

WG3: Algebraic Thinking

CERME 8

WG3

- Group Leaders:
 - Jeremy Hodgen (UK)
 - María C. Cañadas (Spain)
 - Therese Dooley (Ireland)
 - Reinhard Oldenburg (Germany)
- 18 papers / 5 posters / 25 participants
- 14 countries:
 - Canada, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Tunisia, Turkey, UK, USA / Romania

Questions for CERME8

- What are the similarities and differences in the ways in which algebraic thinking is understood in different (national / cultural / ...) contexts and in different theoretical / methodological traditions?
- What are the strengths and weaknesses of different theories / methodologies for researching algebraic thinking?
- What are the implications of the research presented at WG3 for ...
 - future research on algebraic thinking ...
 - Practice in the teaching and learning of algebraic thinking?

Process

- Presentations:
 - Groups of 3 + discussant
 - Posters
 - Short discussions
- Session on improving papers in 3s:
 - Rigour & 'quality'
 - Use of pictures and diagrams
 - Language to communicate

- Entry to algebra
 - Sandra **Gerhard**
 - Previous knowledge of arithmetic & symbolisation
 - Roberto **Tortora**
 - $10^{38}+10^{37}$, sense-making
 - Celia **Mestre**
 - Quasi-variables & the beginning of symbolisation
 - Joana **Mata-Pereira**
 - Generalization: Examples and non-examples
- Equivalence
 - Larissa **Zwetschler**
 - Equivalence
 - Joaquin **Gimenez**
 - Understandings of '=' / equal sign
 - Julia **Pilet**
 - Reference to a mathematical object

- Structural generalisations
 - Heidi **Måsøval**
 - $n^2 + (n-1)^2$, milieu constraints & affordances
 - Therese **Dooley**
 - quadratic strategies & mediation
 - Valentina **Postelnicu**
 - Cartesian connection
 - Tobias **Rolfes**
 - **Covariation**
- Syntactic / Semantic
 - Rahim **Kouki**
 - Limits of syntactic method
 - Reinhard **Oldenburg**
 - Relationship between syntactic and semantic understanding
 - Alexander **Meyer**
 - Contextualised → formalised reasoning

- Teachers and teaching
 - Cecilia **Kilhamn**
 - Differences in teaching variable / expressions
 - Kubra **Çelikdemir**
 - Re-analysis of TIMSS & Teachers' perceptions
 - Sevgi **Sari**
 - Metacognition & Conceptual / Procedural Knowledge
 - Unni **Wathne**
 - Videomat: Different approaches to algebra
 - Ann-Sofi **Røj-Lindberg**
 - Videomat: Methodology

- New directions
 - Jean-Baptiste **Lagrange**
 - Limitations of an *exclusively* functional approach
 - Jerome **Proulx**
 - What does it mean to do algebra mentally?
 - Peter **Kop**
 - Experts' conceptions of functions

A mature discipline

- Revisiting existing research
- What is “new”?
- Developed theories / Entrenched positions
- Young researchers / less developed communities need to re-walk the path / “make it their own”
- Theories – complementary / commensurate?
 - Enactivist ... social ... cognitivist ...

Continuing debates

- Early algebra
- Algebra v algebraic thinking
- What is algebra?

Key issues ...

- Recontextualising / extending “existing” research
- What is algebra / algebraic thinking ...
 - Multiple representations
 - In the doing
 - Structure
 - Transformational / Generational / Meta
- Context & the “motivation” for algebraic thinking
- “Translating” research to practice
 - Digital technology: Why so few papers?
 - Dangers of an exclusive approach (functional v symbolic)

Some issues from the papers ...

- The meaning of algebra
 - Reference (to *one* object)
 - Multiple representations
 - Example space
 - $\equiv, = \dots 3 \times 4, 4 \times 3 \dots x^2 + x, x(x-1) \dots$
 - Framed around “meaning-making”
- Arithmetic \leftrightarrow Algebra
 - Equivalence / equals sign
 - Symbolisation: Do we need to start from number?
- Dualities?
 - Procedural / conceptual understanding
 - Syntactical / semantic

Focus on algebra!

- Variable / unknown / parameter:
 - Advanced as well as early algebra
- Paradigmatic tasks:
 - How effective are they?
- Translating research to the classroom:
 - Bridge the tension between rigorous research & classroom practice
- Understanding student difficulties and misconceptions:
 - What is algebra
- Algebra as an activity
- Research in domain of algebraic thinking is dependent on context & culture BUT to what extent?