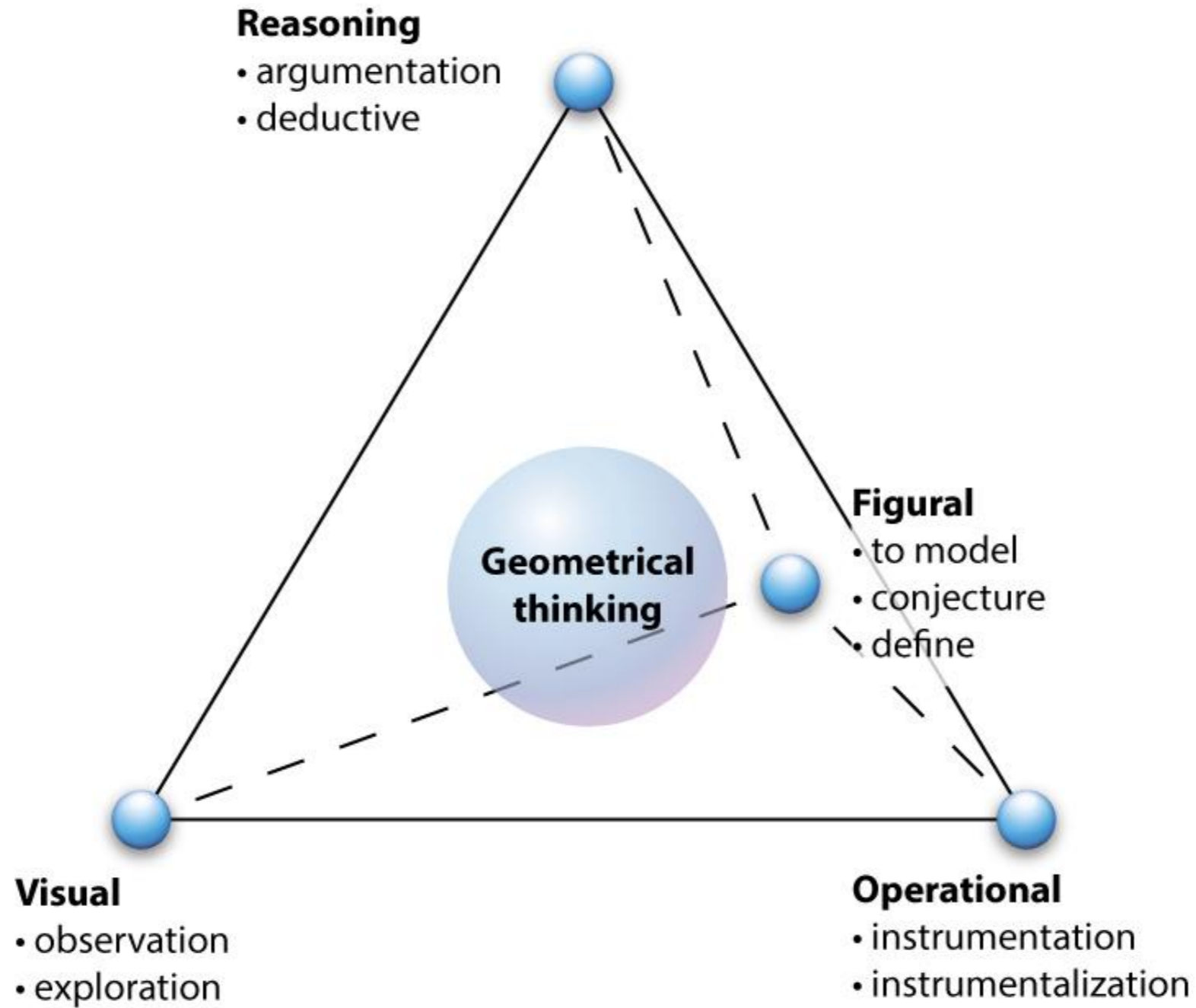# Working Group 4

Geometrical thinking

Leaders

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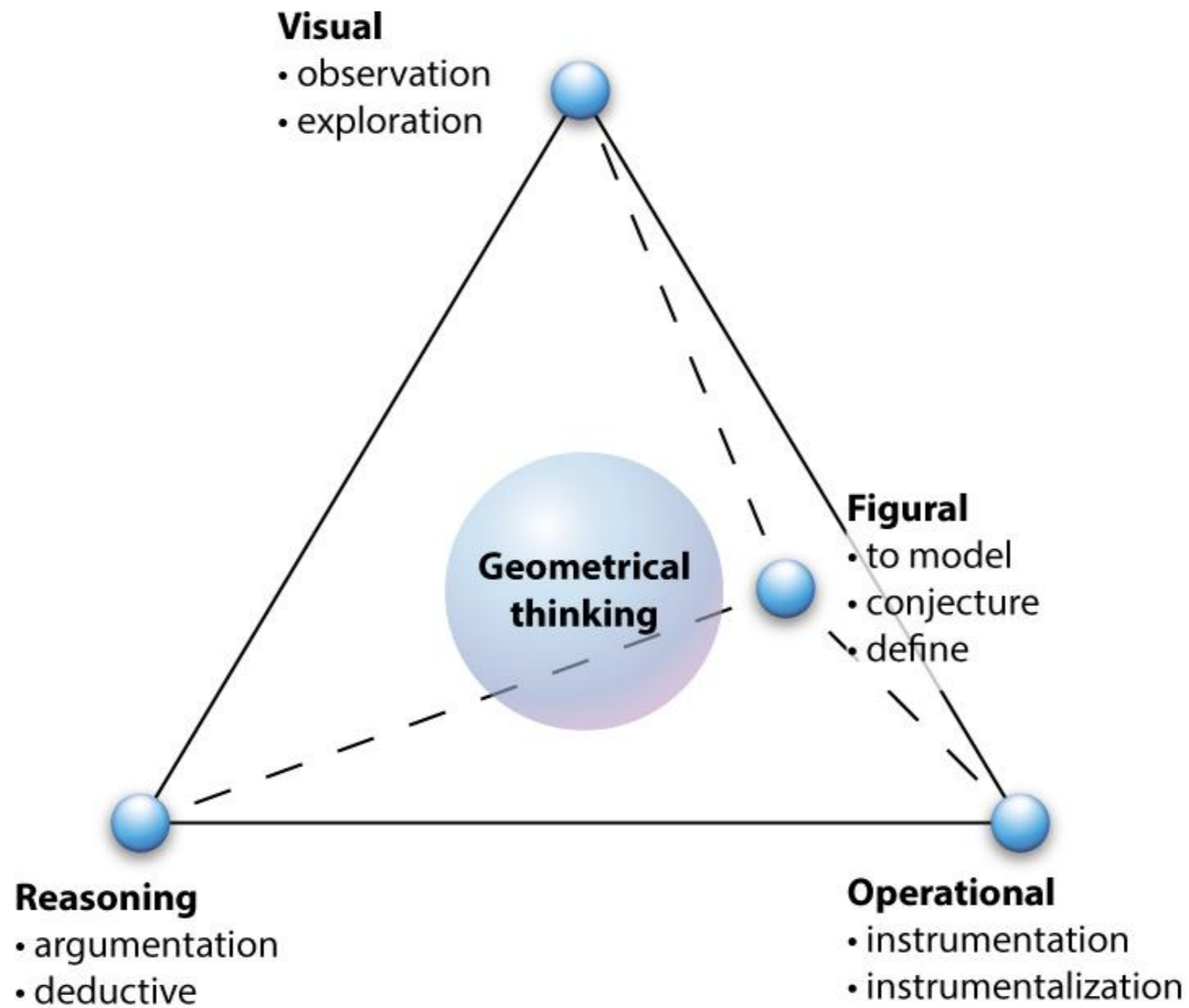
WG4 sessions

Overview of our work

Categories of the WG4 :

1. Descriptions of the competencies we worked about

2. Type of research questions

3. What and how theoretical approaches are used to answer those problems

4. Ongoing questions

5. How the other competencies are involved / link between the questions

Visual pole

Visual
- observation
- exploration

Figural
- to model
- conjecture
- define

Geometrical thinking

Reasoning
- argumentation
- deductive

Operational
- instrumentation
- instrumentalization

1. Description of the competencies we worked about

- Role of visualization in relation to geometry (as the entieties of geometry are not accesable materially)

- visualization as the form of recoding and reporting information (a type of language)

- Habits of seeing vs. field of experience.
- Observation with making-sense entails some action – children start with this 'observation', but learning must involve more than this (and perhaps observation already also involves more than this ….).
- Dynamic visualizations are crucial in engendering strong visualizing skills.

2. Type of research questions or problems

- How to develop student capacity to become inventive visualizers

- Relation of visualization with other ostensive modes of expression

- Approach to space sense by movements (how do students solve the problems that entail topological approach to space)

- Relation beween perception and definition of geometrical concepts

3. What and how theoretical approaches are used to answer those problems

- Theoretical approaches need to do both
  - (1) use constructs or competencies (like the visual and the deductive) to go between the different habits/skills of students and
  - (2) (which we need more of) interrogate the categories/competencies/constructs of deduction etc.

- There is a tension between top-down theories that impose structure and constructs on that which they study, and those studies that look for emergent constructs in the activity.

4. Ongoing questions

To what extent do children take up the pictorial as a resemblance or as a symbolic rendering?

The visual and other modalities are fused rather than translated between – how can we best study this fusion?

What kinds of theories of perception are we assuming? (How physiological/psychological/phenomenological/sociological are we in our approach?)

Operational pole
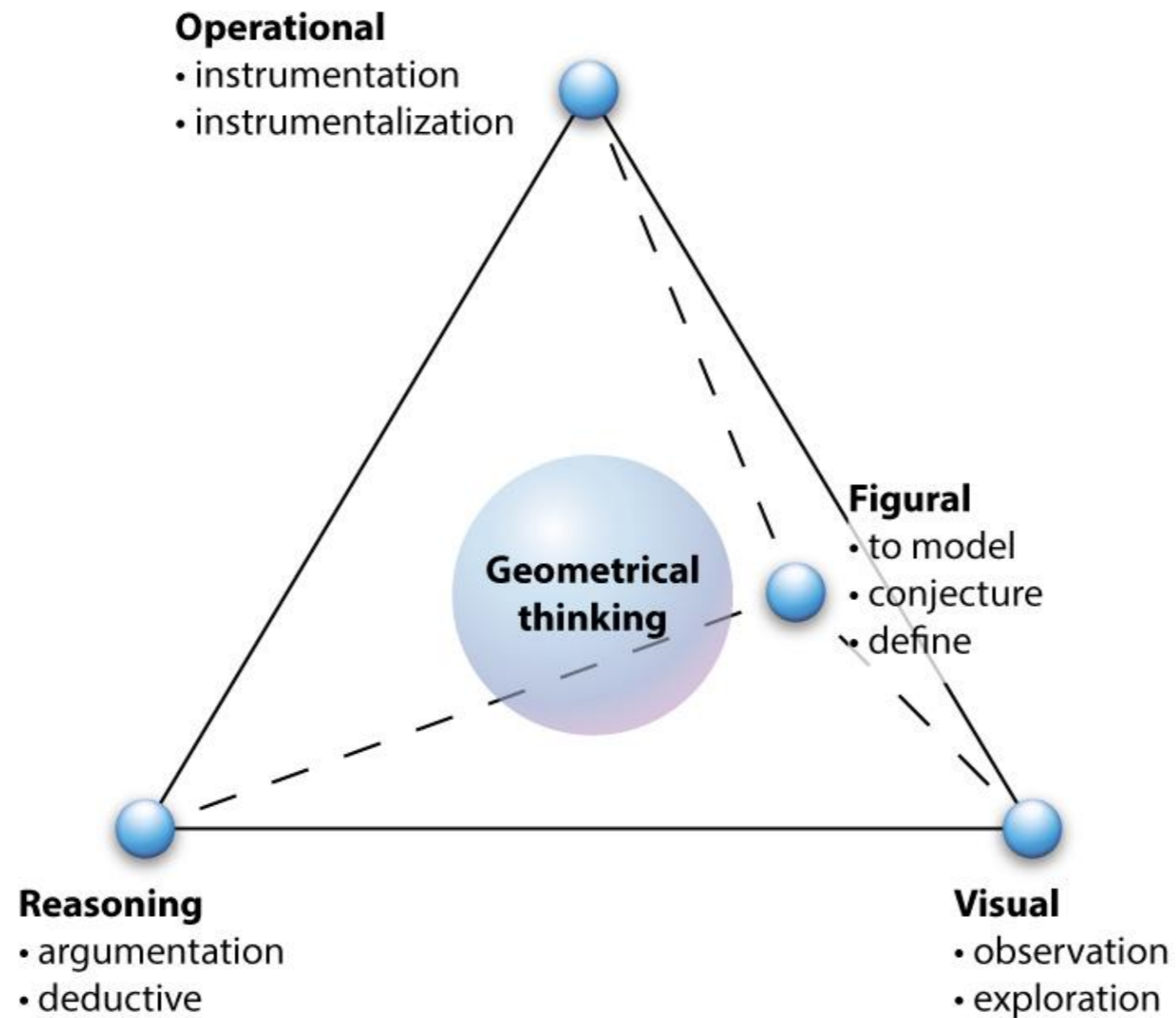
1. Description of the competencies we worked about

to re-organize what is already present cognitively
- to create a new reality through images of changes
- to implement, 'manipulate', transform strategies to create a new "schemata" (not necessarily to achieve a goal)
- to internalize a tool to develop/to foster the conceptual aspects
- to use a tool to answer new questions/to face new tasks
- to use a tool to facilitate to reason at a higher level of abstraction
- to facilitate to link geometrical thinking with other kinds of thinking

2. Type of research questions or problems

Gestures and visualisation

1. In what ways do gestural-haptic modalities factor into students' spatial reasoning as they engage in problems through topological rather than Euclidean concepts?
2. The role of gestures and manipulation in solving geometrical problems
3. Understanding how the use of technology may help to face some of the problems that are related to seeing and the study of spatial geometry.

Transfer of learning and use of tools

3. What and how theoretical approaches are used to answer those problems

1. Reseaches about the role of gestures in problem solving

2. Van Hiele theory

3. Everyday thinking and mathematics thinking (and physicist-geometer's paradigm)

4. Knowing vs seeing

5. Figural concepts

6. Instrumental approach

7. Transfer of learning
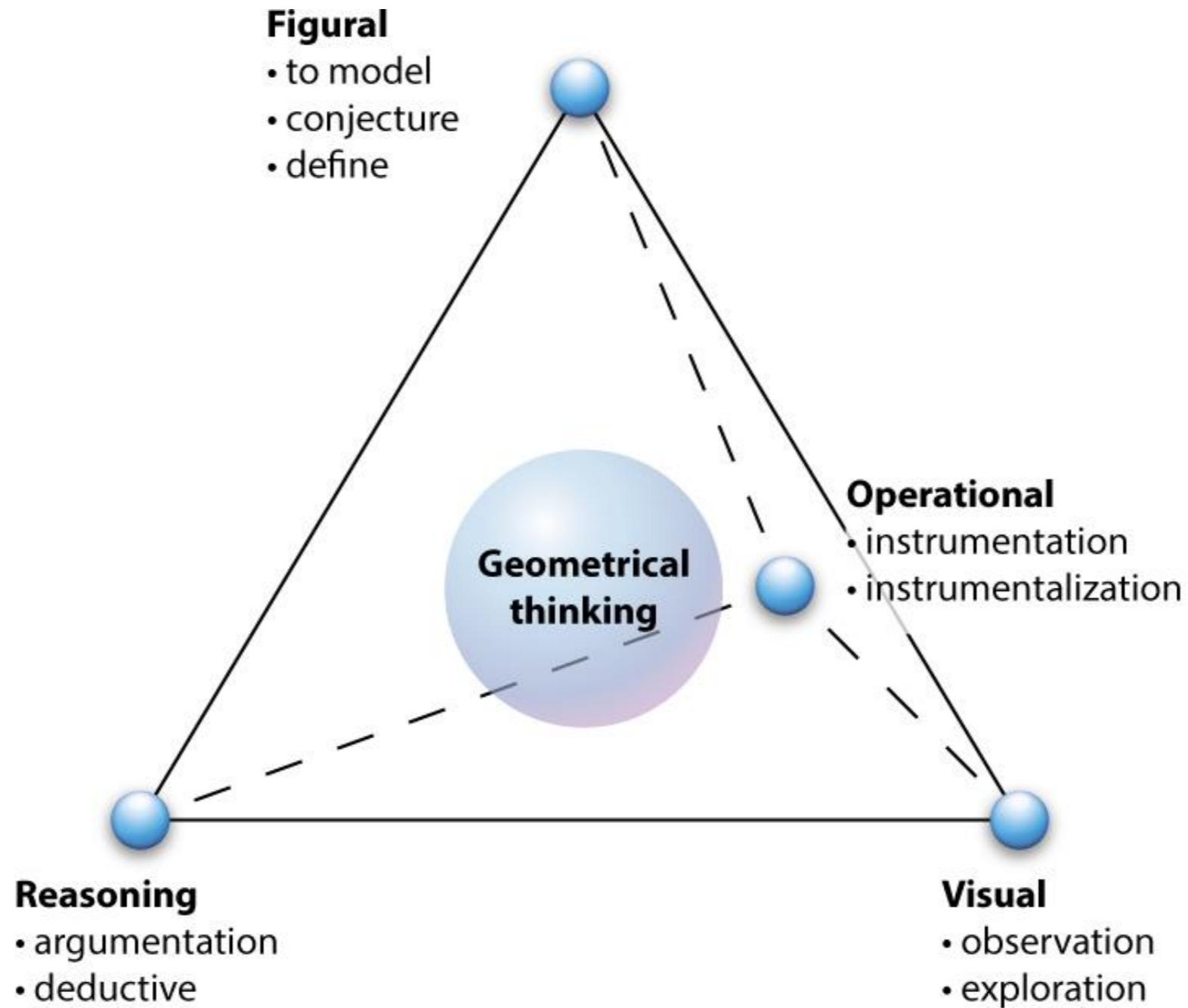
4. Ongoing questions

Uses of tools

1. Does the set square inhibit or delay the development of the interaction between students' figural and conceptual aspects of axial reflection by providing too much assistance? Is the set square a stumbling block for the internalisation of axial reflection?
2. Which kinds of perception do preservice teacher have with respect to the use of tools for geometrical construction ?

Relationship with proof

1. Why proving something that can be verified simply by measuring?

Figural pole

Figural
- to model
- conjecture
- define

Operational
- instrumentation
- instrumentalization

Geometrical thinking

Reasoning
- argumentation
- deductive

Visual
- observation
- exploration

1. Description of the competencies we worked about

Interpretation of the drawing
     a representation, with conceptions about the object
     recognizing its genericity, the status of the geometrical object,
     linking it to a theory, to properties, definitions, theorems, etc.

Linking experimentation on drawings (with or without tools) to proof : exploration on figures, determination of what can be said about it, motivation to proof.

Construction and definition of the geometrical objects. Relationship with 'the real world', geometry as a model, figures as ideal objects.

     Two different perspectives to tackle this competency : the students and the teacher.

2. Type of research questions or problems

•The role of the figural aspects in a given task, at a given level, to determine its type, the kind of geometrical work involved (van Hiele, G_i, etc.)

•General research questions for each of the links between other competencies

•.How tools influence the role of the figural pole

•Measure and approximation: status, role, etc.

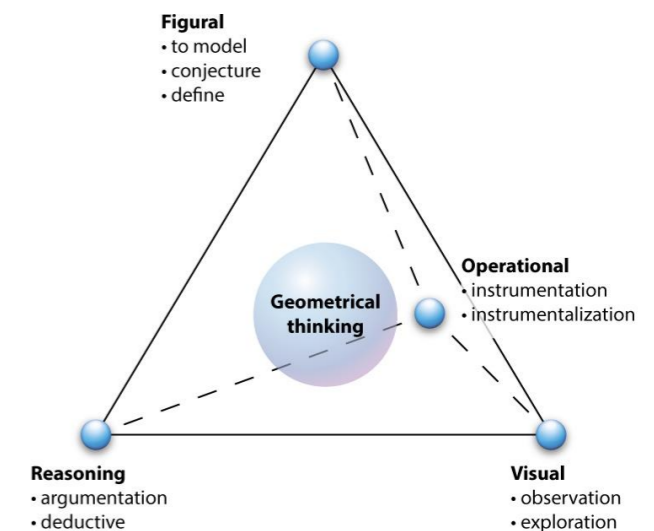# 5. How the other competencies are involved / link between the questions

Two-folded relation between reasonig and the 'face' operational-figural-visual
When the place alloted to the figural grows while working on o-f-r, how is the reasoning pole affected
But deductive activity may have consequences on
•the way students conceive the generecity of figures,
•the design of operations on drawings, on the anticipation capacities of children
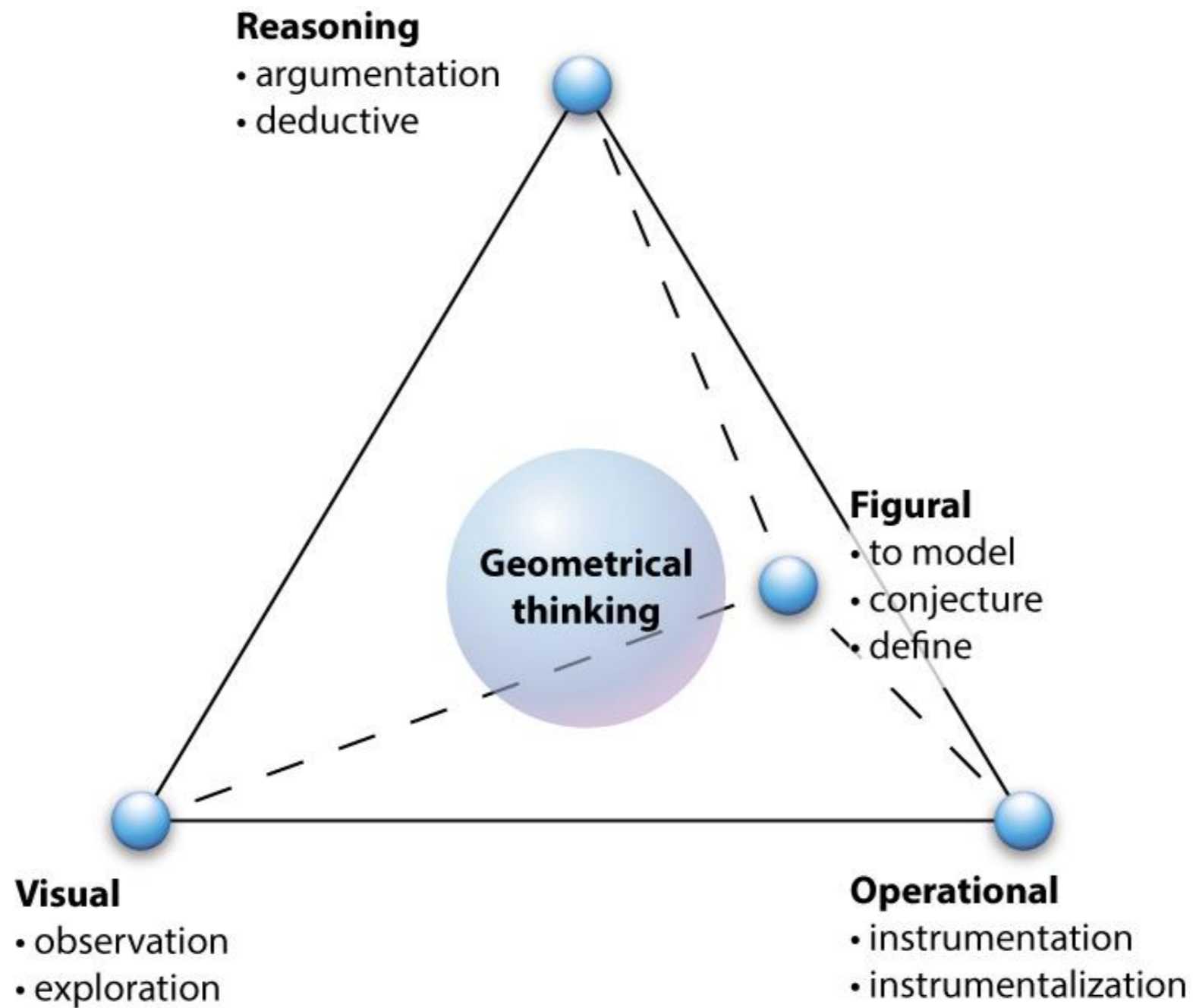
1. Description of the competencies we worked about

The **reasoning competency** is a window in geometrical thinking :

The other competencies are clearly involves in all of the studies, maybe the operational one is not so evident with traditional activities

The classical semiotical and cognitive theories participated at the definition of this window (Duval, Godino, etc.)

A stand point develop in the group is the «reasoning in action» :

Utility of the knowledge model from Balacheff & Margolinas

Discovery and proof processes with origami

To solve unfolding problem with physical and mental images

2. Type of research questions or problems

- Find the space of which conceptions are made possible to develop geometrical thinking from textbooks

  - The use of these spaces to compare textbooks from two different cultures (ex. English and Japanese)

- Manipulations impact on the generating of logically correct deductive arguments

  - Comparing teaching with and without origami

- How the analytical and the visual thinking are cooperating (possible different objectives) and / or collaborating (same objective) in solving problem with unfolding

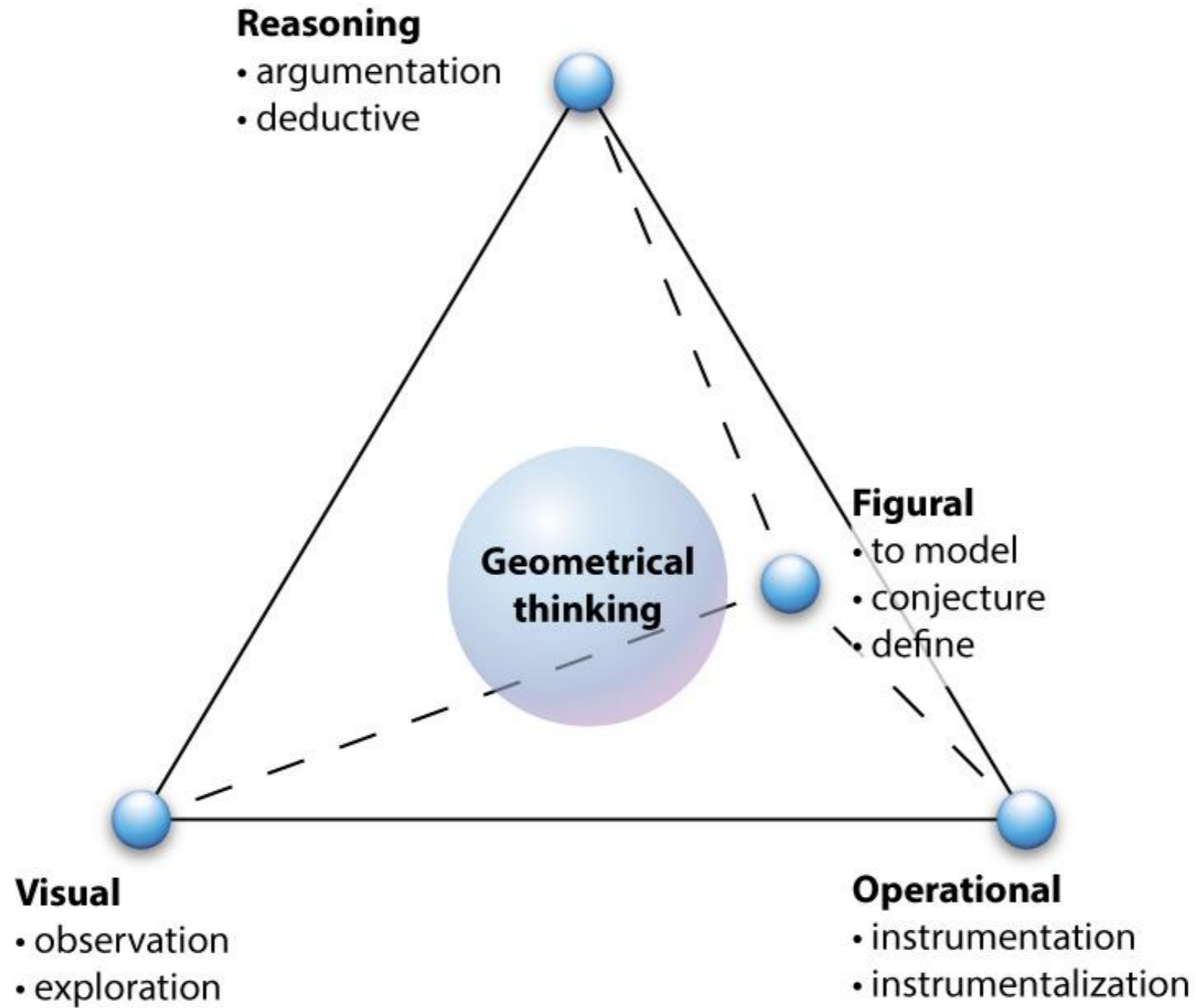**3.** What and how theoretical approaches are used to answer those problems

- Using a knowledge model to compare conceptions with textbooks

- Comparing two teaching styles and using a pretest/postest tool to see the evolution

- Using a semiotical theory to analyse the synergy between two kind of language

4. Ongoing questions

- To complete the a priori studies from real experiments with pupils, student and teacher

- To transfer the acquired competencies with / in other environments (paper-pencil, technological, informatics, ...)

- To orchestrate different forms to teach in the development of the geometrical thinking

5. How the other competencies are involved / link between the questions

- The reasoning competency help to animate, structured and control the visual, figural and operational competencies

  - This is especially helpful in discovery, modelization and validation steps

  - From the geometrical work/working space point of view, in the interaction between the student and the milieu, this competency intervene in the :

    - visuo-figural genesis (visualisation-figuration)

    - instrumental genesis (instrumentation-intrumentalization)

    - discursive-graphic genesis (devolution-institutionalisation)

WG4 thank you